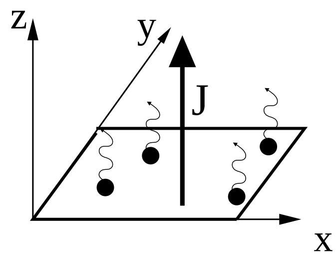
Exercise 1 (5 Points)

Consider a metal with a surface in the plane $z = 0$. Assume that the electrons of the metal are described by an ideal gas of free Fermi particles. The electrons may leave the metal going from $z < 0$ to $z > 0$, i.e. through the metallic surface, but to do that they must overcome a potential barrier W. At finite temperature the so-called thermoemission occurs, because there are electrons with energy larger than W.
The current of electrons through the surface is given by the expression

$$J = \sum_{\vec{v}} e \frac{n_{\vec{v}}}{V} v_z,$$



where e is the electron charge, V is the volume, and $n_{\vec{v}}$ is the number of electrons with a given velocity $\vec{v} = \{v_x, v_y, v_z\}$. (Hint: Note that due to the barrier only the electrons with a sufficiently large positive velocity $v_z \geq v_z^c$ can leave the metal.)

For a given temperature calculate the thermoemission current J for

- $T \ll T_F$, where T_F is the Fermi temperature. Assume in the calculation that $W - \epsilon_F \gg k_B T$, where ϵ_F is the Fermi energy.
- The classical regime $T \gg T_F$, for which the Fermi statistics may be approximated by a Boltzmann statistics.

Exercise 2 (5 Points)

Consider a two-dimensional gas of N spinless bosons. Assume that they have the usual spectrum for non-relativistic free particles ($\epsilon_{\vec{p}} = p^2/2m$), but that in addition there is a bound state with negative energy $\epsilon_B = -\Delta$. (Hint: note that now this is the lowest single-particle level, and not any more $p = 0$, so this is the term that you have to extract from the rest of the sum).

- Obtain the equation $N = N(\mu)$ that relates the number of particles N and the chemical potential μ.
- Find out whether there is Bose-Einstein condensation in this system, and, in case there is, find the equation $N = N(T_c)$ which determines the critical temperature T_c.