Exercise 1 (6 Points)

Consider the following distribution function:

$$f(\vec{r},\vec{v},t) = \exp\left[\alpha(\vec{r},t) - \frac{m}{2}\beta(\vec{r},t)(\vec{v}^2 - 2\vec{u}\cdot\vec{v})\right].$$

- Show that for this distribution the collisional term of the Boltzmann equation is zero.
- Show also that this distribution is in general not a solution of the Boltzmann equation and obtain the conditions that α, β, and $\vec{u}$ must fulfill such that f satisfies the Boltzmann equation.
- Calculate the density $n(\vec{r},t)$. Using the result, calculate the average velocity $\bar{\vec{v}}(\vec{r},t)$ which is defined *via* the average particle current density $\vec{j}(\vec{r},t)$ as $n(\vec{r},t)\bar{\vec{v}}(\vec{r},t) := \vec{j}(\vec{r},t) := \int d^3v\vec{v}f$. Calculate also the average energy per particle $\epsilon(\vec{r},t)$ which is defined *via* $n(\vec{r},t)\epsilon(\vec{r},t) := \int d^3v\frac{m}{2}\vec{v}^2 f$.
- Consider $\vec{F} = -\vec{\nabla}V(\vec{r})$, with $V(\vec{r}) = m\omega^2 r^2/2$. Show that if f fulfills the Boltzmann equation, then $\beta(\vec{r},t) = \beta(\vec{r},t+2\pi/\omega)$.

Exercise 2 (4 Points)

Consider a dilute gas, infinite in extension, and composed of charged particles, each of charge q and mass m, at equilibrium. In the absence of an external electric field the equilibrium distribution is the Maxwell-Boltzmann distribution:

$$f^{(0)}(\vec{v}) = n\left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mv^2/2k_BT},$$

where n and T are constants. A weak uniform electric field $\vec{E}$ is then switched on, leading to a new equilibrium distribution f. Assume that for the new distribution f:

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \frac{f^{(0)} - f}{\tau},$$

where τ is some collision time. Calculate the new equilibrium function f up to first order in the (weak) applied field $\vec{E}$. Calculate the electrical conductivity (σ), defined as

$$nq < \vec{v} >= \sigma\vec{E},$$

where $< \vec{v} >$ is the average velocity.