Exercise 1 (4 Points)

Consider electrons under a given magnetic field $\vec{B} = B\vec{e}_z$. As we discussed in the class, the single-particle eigenenergies for these system are of the form

$$\epsilon_{ns}(p_z) = \frac{p_z^2}{2m} + 2\mu_B B(n + 1/2) - s\mu_B B,$$

where μ_B is the Bohr magneton, and $s = \pm 1$. The states have a degeneracy $g(B) = \frac{eBL_xL_y}{2\pi\hbar}$, where we employ the same notation as in the class.

- Calculate for $V \to \infty$ the expression for $\frac{ln\mathcal{T}}{V}$, where $\mathcal{T}$ is the grand-canonical partition function. (You do not have to solve for the integrals and sums.)
- Calculate $\frac{ln\mathcal{T}}{V}$ for large T. (Now you have to solve for the integrals and sums.)
- Consider $\mu_B B/k_B T \ll 1$. Find the magnetization per unit volume, and the susceptibility χ.
- Compare the value of χ to that of the Pauli paramagnetism and the Landau diamagnetism. What is your conclusion?

Exercise 2 (6 Points)

Consider spinless fermions in a magnetic field $\vec{B} = B\vec{e}_z$ at zero temperature. Consider the 2D problem, in which the fermions can just move in the $x - y$ plane. In that case the Landau levels are $\epsilon_n = 2\mu_B B(n + 1/2)$, with degeneracy $g(B) = eBL_xL_y/2\pi\hbar$.

- Calculate the critical magnetic field B_c, such that if $B > B_c$ then all particles are in the lowest Landau level.
- Calculate the regime of values of the magnetic field for which the levels $n = 0, \dots, j$ are populated, but $n > j$ are empty.
- Calculate the energy per particle (E_{total}/N) for $B > B_c$, and for the regime of the second point. (Remember that $T = 0$.)
- Using the expression for the magnetization per unit volume in the ground-state $\mathcal{M} = \frac{-1}{v}\frac{\partial(E_{\text{total}}/N)}{\partial B}$, calculate the susceptibility χ. Plot χ as a function of B/B_c.

Suppose that now the fermions have spin 1/2.

- How are the eigenenergies? Which is the degeneracy of the eigenenergies now? Compare with the case without spin.
- Calculate the critical magnetic field B_c, such that if $B > B_c$ then all particles are in the state with the lowest energy.
- Calculate the regime of values of the magnetic field for which levels of the first k different eigenergies are populated, but levels of higher eigenenergies are empty.