Strongly-correlated atomic gases (WS 2011/2012)

1. Bose-Einstein condensates

- Weakly-interacting Bose-gases: Hamiltonian
- Off-diagonal long-range order, the order parameter
- The Bogoliubov approximation, condensate wavefunction
- Chemical potential
- The Gross-Pitaevskii equation
- The time-independent GPE
- Hydrodynamics
- Josephson junction: mean-field analysis, pendular equations
- Beyond mean-field: Josephson Hamiltonian, coherence factor,number squeezing
- Bose-Hubbard Hamiltonian: quantum depletion, fragmentation

2. Atoms in optical lattices (I)

- Lattice potential, band structure, Bloch and Wannier functions
- Bose-Einstein condensates in optical lattices: coherence, DNLSE
- Josephson-arrays of BECs: pendular equations, classical insulator transition
- Interference fringes in time-of-flight pictures
- Bose-Hubbard Hamiltonian
- Mott-insulator to superfluid transition
- Understanding the phase diagram for bosons in a lattice: energy gap
- Overall harmonic trapping: local-density approximation, wedding-cake structure
- Experiments: Mott insulator and Mott shells

3. Atoms in optical lattices (II): Spin-1/2 lattice gases

- Fermi-Hubbard Hamiltonian: two-component versus one-component Fermi gas, band-insulator, Mott-metal transition, Experiments
- Super-exchange: anti-ferromagnetic Heisenberg model for spin-1/2 fermions
- Ferromagnetic exchange for spin-1/2 bosons
- Observation of super-exchange in double-well experiments
- Weak coupling regime and (T,U) phase diagram
- Entropy , Pomeranchuk effect
- Cooling in optical lattices
  - Filter cooling
  - Inmersion cooling
  - Demagnetization cooling
- Away from half-filling: d-wave superfluidity

4. Properties of the Heisenberg model

- Ferromagnetic state
- Antiferromagnetic state
- Holstein-Primakoff bosons
- Spin-wave theory for ferromagnets, Mermin-Wagner theorem
- Spin-wave theory for anti-ferromagnets

5. Atoms in optical lattices (III): Other lattice models and control knobs

- Hard core regime: mapping occupation into a spin
    - XY model
    - Modification of the hopping sign in periodically modulated latices
- Polar lattice gases
    - Polar molecules
    - Dipole-dipole interaction
    - Polar molecules in optical lattices: extended Hubbard model
    - Hard-core regime: XXZ Heisenberg model
- Getting the XXZ model using super-exchange
- Spinor lattice gases
    - Spinor gases
    - Short-range interactions: spin-changing collisions
    - Mott regime, effective super-exchange Hamiltonian
    - Spin-1 bosons: effective bilinear-biquadratic spin Hamiltonian
    - Quadratic Zeeman effect: single-ion anisotropy
- Lattice geometry
    - Triangular lattices: creation and control of tunneling
    - Condensate phases in triangular lattices, frustration
    - Kagome lattice
    - P-bands: Orbital superfluidity
- Effective three-body hard-core using three-body losses

6. One-dimensional spin systems

- One-dimensional XXZ Heisenberg model
    - Ferromagnetic phase
    - Neel phase
    - XY phase. Jordan-Wigner transformation
- J1-J2 model. Majumdar-Ghosh limit. Dimerization.
    - Majumdar-Ghosh limit
    - Spontaneous dimerization
    - Models with explicit dimerization
- Spin-1 Haldane chain: string order. Anisotropic S=1 chain
- Polar bosons in 1D optical lattices: Haldane-insulator phase
- AKLT model: Valence-Bond solids, Edge states, Matrix product states
- Bilinear-biquadratic spin-1 chain