Exercise 1 (4 Points)

We consider a substance such that the entropy can be expressed in the form

$$S = Nk_B \frac{V_0}{V} \left(\frac{T}{T_0}\right)^a ,$$

where N, V_0, T_0, and a are fixed constants. If we perform for this substance a reversible isothermal expansion at $T = T_0$ from V_0 to V, the work produced is

$$W = Nk_B T_0 \ln \frac{V}{V_0}.$$

Using this information determine:

- the Helmholtz free energy.
- the equation of state.
- the work done in a reversible isothermal expansion at an *arbitrary* (constant) temperature T.

Exercise 2 (6 Points)

The Helmholtz free energy for an ideal gas can be written as:

$$A_{\text{id}} = Nk_B T \left[\ln \left(\frac{N}{V n_Q(T)} \right) - 1 \right] ,$$

where $n_Q(T) = \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}$.

- Obtain from A_{id} the equation of state for an ideal gas.

The equation of state for a real gas is (for the van der Waals model):

$$\left(P + \frac{a}{V^2}\right)(V - b) = Nk_B T,$$

where a and b are constants. Obtain from the equation of state

- The free energy A_{r} imposing that for $a = b = 0$ you must retrieve the expression for an ideal gas A_{id}.

Once you have A_{r}, determine:

- the entropy S.
- the internal energy U.
- the isothermal compressibility κ_T.
- the thermal expansion coefficient α.
- the specific heats at constant volumen (C_V) and constant pressure (C_P).