Exercise 1 (5 Points)

Consider the Bose-Einstein condensation of an ideal three-dimensional Bose-gas in free space. In the lecture we have seen that the Bose-Einstein condensation is indeed a first-order phase transition, as it can be clearly seen in a P-v diagram.

- We showed in the class that in a P-v diagram there is a critical transition curve, which separates the purely non-condensed region (at the right of the curve) from the coexistence region (at the left of the curve). Calculate the relation $P(v)$ that characterizes this transition curve.
- For a given temperature obtain the vapour pressure that characterizes the coexistence region.
- Write down the coresponding Clausius-Clapeyron equation.
- Obtain the latent heat at a given temperature T.

Exercise 2 (5 Points)

Consider a Bose-gas in two-dimensions in a harmonic oscillator potential of frequency ω.

- Calculate the corresponding density of states, and compare it to the density of states of a 2D Bose-gas in free space.
- Calculate the number of non-condensed atoms as a function of T and z. Compare again the result with the case of a 2D Bose-gas in free space. Is there condensation?
- In case there is condensation, calculate the critical temperature for condensation, T_c, and the condensate fraction N_0/N as a function of T/T_c.

Hint: You may need to know that $\int_0^\infty dx x \frac{ze^{-x}}{1-ze^{-x}} = g_2(z) \leq g_2(1) = \zeta(2) \simeq 1.65$.

The team of the Statistical Physics lecture wishes you a merry Christmas and a happy new year!