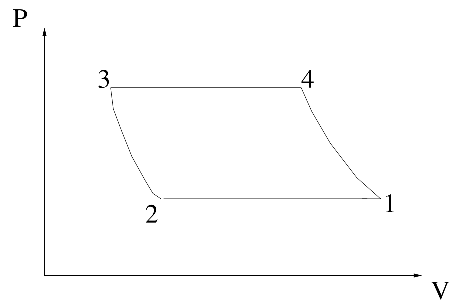
*Exercise 1* (4 Points)

Consider an ideal gas that undergoes the thermodynamic cycle of the figure (Claude Process) formed by the following reversible processes: $1 \to 2$ (at constant pressure), $2 \to 3$ (at constant temperature), $3 \to 4$ (again at constant pressure), and $4 \to 1$ (at constant entropy). Use in the following that $C_V = 3k_B N/2$, and that $C_P = \gamma C_V$ with $\gamma = 5/3$.




- Express the entropy of an ideal gas as a function of $(V,T)$, $(P,V)$, and $(P,T)$. This will be useful for the rest of the exercise.
- Depict the process in the $T-S$ diagram, discussing why you depict it as you do.
- Demonstrate with a simple argument that the area of the cycle (actually of any cycle) must be the same in both representations $P-V$ and $T-S$.
- Calculate the heat change in every step of the cycle. Express the results of this point, and also of the following points only as a function of the three different temperatures you have in the cycle.
- Calculate the work in every step of the cycle. Check that the total work coincides with the total heat.
- Obtain the efficiency of the cycle.

*Exercise 2* (3 Points)

Consider a lattice with $N_0$ sites. Consider $N \leq N_0$ particles in the lattice. Every site can be filled by at most one particle with a binding energy $u$. Considering the $N_0$ sites on a surface this model constitutes for $u < 0$ a simple model for adsorption. The energy of $N$ particles on the lattice is then $E = Nu$.

- Note that there are more than one way to fill $N$ particles in $N_0$ sites. This means that the energy eigenstates are therefore degenerated. Calculate this degeneration.
- Calculate the grand-canonical partition function.
- Obtain the average number of particles $\langle N \rangle$ in the lattice for a given temperature $T$.

- Obtain the relation $\mu(\rho)$ between the chemical potential $\mu$ and the average lattice occupation. $\rho \equiv \langle N \rangle / N_0$.
- How is $\rho$ for $u = 0$, $u \gg 0$, $u \ll 0$? What is your physical interpretation?
- Depict $\rho(u)$ for very low temperature, and for large temperature.

*Exercise 3* (3 Points)

Consider a classical one-dimensional gas of $N \gg 1$ particles at a temperature $T$. The particles move along a line of length $L$, and are assumed to be hard spheres of diameter $b$, i.e. they cannot penetrate each other. Of course, this reduces the effective "volume" (in this case volume must be substituted by length, since we are in one dimension!).

- Calculate the canonical partition function.
- Obtain the Helmholtz free energy.
- Calculate the internal energy.
- Obtain the equation of state.