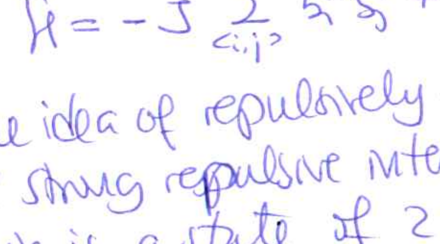
* Non-Equilibrium Dynamics

* We will finish these lectures with a necessarily brief comment on non-equilibrium dynamics, which constitute one of the most active research topics in the area of ultra-cold gases nowadays.
* The unique degree of tunability, isolation, and long coherence times in ultra-cold gases experiments have enabled the "clean" exploration of phenomena previously unaccessible in condensed-matter physics (since the latter are characterized by a strong coupling to the environment and rapid decoherence).
* A first example of a phenomenon absent in systems in thermal equilibrium is that of repulsively-bound pairs, first studied experimentally by the Denschlag/Grimm group in Innsbruck in 2006 [Winkler et al., Nature 441, 853 (2006)].

Let's consider a Bose gas in a lattice, described by the Bose-Hubbard Hamiltonian

$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{b}_i^+ \hat{b}_j + \frac{U}{2} \sum_j \hat{n}_j (\hat{n}_j - 1)$$

The idea of repulsively-bound pair is best understood in the regime of strong repulsive interactions: $U \gg J$, where the repulsively bound pair is a state of 2 atoms at the same site: $|2\rangle_j \equiv \frac{(\hat{b}_j^+)^2}{\sqrt{2}} |VAC\rangle$. This state has an energy offset U with respect to states where the atoms are separated (see the figure).



* The key point is that the pair is unstable to decay by converting the interaction energy into kinetic energy, because the Bloch band allows a maximal kinetic energy for 2 atoms given by $8J$ (i.e. twice the band width). Hence if U surpasses that, then the pair cannot break without violating energy conservation. Since we assume that energy is conserved, hence the pair cannot break!

* Note that contrary to the case $U<0$ where pairs can form in equilibrium, here (for $U>0$) pairs are energetically expensive. However, here they are maintained (hence in a non-equilibrium state) due to the absence of energy dissipation.

* Note also that pairs can move through second-order processes:

[ $\sim J^2/U$]

but the atoms cannot move independently.

[Note also that singly-occupied sites (singlons) may move through doublons (we will call so the doubly-occupied sites), by a simple swap]

* In the Innsbruck experiments the initial state is prepared from a pure sample of Rb_2 Feshbach molecules (each lattice site is occupied by no more than a single molecule). Sweeping across the Feshbach resonance they obtain a deep lattice (35 E_{rec}) with 2×10^4 dimers (filling factor $\sim 1/3$). The individual Rb atoms far from the resonance interact repulsively ($a \simeq 100 a_B$).

They then reduce the lattice depth in one direction (to 10 E_{rec}) which allows, in principle, for a separation of the pairs. However $J \sim 63$ Hz whereas $U \sim 2.2$ kHz, and hence $U \gg J$. So one expects to be in the repulsively-bound pair regime.

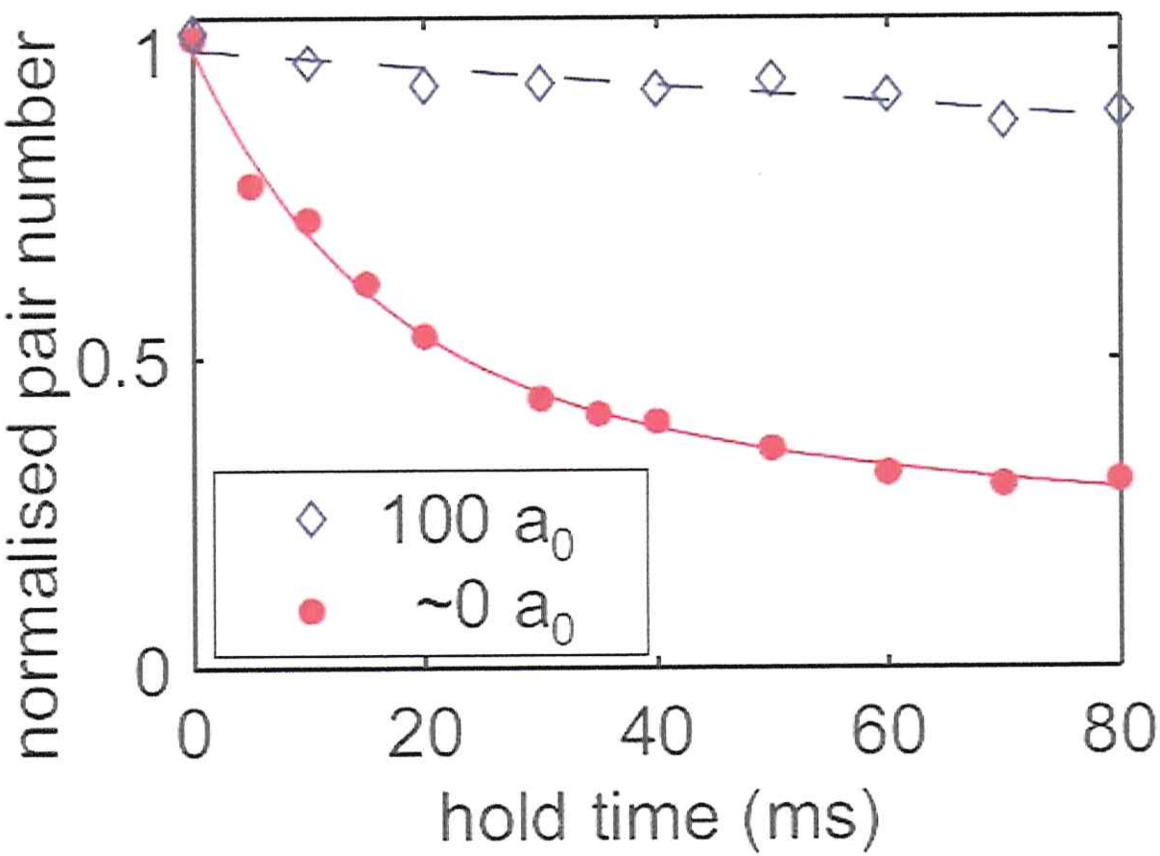
After a holding time, they adiabatically ramp-up the lattice back to 35 E_{rec}, and convert back to Feshbach molecules (with nearly 100% efficiency). A purification pulse removes those atoms which are not in Feshbach molecules (i.e. coming from dissociated pairs). Then they convert back into atoms, and measure their final number via conventional absorption imaging.

The results of the lifetime are shown in p. 155. Note that for $a = 100\, a_{Bohr}$, the lifetime ~ 700 ms (mainly limited by inelastic scattering from lattice photons), that greatly exceed the $h/4J \sim 4$ ms associated to a single tunnel event to a nearest neighboring site.

Note that for $a = 0$ a much faster decay occurs, showing that the pairs are stabilized by the on-site repulsion U.

Repulsively-bound pairs

[Winkler et al., Nature **441**, 853 (2006)]



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* Studying out-of-equilibrium dynamics can help also gain a better understanding on statistical mechanics of isolated systems. One can study for example the effect of integrability in the properties of an isolated gas out-of-equilibrium, a problem that prior to ultra-cold gases could have been deemed as academic.

This is particularly the case of one-dimensional systems. A particularly interesting experiment in this context was performed at D. Weiss' lab at Penn State University [Kinoshita et al., Nature 440, 900 (2006)]. In that experiment they studied the relaxation dynamics of a 1D Bose gas, showing that as long as the system remained 1D no relaxation occur towards the expected thermal state. Let's have a brief look to that experiment.

* They employ a strong 2D lattice to induce an array of parallel Bose gases. The dynamics within each tube is strictly 1D due to the very large transversal confinement at each site. There's in addition a weak axial confinement.

To study the 1D gases, they allow the atoms to expand in one dimension before taking an absorption image from the transverse direction. When one integrates the image transverse to the tubes one gets a 1D spatial distribution that corresponds to the momentum distribution after expansion.

To create non-equilibrium momentum distributions they pulse a 1D lattice that depletes the zero momentum state and transfer atoms to $\pm 2\hbar k$ peaks. The cloud hence splits into two (see p. 158) and the two collide with each other in the center twice per cycle.

In p. 159 one can see the expanded momentum distribution for different values of the 1D coupling strength γ.

[Note: $\gamma = \left|\frac{2}{a_{1D} n_{1D}}\right|$, where n_{1D} is the 1D density and $a_{1D} \cong \frac{l_{HO}^2}{2a}$ with l_{HO} the transverse oscillator width and a the 3D scatt. length. The larger γ the closer one is to the so-called Tonks-Girardeau regime of infinitely repulsive interactions.]

* The different curves for each case show the result after a different holding time. The key point is that for a thermalized gas one would expect a gaussian distribution. It's clear that although there's some evolution there's no thermalization here! Remarkably this was the case in spite of the fact that ~~~~ each atom had collided thousand of times (this must be compared with the ~3 collisions/particle that characterized 3D thermalization).
* The crucial point that explains this lack of thermalization observed from the Tonks-Girardeau limit to the intermediate coupling regime, is the (almost) integrable character of the 1D gas.

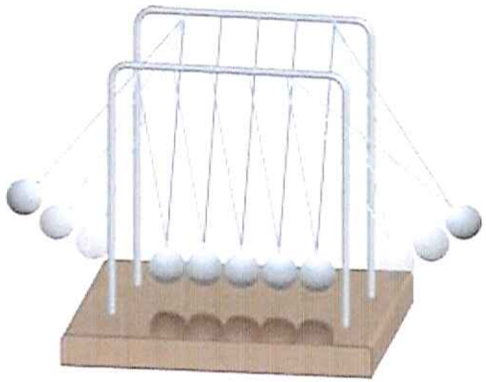
[Note: 1D gases, in the absence of external confinement, formed by δ-interacting particles formed an integrable system, as shown by Lieb and Liniger already in 1963.]

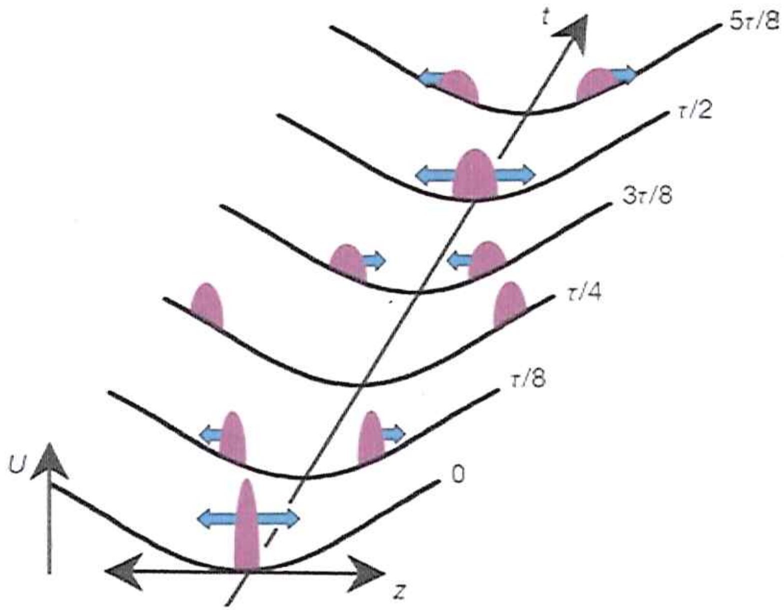
[Note II: The Tonks-Girardeau limit is integrable, using Girardeau's Bose-Fermi mapping, also for the case of an external confinement. Interestingly, in that regime, few-body observables after relaxation may be still described by a generalization of the Gibbs ensemble, the so-called generalized-Gibbs ensemble, introduced by Rigol et al in PRL 98, 050405 (2007).]

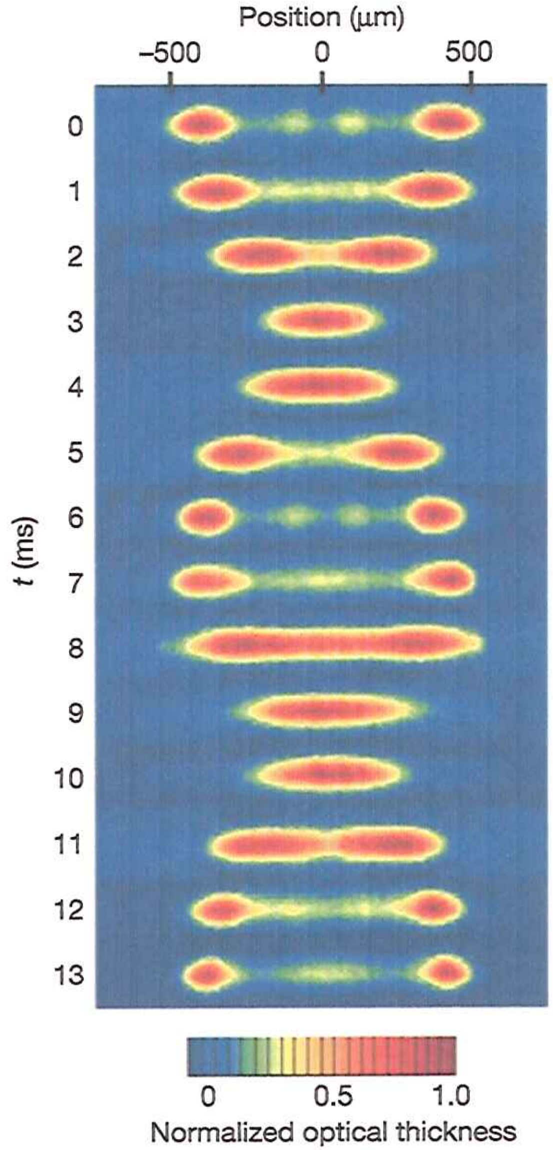
· The integrability of the gas impose severe constraints to the dynamics due to conserved quantities. This prevents thermalization ~~~~.
Departing from integrability ~~~~ may lead to thermalization as it was inferred in experiments on 2-wire gases in the group of J. Schmiedmayer (Vienna) [Hofferberth et al., Nature 449, 324 (2007)].
However even for non-integrable systems equilibration is far from guaranteed as we will discuss momentarily.

Quantum Newton's cradle

[Kinoshita et al., Nature **440**, 900 (2006)]







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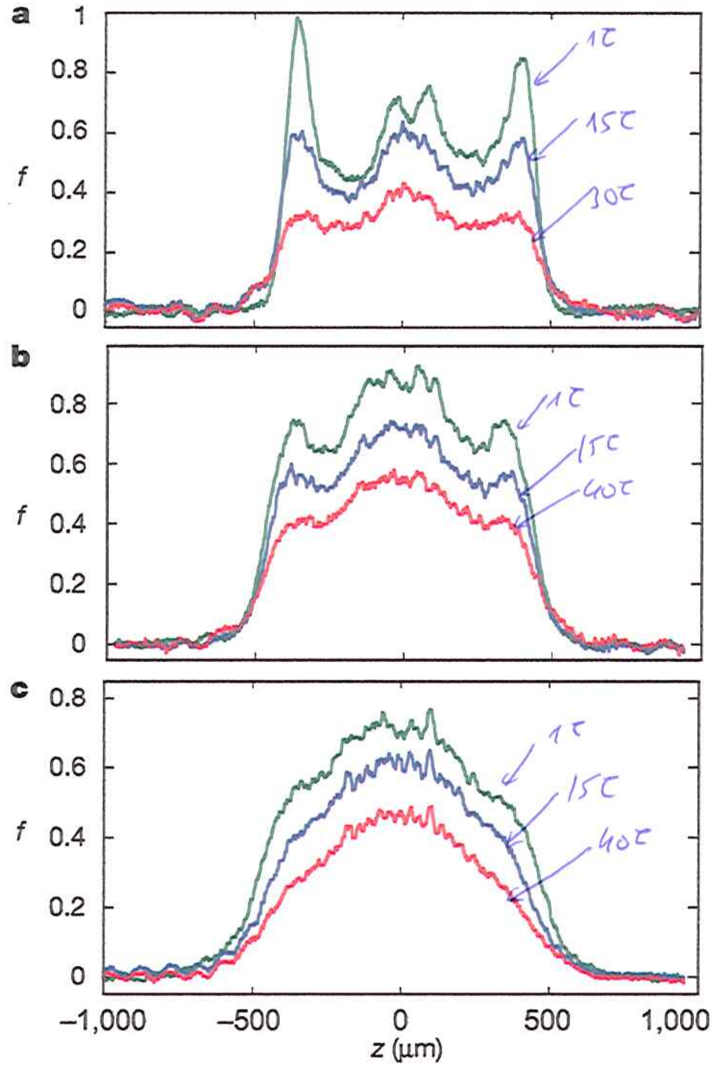
Quantum Newton's cradle

[Kinoshita et al., Nature **440**, 900 (2006)]



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* Transport vs interactions

* In p. 153 we studied the idea of repulsively-bound pairs in Bose gases. Actually very similar arguments work as well for fermions, and indeed the equivalent of the repulsively-bound pairs in Fermi gases was studied by the group of T. Esslinger in 2010 [Strohmaier et al., PRL 104, 080401 (2010)].

In those experiments they measured the doublon lifetime as a function of U/J

[Note: Doublons were created by lattice depth modulation with frequency U/J and then measured by encoding double occupancy into a previously unpopulated spin state using RF spectroscopy] (see p. 161)

They showed that the doublon lifetime scales exponentially with U/J

[Note: This exponential scaling results from higher-order scattering processes involving several single particles per doublon (similar to a multiphoton process that bridges the U gap].

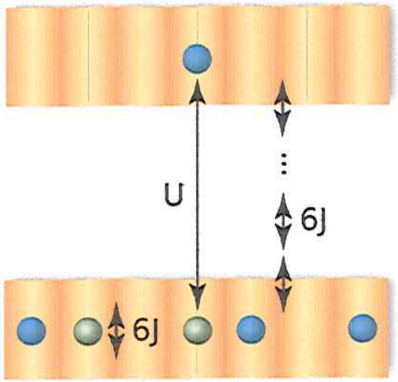
* Interestingly interactions may strongly affect the dynamics of Fermi gases out of equilibrium even for ratios U/J relatively small. This was clearly shown in a recent experiment of 2012 in I. Bloch's lab [Schneider et al., Nature Physics 8, 213 (2012)].

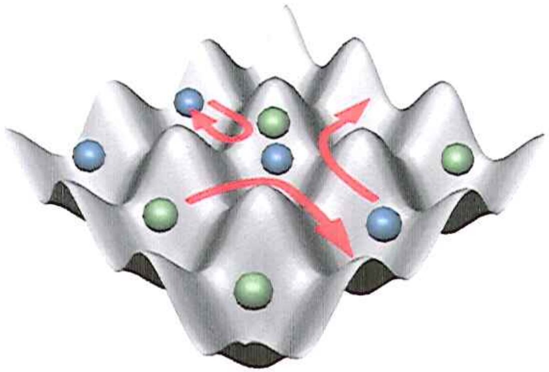
In that experiment they study transport in a homogeneous Fermi-Hubbard model by allowing an initially confined atomic cloud with variable interaction to expand freely within a homogeneous lattice. They showed that even small interactions may lead to a severe reduction of mass transport. Let's have a closer look to that experiment.

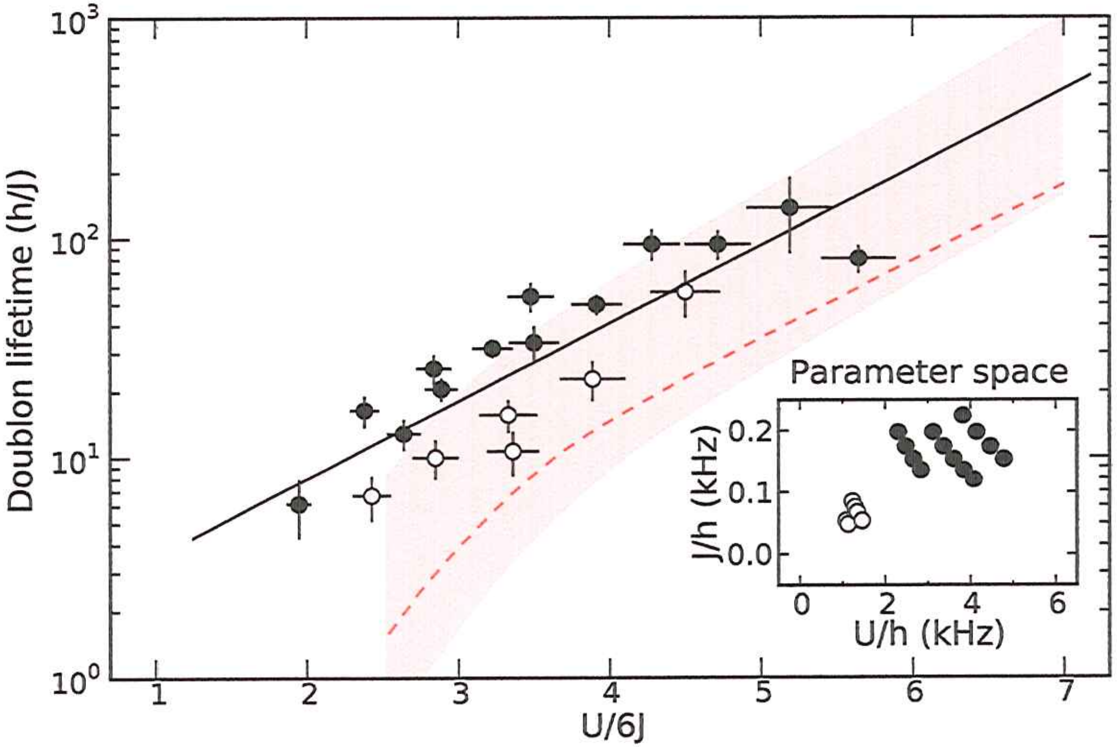
They prepare a band insulating state of fermionic K atoms in a combination of a blue-detuned 3D lattice and a red-detuned dipole trap. Subsequently the expansion is initiated by suddenly eliminating all confining potentials in the horizontal direction (see p. ( ))

They first studied the non-interacting case ($H_J = -J\sum_{\langle i,j\rangle} a_i^\dagger a_j$). This Hamiltonian gives rise to a ballistic expansion (i.e. the spatial distribution width

Repulsively-bound pairs in Fermi gases [Strohmaier et al., PRL **104**, 080401 (2010)]







increases linearly in time. Each initially localized particle expands independently with a constant quasi-momentum distribution. Since a localized single-particle state (a Wannier function) is an equal superposition of all Bloch waves in the 1st Brillouin zone, the velocity distribution inherits the square symmetry of the Brillouin zone. There's hence a transition from the rotational symmetry of the initial density distribution to a square symmetry (see p. 164)

* In the interacting case (using 2 hyperfine states, and a Feshbach resonance to modify their interactions) the expansion is qualitatively different. The dynamics gradually changes from a purely ballistic expansion in the non-interacting case into an almost bimodal expansion for interacting atoms. When $|U|$ grows, larger and larger parts of the cloud remain spherical, and only a small fraction of atoms in the tails of the cloud displays a square distribution (see p. 164).

The spherical core shape results from frequent collisions at the cloud center, which drive the system close to local thermal equilibrium. Thus the central region is characterized by a diffusive dynamics. However the cloud expansion is not given by this diffusion but instead by the physics in the tails of the cloud, where density is low and local equilibrium cannot be reached. At the wings scattering is so rare that the expansion is ballistic. Therefore the tails show the square symmetry characteristic for freely expanding particles.

* Using phase-contrast imaging they determined the HWHM of the central core $R_c(t)$ which may be fitted as:

$$R_c(t) = \sqrt{R_{c0}^2 + v_c^2 t^2}$$

[Note: the initial "ballistic fraction" decreases with U. Note however that during the expansion the density reduces and hence, for infinite expansion times, all atoms are expected to become ballistic]

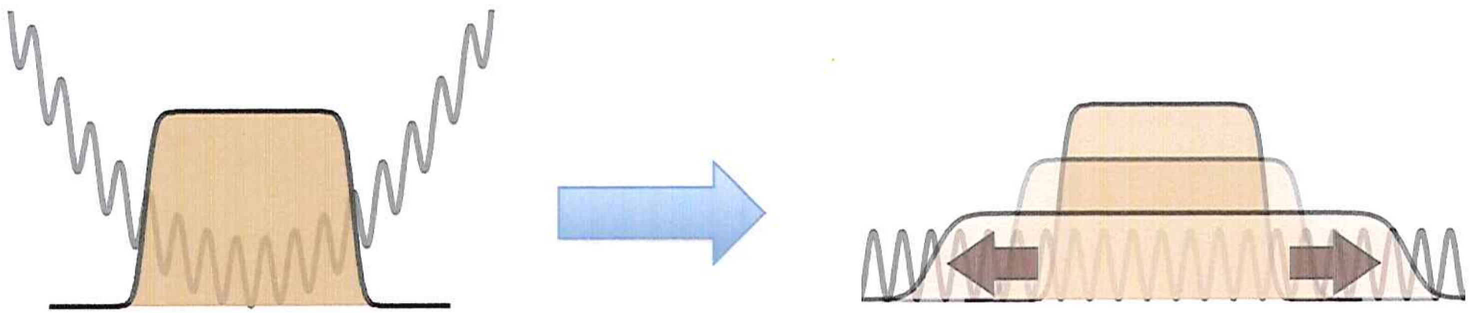
In this way the extract the ~~core~~ core expansion velocity v_c. Surprisingly v_c decreases dramatically already for interactions much smaller than the band width $8J$, which highlights the strong impact of moderate interactions in these systems. (see p. 165)
For $|U/J| \gtrsim 3$ the core shrinks instead of expanding! The expansion of the diffusive core is strongly suppressed and the frozen core dissolves by emitting ballistic particles.
[Note: for large $|U|$, one forms basically repulsively-bound pairs at the core, which are basically immobile]
[Note II: The separation between ballistically expanding atoms carrying high entropy and the high density core in the center could be used to locally cool the atoms, via the so-called quantum distillation. This effect is based on the swap $21 \to 12$, which allows singlons to scape the core].

• Note finally that there's an identical evolution for $U<0$ and $U>0$. This may be easily understood as follows.
For $U<0$, the total energy is negative and low momentum states became more populated during expansion.
For $U>0$, the total energy is positive, and hence one has an enhanced occupation of higher momentum states (in the vicinity of $(\pi,\pi,\pi)/d$)
For the Hubbard model the group velocity $v(\vec{q}) = \frac{1}{\hbar}\frac{dE}{dq}$ for $\vec{q}$ and $(\pi,\pi,\pi)/d - \vec{q}$ are the same ($v_i \sim \sin q_i d$) leading hence to the same expansion for U and $-U$. (see p. 165).

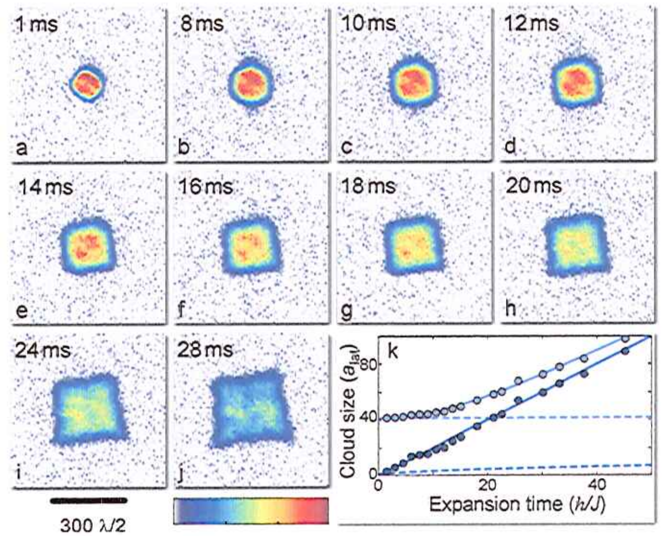
* As an important corollary of this experiment, note that the surprisingly large observed timescales of mass transport set lower limits on the timescales need both to adiabatically load the atoms in the lattice and to cool the system in the lattice. This is extremelly important for all attempts to create complex strongly-correlated many body states in these systems, as e.g. the Néel antiferromagnetism.

Expansion of a Fermi gas

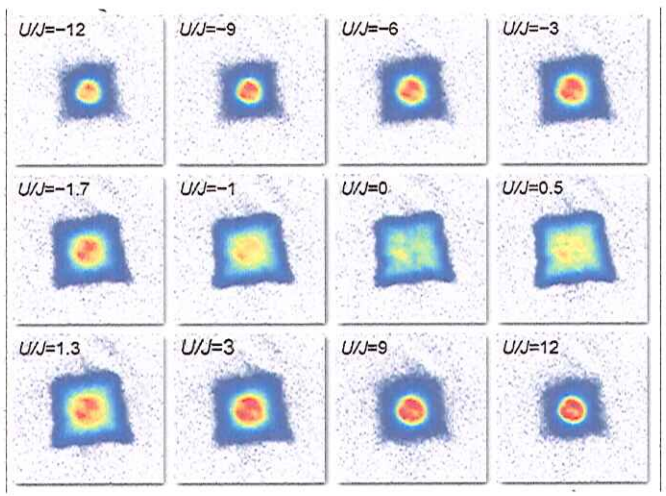
[Schneider et al., Nature Physics **8**, 213 (2012)]



Non-interacting case

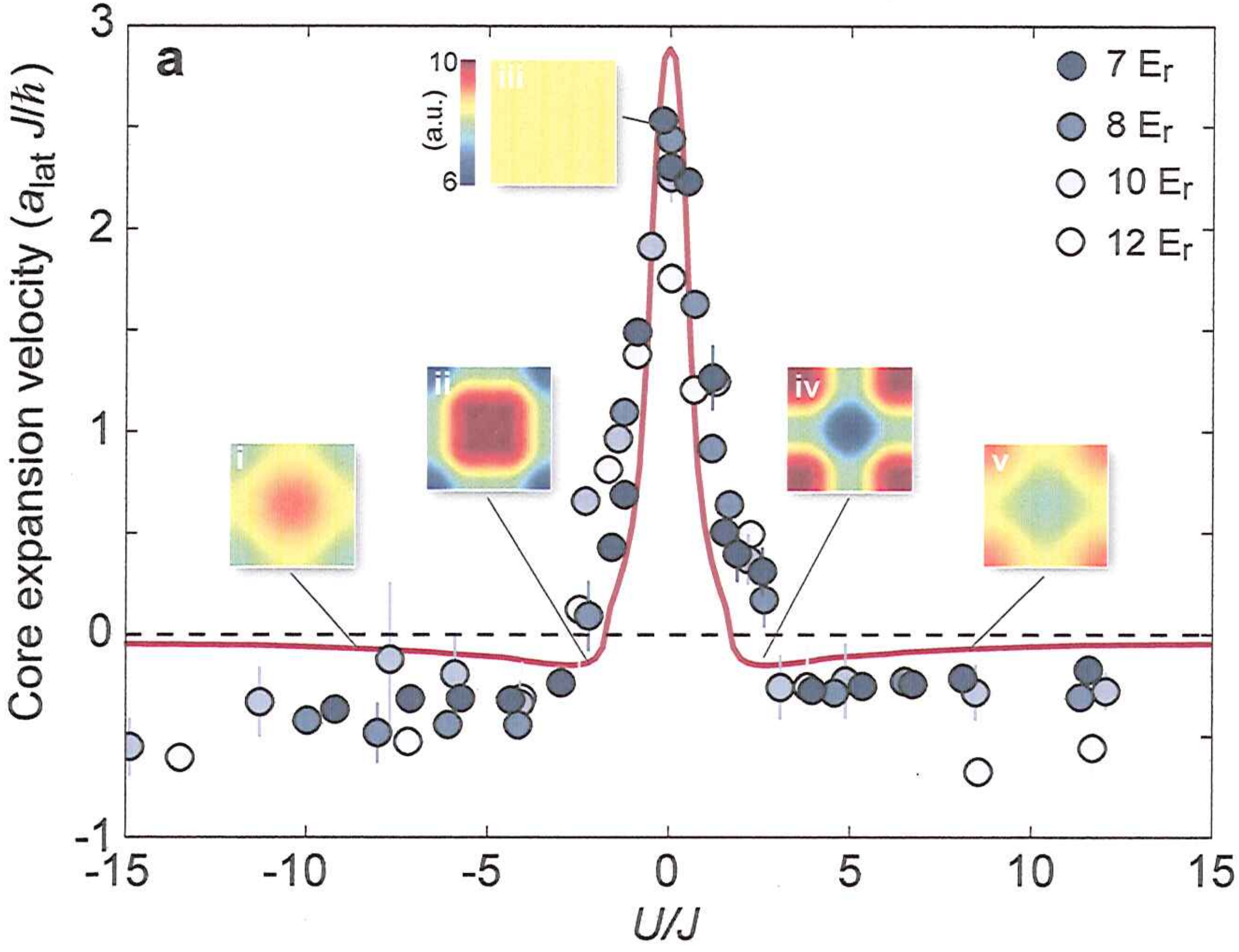


Interacting case



Expansion of a Fermi gas

[Schneider et al., Nature Physics **8**, 213 (2012)]



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* Disorder vs. transport

* Anderson localization was proposed back in 1958 to understand how disorder can lead to the cancellation of conduction. It's a purely quantum single-particle effect, resulting from the interference between the various amplitudes of the scattering paths of a matter-wave propagating among impurities.
* Anderson localization occurs in 1D for any disorder strength (any!) but in 3D there's a mobility edge, i.e. an energy threshold that separates localized from extended states (insulator from conductor).
* Anderson localization has been studied in atoms in optical lattices, where the disorder may be induced by speckle light or a non-commensurate secondary lattice.
[Note: whereas speckle leads to a rather "good" disorder, although not completely uncorrelated, bichromatic lattices lead actually to quasidisorder, which in 1D is given by the Harper model we saw when discussing the Hofstadter butterfly in p. 102.]
* Experiments have been performed in both 3D and 1D lattices. Let me briefly comment here about the experiment performed at Alain Aspect's lab at Palaiseau [Jendrzejewski et al., Nature Physics 8, 398 (2012)]. In that experiment they employ speckle in a 3D optical lattice. Fluorescence imaging of the expanding cloud yields density profiles composed of a steady localized part and a diffusive part. (The expansion occurs after releasing the cloud from an initial confinement.)
The observed localization can't be explained as classical trapping (due to the average energy of the atoms) or by quantum trapping in local potential minima (which don't support stationary states). The localization is hence Anderson-like.

* In p. 168 one sees the evolution of the column density (integrated along one axis) as a function of time for two different strengths of the disorder.
For weak disorder we observe a diffusive expansion (i.e. the width scales as $\sqrt{t}$). After some time the central density is so small that it can't be measured any more. In contrast for the larger disorder strength, the diffusive expansion is much slower, and an almost steady peak survives at the center for very long times.

* These observations suggest indeed a mobility edge between the localized and diffusive components.

* The interplay of disorder and interactions constitutes a challenging problem. In principle a weak repulsion in a Bose system competes again disorder. It basically broadens the "Anderson islands" of exponentially localized states, progressively establishes coherence, and leads eventually to a superfluid.
However strong interactions bring the superfluid into the strongly-correlated regime where disorder and interactions cooperate, leading to a new insulator (the so-called Bose-glass)

* This insulator extending from weak to strong interactions was nicely observed (in a 1D bichromatic lattice) by the group of Giovanni Modugno and Massimo Inguscio at LENS, Florence, in 2014. [D'Errico et al., PRL 113, 095301 (2014)]. To a good approximation the system is described by a Harper Hamiltonian:

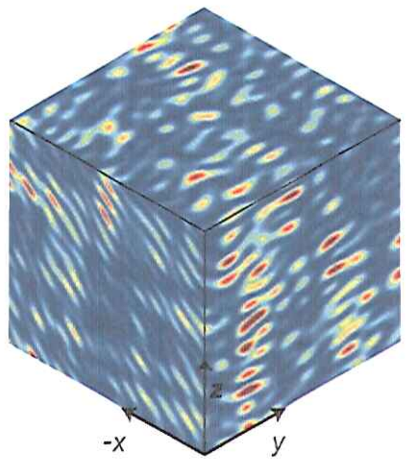
$$H = -J\sum_j (b_j^\dagger b_{j+1} + h.c.) + \underset{\substack{\uparrow \\ \text{quasidisorder strength}}}{\Delta} \sum_j \cos(2\pi\beta j)\, n_j + \frac{U}{2}\sum_j n_j(n_j-1)$$

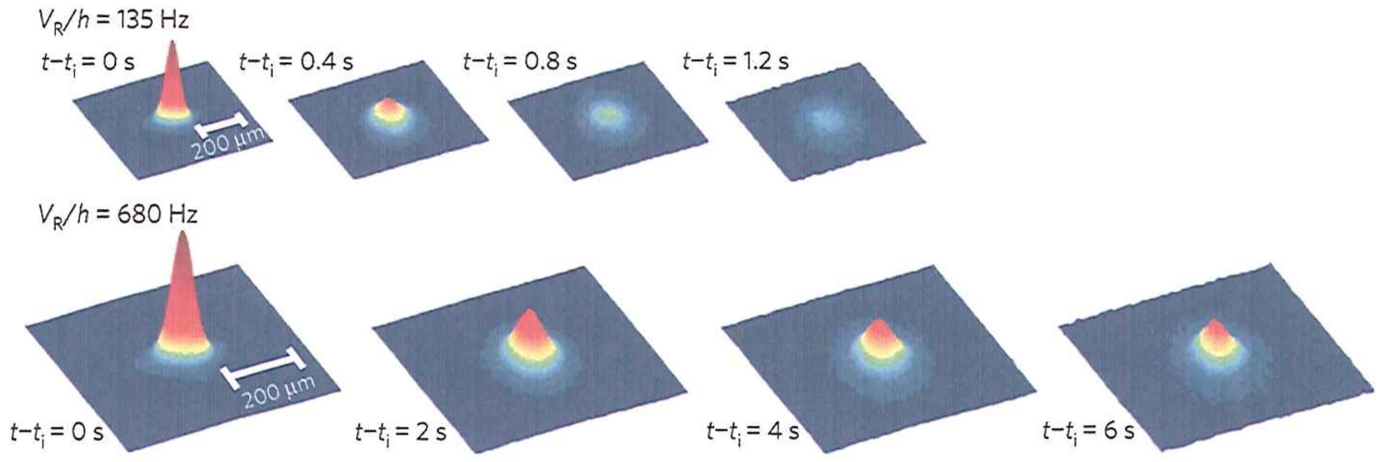
$\beta = \frac{\lambda_1}{\lambda_2} = 1.243$

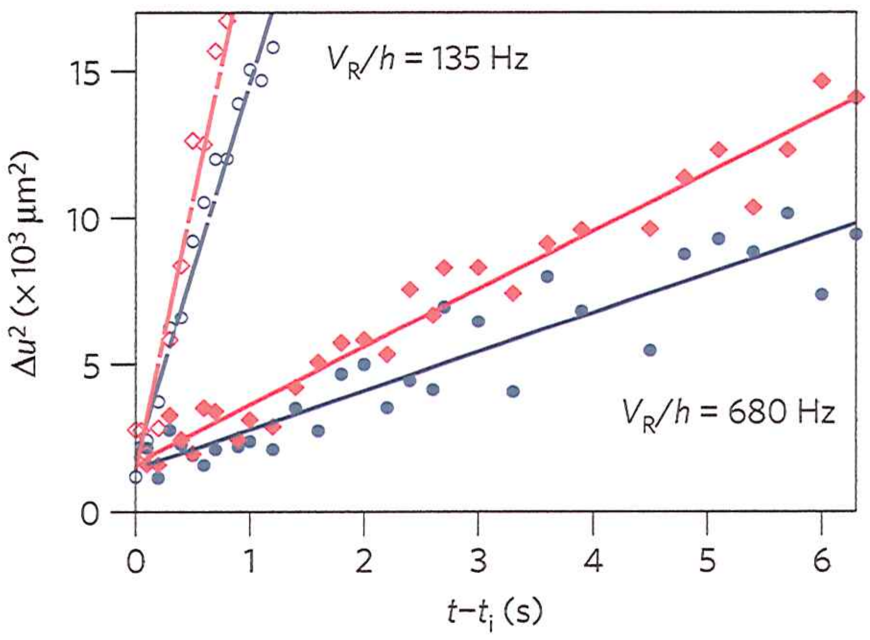
ratio of wavelengths of the 2 lattices.

Anderson localization

[Jendrzejewski et al., Nature Physics **8**, 398 (2012)]







* For $U=0$ all eigenstates are localized for $\Delta > 2J$ (Note: this is because this is a quasi-disorder).

~~the 1D system~~ In Florence they studied the behavior of the system for different ratios Δ/J, U/J.

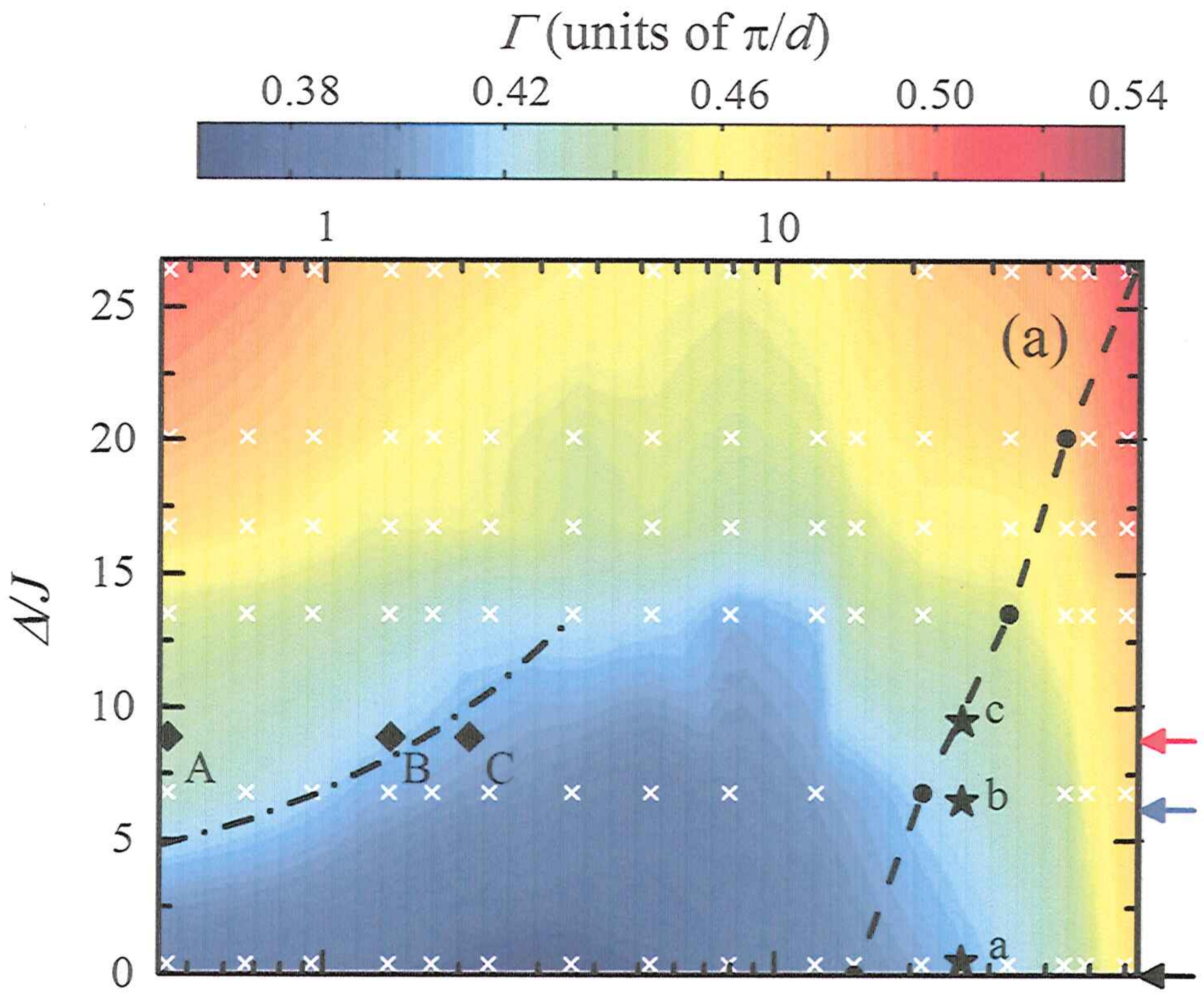
· A good indication of the nature of the system ~~[illegible]~~ (superfluid vs insulator) is given by the momentum distribution $P(k)$, which is obtained in time-of-flight. The root-mean-square width Γ of $P(k)$ is a good measure of the coherence of the system (recall the discussion in p. 3).

One can see the results for Γ in p. 170. It clearly shows a coherent regime (blue) and an incoherent regime (with broad Γ) (red).

* The identification of these regimes with SF and Bose glass is complemented by transport measurements and by lattice modulation spectroscopy, but here we will not discuss this in detail.

Disorder + Interactions

[D'Errico et al., PRL **113**, 095301 (2014)]



* Many-body localization

* A very similar set-up as that of LENS has been recently employed at I. Bloch's group in Munich to study many-body localization. Before moving to the experiment itself, let me comment a little bit about the idea.

* The ergodic hypothesis is a cornerstone of statistical mechanics. In an ergodic system local degrees of freedom become fully entangled with the rest of the system. Or in other words, the system can explore the Hilbert space. This allows to neglect quantum correlations in the dynamics of large many-body systems, forming the basis of the emergence of thermal equilibrium in isolated quantum systems
[Note: recall that in quantum statistical mechanics we assume equal a priori probability for all states with the same energy, and we assume the dephasing hypothesis (only diagonal terms in the density matrix are relevant).
This is based on the ergodic assumption!]

* One path ~~to~~ breaking ergodicity is using integrable systems (recall our previous discussion in p. 156). However integrable models represent very special, fine-tuned situations.
Many-body localization (MBL) in disordered isolated quantum systems constitute a more generic alternative to thermalization dynamics. A MBL transition is something quite remarkable. One one side of the transition ergodicity prevails and quantum effects decay at long times (no memory effects). On the other side quantum correlations persist indefinitely. Hence the MBL transition sets a sharp boundary between a macroscopic world showing quantum phenomena and one governed by classical physics!

* As mentioned above one such a transition has been recently studied by I. Bloch's lab [Schreiber et al., arXiv:1501.05661]. The set-up is similar as in LENS (i.e. they use a bichromatic lattice to induce quasi-disorder).

In the experiment see p. 173 they employ a superlattice to create an initial density wave, then they remove the superlattice, and they study the subsequent dynamics. The key question is: does the system become uniform or on the contrary it retains information about the initial state?

The main observable they employ is the sublattice imbalance

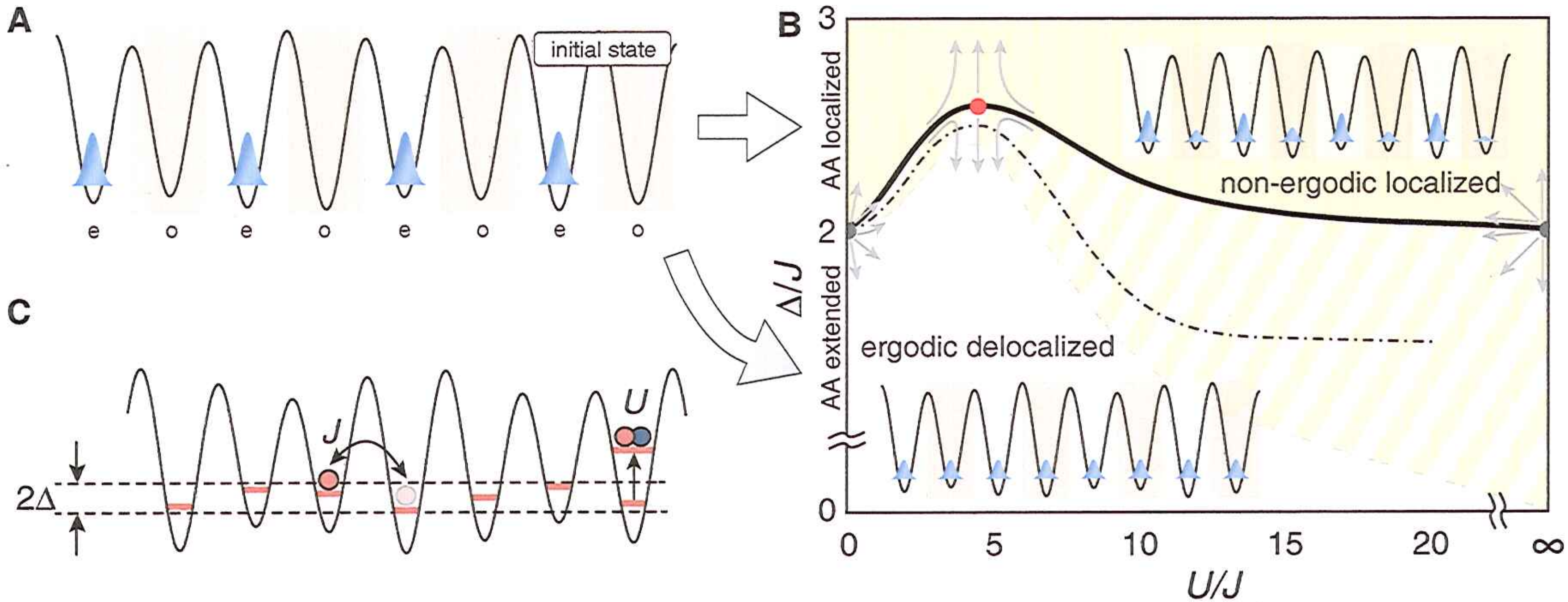
$$I = \frac{N_e - N_o}{N_e + N_o}$$ where $N_{e,o}$ = atom number on even/odd sites.

For the initial density wave $I \simeq 0.9$. If the system relaxes into uniformity then $I \to 0$, but not when ergodicity is broken. Intuitively, if the system is strongly localized, all particles will stay close to their original positions during the evolution and hence the density wave will be only smeared out a little bit.

The long-time stationary value of I thus serves as an order parameter for the MBL phase, and hence allows to map the phase boundary between the ergodic and non-ergodic phases in the phase space (Δ, U) (i.e. disorder strength / interactions).
(see p. 174)

Many-body localization

[Schreiber et al, arXiv:1501.05661]



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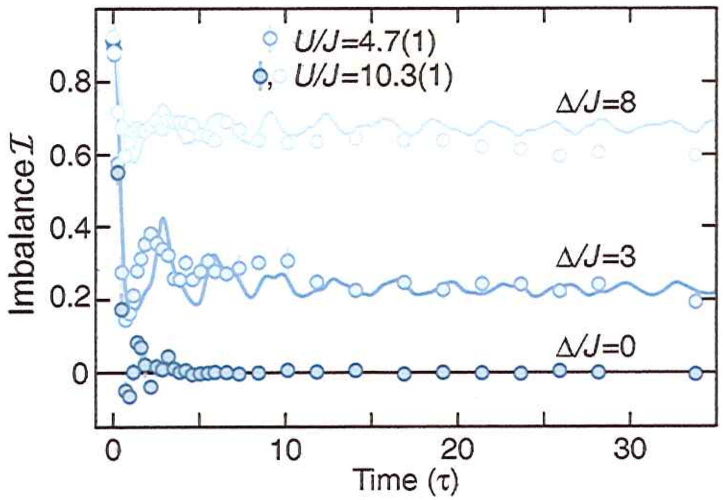
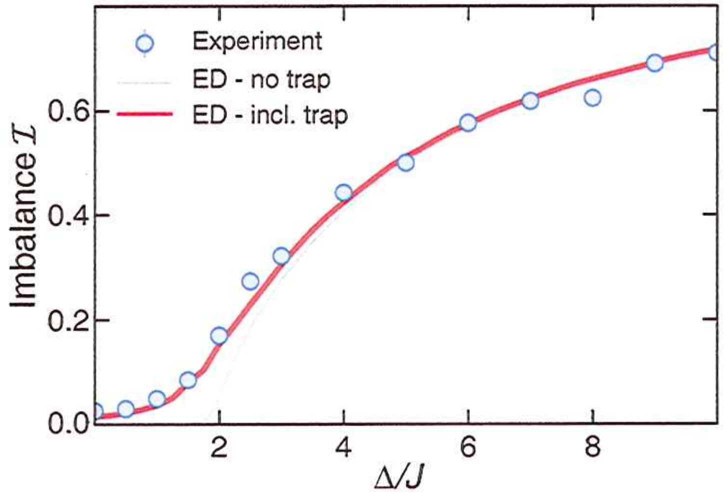
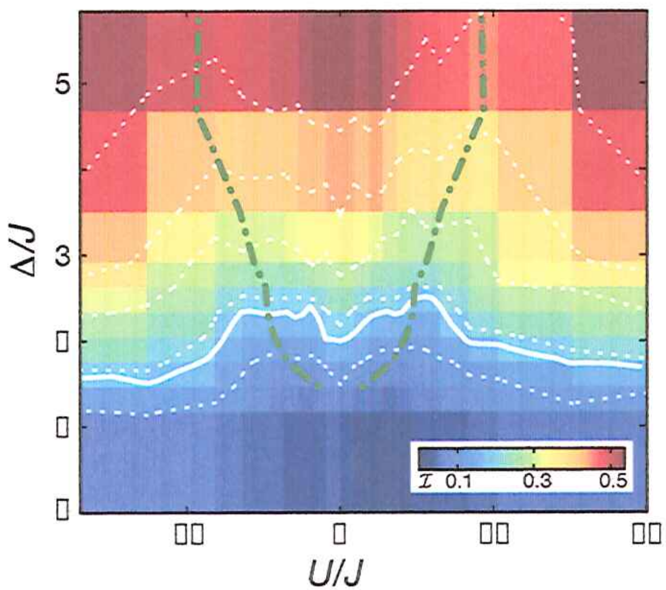
Many-body localization

[Schreiber et al, arXiv:1501.05661]







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* Just a final comment about MBL. Is it really necessary to have external disorder (as it is the case in Munich experiments) to have MBL? This is a rather interesting question because it may mean that ergodicity breaking may be (in optical lattices) much more general than expected. This of course may have very fundamental consequences for cooling and state preparation in optical lattices.

* The answer seems to be that disorder is not necessary! There are systems that present MBL (or at least extremely slow transport) in the absence of disorder.

* One of these systems is actually polar lattice gases, as we recently studied (Barbiero et al., arXiv: 1505.02028). In this system a very similar experiment as that of J. Bloch would show MBL in the absence of disorder due to the long-range character of the dipolar interactions (which lead to cluster formation, similar to the repulsively-bound pairs of p. 153, and effective blockade repulsion even for attractive dipolar interactions). I will not enter into details here, but this opens quite exciting perspectives.

* So as a final word on out-of-equilibrium dynamics. It should be clear to you by now, that transport in optical lattices may be seriously handicapped by integrability, disorder and interactions (and the finite band-width of the lattice motion). This makes state preparation in lattices a rather non-trivial and challenging subject, that will be one of the most active and exciting research topics on cold gases in the nearest future!