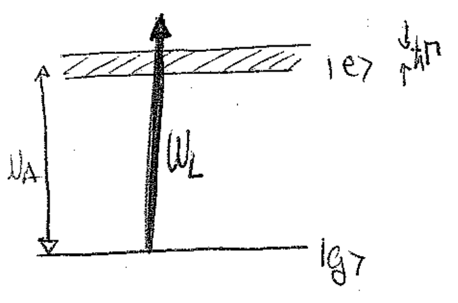
• ATOM - LIGHT INTERACTION

• In this lecture we will learn some basics of the interaction between light and atoms. It is important to understand some key features of this interaction, because it is in the basis of how atoms are cooled (cooling) and how atoms are manipulated (dipole potential). Our discussion will be relatively short and summarized, since a thorough discussion would take us too much time (for more details and a much more rigorous treatment have a look in my lectures on Theoretical Quantum Optics of WS 07/08).

• THE TWO-LEVEL ATOM

* We will model in this lecture the internal electronic structure of the atom as a two-level system, with a ground state $|g\rangle$ and an excited state $|e\rangle$. The latter has a lifetime $1/\Gamma$ and may spontaneously decay onto $|g\rangle$. The transition frequency $\omega_A = \frac{1}{\hbar} \times$ [energy difference between $|e\rangle$ and $|g\rangle$]



* Note (see figure) that due to the uncertainty principle the upper state $|e\rangle$ is broadened.

* We assume that a laser (of frequency ω_L) excites the transition. We define the detuning $\delta = \omega_L - \omega_A$.
(Note: the two-level approximation is valid only if $|\delta| \ll \omega_A, \omega_L$)

* The physics of the atom alone is given by the Hamiltonian

$$\hat{H}_A = \underbrace{\frac{\hat{P}^2}{2m}}_{\text{kinetic energy of the center of mass (we assume no potential energy)}} + \underbrace{\hbar\omega_A |e\rangle\langle e|}_{\text{internal energy (we set the } |g\rangle \text{ energy as zero)}}$$

* The laser may be treated in a semi-classical approximation, and it may be described by a time-dependent electric-field

$$\vec{E}_L(\vec{r},t) = \frac{1}{2}\,\mathcal{E}_L(\vec{r})\left[\vec{e}_L(\vec{r})\,e^{-i\phi(\vec{r})-i\omega_L t} + c.c.\right]$$

(Note: for a plane wave $\phi(\vec{r}) = -i\vec{k}_L\cdot\vec{r}$)

* We will consider the interaction between the atom and the laser in the so-called electric dipole approximation. This approximation is valid if the typical size of the atom is much smaller than the laser wavelength.
In that case the interaction reduces to

$$\hat{H}_{AL} = -\hat{D}\cdot\vec{E}_L(\vec{R},t)$$

where $\vec{R}$ is the center of mass position of the atom and $\hat{D}$ is the electric dipole of the transition

$$\hat{\vec{D}} = \vec{d}\,|e\rangle\langle g| + h.c.$$

where $\vec{d} = -e\,\langle e|\vec{r}'|g\rangle = -e\int d^3r'\,\phi_e^*(\vec{r}')\,\vec{r}'\,\phi_g(\vec{r}')$

* In the so-called rotating-wave-approximation we neglect all terms except those quasi-resonant. This means that, in principle we would have terms $|e\rangle\langle g|\,e^{i\omega_L t}$, $|g\rangle\langle e|\,e^{-i\omega_L t}$ In interaction picture with H_A, these terms go respectively as $e^{i(\omega_L+\omega_A)t}$ and $e^{-i(\omega_L+\omega_A)t}$. Since $\omega_L \approx \omega_A \sim 10^{15}$ Hz this means that these terms oscillate very very fast, and hence they average to zero. (This is nothing else as the conservation of the energy, and it's just violated for ultrashort fs pulses).

The only terms that remain in $\hat{H}_{AL}$ are:

$$\hat{H}_{AL} \cong \underbrace{\frac{\hbar}{2}\Omega_1(\vec{R})}_{\text{Rabi frequency}} \Big[\underbrace{|e\rangle\langle g| e^{-i\omega_L t} e^{-i\phi(\vec{R})}}_{\text{Absorption}} + \underbrace{|g\rangle\langle e| e^{i\omega_L t} e^{i\phi(\vec{R})}}_{\text{Stimulated emission}} \Big]$$

$$\hbar\Omega_1(\vec{R}) \equiv -(\vec{d}\cdot\vec{\varepsilon}_L(\vec{R}))\,\mathcal{E}_L(\vec{R})$$

(we may assume $\Omega_1(\vec{R})$ as real, since we can include all phases into ϕ)

* The absorption and stimulated emission process may be easily understood graphically

Note: Ω_1^2 is basically the laser intensity.

Absorption

$\Downarrow$

Stimulated emission

$\Downarrow$

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* For the moment we will forget the center of mass physics and just consider an atom stopped. The internal physics of the atom is provided by the so-called optical Bloch equations. Let's see them.

* We describe the internal state of the atom as

$$|\psi(t)\rangle = a_g(t)|g\rangle + a_e(t)|e\rangle$$

This atom is governed by the Hamiltonian $\hat{H} = \hat{H}_A + \hat{H}_{AL}$

$$\hat{H} = \hbar\omega_A |e\rangle\langle e| + \frac{\hbar}{2}\Omega_1 \left[|e\rangle\langle g| e^{-i\omega_L t} e^{-i\phi(\vec{R})} + |g\rangle\langle e| e^{i\omega_L t} e^{i\phi(\vec{R})} \right]$$

* Then, $|\psi(t)\rangle$ is provided by the Schrödinger equation

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

$$i\hbar \dot{a}_g(t)\,|g\rangle + i\hbar \dot{a}_e(t)\,|e\rangle = \hbar\omega_A\, a_e(t)\,|e\rangle + \frac{\hbar}{2}\Omega_1 e^{-i\omega_L t} e^{-i\phi(\vec{R})} a_g(t)\,|e\rangle + \frac{\hbar}{2}\Omega_1 e^{i\omega_L t} e^{i\phi(\vec{R})} a_e(t)\,|g\rangle$$

Then:

$$i\hbar \dot{a}_g(t) = \frac{\hbar}{2}\Omega_1 e^{i\omega_L t} e^{i\phi(\vec{R})} a_e(t)$$

$$i\hbar \dot{a}_e(t) = \hbar\omega_A\, a_e(t) + \frac{\hbar}{2}\Omega_1 e^{-i\omega_L t} e^{-i\phi(\vec{R})} a_g(t)$$

Let's introduce the transformation

$$c_g(t) = a_g(t)$$

$$c_e(t) = a_e(t)\, e^{-i\omega_L t}$$

(Note: This means that we move into a rotating frame with the laser frequency ω_L)

Then:

$$\dot{c}_g(t) = -\frac{i}{2}\Omega_1 e^{i\phi(\vec{R})} c_e(t)$$

$$\dot{c}_e(t) = -\frac{i}{2}\Omega_1 e^{-i\phi(\vec{R})} c_g(t) + i\delta\, c_e(t)$$

Let $\underbrace{\rho_{gg} \equiv c_g^* c_g}_{\text{Population in the ground state}}$, $\underbrace{\rho_{ee} \equiv c_e^* c_e}_{\text{Population in the excited state}}$, $\underbrace{\rho_{ge} = c_g^* c_e}_{\text{Coherence}}$ ($\rho_{ge} = \rho_{eg}^*$)

(Note: $\hat{\rho}$ is the so-called density matrix). Then:

$$\dot{\rho}_{gg} = \frac{i}{2}\Omega_1 \left[e^{-i\phi}\rho_{eg} - e^{i\phi}\rho_{ge}\right]$$

$$\dot{\rho}_{ee} = -\frac{i}{2}\Omega_1 \left[e^{-i\phi}\rho_{eg} - e^{i\phi}\rho_{ge}\right]$$

$$\dot{\rho}_{ge} = \frac{i}{2}\Omega_1 e^{-i\phi}\left[\rho_{ee} - \rho_{gg}\right] + i\delta\,\rho_{ge}$$

* Up to now we have not considered the spontaneous emission. The rigorous treatment of the spontaneous emission demands the use of the master equation formalism (for more details see my script of Theoretical Quantum Optics in WS 07/08)*, but at the level of the master equation it may be added easily.

The spontaneous emission leads to an exponential decay of the population in $|e\rangle \Rightarrow \dot{\rho}_{ee} \propto -\Gamma \rho_{ee}$. Since the population isn't lost $\Rightarrow \dot{\rho}_{gg} \propto \Gamma \rho_{ee}$.

The whence $\dot{\rho}_{ge} \propto -\Gamma/2\, \rho_{ge}$.

The latter may be understood with a hand-waving argument. (Note: the latter may be understood with a hand-waving argument: note that $\dot{\rho}_{ee} = \dot{c}_e^* c_e + c_e^* \dot{c}_e$, hence $\dot{\rho}_{ge} = \dot{c}_g^* c_e + c_g^* \dot{c}_e$; note that $\dot{c}_e \sim \frac{-\Gamma}{2} c_e$; c_g is also affected by the decayed electron but after the decay the electron gets a random phase, since the spontaneous emission is an stochastic process. In average hence the contribution to $\dot{c}_g$ of the spontaneous emission is hence zero, since the random phase factor averages to zero.

Hence $\dot{\rho}_{ge} \sim c_g^* \dot{c}_e \sim -\Gamma/2\, \rho_{ge}$.)

* The final set of <u>Bloch eqs.</u> is hence:

$$\dot{\rho}_{ee} = -\frac{i}{2}\Omega_1 \left[e^{-i\phi}\rho_{ge}^* - e^{i\phi}\rho_{ge}\right] - \Gamma \rho_{ee}$$

$$\dot{\rho}_{ge} = \frac{i}{2}\Omega_1 e^{-i\phi}(\rho_{ee} - \rho_{gg}) + (i\delta - \Gamma/2)\rho_{ge}$$

and $1 = \rho_{gg} + \rho_{ee}$ by conservation of the population.

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* Note: Rigorously, you introduce a master equation $\dot{\hat{\rho}} = \frac{i}{\hbar}[\hat{\rho}, \hat{H}] + \mathcal{L}_{sp}\hat{\rho}$, where we add the so-called superoperator $\mathcal{L}_{sp}\hat{\rho} = \frac{\Gamma}{2}\{2|g\rangle\langle e|\hat{\rho}|e\rangle\langle g| - |e\rangle\langle g|g\rangle\langle e|\hat{\rho} - |e\rangle\langle g|g\rangle\langle e|\hat{\rho}\} = \frac{\gamma}{2}\{\rho_{ee}|g\rangle\langle g| - |e\rangle\langle e|\hat{\rho} - \hat{\rho}|e\rangle\langle e|\}$. If you take now the different components $\hat{\rho}_{ij}$ you get the eqs. above.

* Let
$$u(t) = \frac{1}{2}\left[e^{-i\phi}\rho_{ge}^{*} + e^{+i\phi}\rho_{ge}\right]$$
$$v(t) \equiv \frac{1}{2i}\left[e^{-i\phi}\rho_{ge}^{*} - e^{+i\phi}\rho_{ge}\right]$$
$$w(t) \equiv \frac{1}{2}(\rho_{ee} - \rho_{gg})$$

Then after a straightforward calculation (I leave you that as exercise!):

$$\left.\begin{aligned}
\frac{du}{dt} &= -\frac{\Gamma}{2}u + \delta v \\
\frac{dv}{dt} &= -\delta u - \frac{\Gamma}{2}v - \Omega_1(\vec{R})\,w \\
\frac{dw}{dt} &= \Omega_1(\vec{R})\,v - \Gamma w - \Gamma/2
\end{aligned}\right\} \quad \underline{\text{Optical Bloch Eqs.}}$$

* We shall be interested in the stationary regime ($\dot{u} = \dot{v} = \dot{w} = 0$)
This is because the internal dynamics is typically much faster
than the external (center-of-mass) dynamics. We should
not forget that our goal is ultimately to discuss the
mechanical effects of light on atoms.

* The stationary solutions are of the form (again you may try to get them as an exercise!)

$$\left.\begin{aligned}
u_{st} &= \frac{\delta}{\Omega_1(\vec{R})}\,\frac{s(\vec{R})}{1+s(\vec{R})} \\
v_{st} &= \frac{\Gamma}{2\Omega_1(\vec{R})}\,\frac{s(\vec{R})}{1+s(\vec{R})} \\
w_{st} &= -\frac{1}{2}\,\frac{1}{1+s(\vec{R})}
\end{aligned}\right\}$$

where we have introduced the <u>saturation parameter</u>

$$s(\vec{R}) = \frac{\Omega_1^2(\vec{R})/2}{\delta^2 + \Gamma^2/4}$$

* For the calculation we will do soon we will be interested on the stationary average value of the dipole operator $\hat{D}$:

$$\langle \hat{\vec{D}} \rangle_{st} = \mathrm{Tr}\{\hat{\vec{\rho}}\, \hat{\vec{D}}\} = \vec{d}\, \mathrm{Tr}\{\hat{\rho}\,(|e\rangle\langle g| + |g\rangle\langle e|)\}$$

$\uparrow$ density matrix in the non-rotating frame

$$= \vec{d}\,\{a_g^* a_e + a_e^* a_g\} = \vec{d}\,\{\rho_{ge}\, e^{-i\omega_L t} + \rho_{ge}^*\, e^{i\omega_L t}\}$$

$$= \vec{d}\,\{[\rho_{ge} e^{i\phi}]\, e^{-i\omega_L t} e^{-i\phi} + [\rho_{ge}^* e^{-i\phi}]\, e^{i\omega_L t} e^{i\phi}\}$$

Note that
$$u_{st} + i v_{st} = e^{-i\phi} \rho_{ge}^*$$
$$u_{st} - i v_{st} = e^{i\phi} \rho_{ge}$$

$$\langle \hat{\vec{D}} \rangle_{st} = \vec{d}\,\{(u_{st} - i v_{st})\, e^{-i\omega_L t} e^{-i\phi} + (u_{st} + i v_{st})\, e^{i\omega_L t} e^{i\phi}\}$$

$$= \vec{d}\,\{2u_{st} \cos(\omega_L t + \phi) - 2v_{st} \sin(\omega_L t + \phi)\}$$

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* <u>FORCE EXERTED BY THE LASER ON THE ATOM</u>

* Let's return now to our Hamiltonian

$$\hat{H} = \frac{\hat{\vec{P}}^2}{2m} + \hbar\omega_A |e\rangle\langle e| + \frac{\hbar}{2}\Omega_1(\vec{R}) \left[|e\rangle\langle g|\, e^{-i\omega_L t} e^{-i\Phi(\vec{R})} + |g\rangle\langle e|\, e^{i\omega_L t} e^{i\Phi(\vec{R})} \right]$$

The force operator is defined by following an analogy with Newton's mechanics:

$$\hat{\vec{F}} = \frac{d\hat{\vec{P}}}{dt} = \quad \text{(Heisenberg picture)}$$

$$= -\frac{i}{\hbar}[\hat{\vec{P}}, \hat{H}] = -\frac{i}{\hbar}[\hat{\vec{P}}, \hat{H}_{AL}(\vec{R})]$$

(the other Hamilton eq. is obviously $\frac{d\vec{R}}{dt} = \vec{P}/m$)

* Since $\vec{P} = -i\hbar \vec{\nabla}_R$

$$\hat{\vec{F}} = -\vec{\nabla} \hat{H}_{AL}(\vec{R}) = +\vec{\nabla}\left[\hat{\vec{D}} \cdot \vec{E}_L(\vec{R},t)\right]$$

$$= \sum_{i=x,y,z} \hat{D}_i \, \vec{\nabla} E_L^{(i)}(\vec{R},t)$$

* (I) → <u>Note</u>: the semiclassical approach is just valid if the width of the atomic wavepacket $\ll \lambda_L$. Wavepacket spreading would eventually lead to a violation of this condition, but spontaneous emission kills center-of-mass coherence, relocalizing the atom. The semiclass. approx. is valid hence if the so-called recoil frequency $\omega_{rec} = \frac{\hbar k_L^2}{2m} \ll \Gamma$

We are interested in the averaged force

$\vec{F} = \langle \hat{\vec{F}} \rangle =$ ← since the spatial extension of the atom $\ll \lambda_L$ the $\langle \vec{E}_L(\vec{R}) \rangle \simeq \vec{E}_L(\langle \vec{R} \rangle)$, $\langle \vec{R} \rangle = \vec{R}(t)$

$$\cong \sum_{i=x,y,z} \langle \hat{D}_i \rangle \, \vec{\nabla} E_L^{(i)}(\vec{R}(t),t) \simeq \sum_{i=x,y,z} \langle \hat{D}_i \rangle_{st} \, \vec{\nabla} E_L^{(i)}(\vec{R}(t),t)$$

$\Uparrow$ <u>semiclassical approach</u> *(I) (see above)

(the atomic motion is classical but the internal dynamics is treated quantum-mechanically)

<u>Note</u>: From p. 2:
$\vec{E}_L = \sum_i \vec{e}_i E_{Li}^0(\vec{r}) \cos[\phi(\vec{r}) + \omega_L t]$
$\sum \langle \hat{D}_i \rangle_{st} \vec{\nabla} E_{Li} = 2\left[u_{st} \cos[\omega_L t + \Phi] - v_{st} \sin[\omega_L t + \Phi]\right] \sum_i d_i \vec{\nabla}\left[E_{Li}^0 \cos(\phi(\vec{r}) + \omega_L t)\right]$
but $\vec{d} \cdot \vec{E}_L^0 = -\hbar \Omega_1(\vec{r})$ → then we get the result for the force written below.

Hence (from page 7), and the definition $\Omega_1(\vec{R}) = -(\vec{d} \cdot \vec{e}_L(\vec{R}))\mathcal{E}_L(\vec{R})$

$$\vec{F}(\vec{R}) = -2\hbar\left[u_{st} \cos(\omega_L t + \Phi(\vec{R})) - v_{st} \sin(\omega_L t + \Phi(\vec{R}))\right] \cdot \vec{\nabla}\left[\Omega_1(\vec{R}) \cos(\omega_L t + \Phi(\vec{R}))\right]$$

$$= -2\hbar \Big\{ u_{st} \vec{\nabla}\Omega_1 \cos^2(\omega_L t + \Phi) + v_{st} \Omega_1 \vec{\nabla}\phi \sin^2(\omega_L t + \phi) + \left[-u_{st} \Omega_1 \vec{\nabla}\phi - v_{st} \vec{\nabla}\Omega_1\right] \sin(\omega_L t + \phi) \cos(\omega_L t + \phi) \Big\}$$

time-average over one period of the laser ($\cos^2 \to 1/2$, $\sin^2 \to 1/2$, $\cos \sin \to 0$)

$$\cong -2\hbar \left\{ u_{st} \vec{\nabla}\Omega_1 \frac{1}{2} + v_{st} \Omega_1 \vec{\nabla}\phi \frac{1}{2} \right\}$$

$$= -\hbar \Omega_1(\vec{R}) \left\{ u_{st} \frac{\vec{\nabla}\Omega_1}{\Omega_1} + v_{st} \vec{\nabla}\phi \right\}$$

$$= -\hbar \frac{s(\vec{R})}{1+s(\vec{R})} \left\{ \delta \frac{\vec{\nabla}\Omega_1(\vec{R})}{\Omega_1(\vec{R})} + \frac{\Gamma}{2} \vec{\nabla}\phi(\vec{R}) \right\}$$

* Hence the force exerted by the laser on the atom may be split into 2 parts: $\vec{F}(\vec{R}) = \vec{F}_R(\vec{R}) + \vec{F}_D(\vec{R})$

  * Radiation pressure: $\vec{F}_R(\vec{R}) = \dfrac{-\hbar s(\vec{R})}{1+s(\vec{R})}\,\dfrac{\Gamma}{2}\,\vec{\nabla}\phi(\vec{R})$

  * Dipolar force: $\vec{F}_D(\vec{R}) = \dfrac{-\hbar s(\vec{R})}{1+s(\vec{R})}\,\delta\,\dfrac{\vec{\nabla}\Omega_1(\vec{R})}{\Omega_1(\vec{R})}$

* Let's analyze in some detail each one of these forces.

• RADIATION PRESSURE

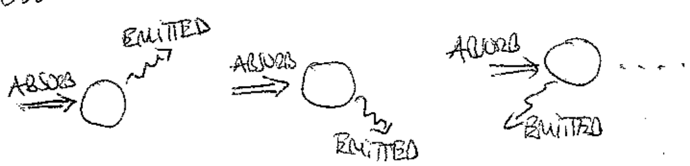
* This force depends on the phase gradient $\vec{\nabla}\phi(\vec{R})$.
* Let's consider the simplest case, i.e. a plane wave $\phi(\vec{R}) = -\vec{k}_L\cdot\vec{R}$

Then $\vec{F}_R(\vec{R}) = \dfrac{\hbar\vec{k}_L}{2}\,\Gamma\,\dfrac{s(\vec{R})}{1+s(\vec{R})}$

* We may write this force in the form $\vec{F}_R = \hbar k_L \Gamma \rho_{ee}^{(st)}$, which allows for a simple physical interpretation.

At every absorption (PHOTON) the atom absorbs a momentum $\hbar\vec{k}_L$. If the atom then emits via stimulated emission ( ) then due to momentum conservation the atom gets a recoil momentum $-\hbar\vec{k}_L$ (this is why the velocity $\hbar k_L/m$ is called recoil velocity) and hence nothing happens in a cycle of absorption/stimulated emission.

The situation is very different if after an absorption there's a spontaneous emission. In that case the emitted photon is emitted in an arbitrary direction. As a result the averaged effect of the spontaneous emission is zero, and only the net effect of the absorption is important.



* The typical time-scale of an absorption / spontaneous emission cycle is $(\Gamma \rho_{ee}^{st})^{-1}$, hence the resultant force (I recall $\text{Force} = \frac{\text{Momentum}}{\text{time}}$) is $\vec{F}_R = \hbar \vec{k}_L (\Gamma \rho_{ee}^{st})$, as above.

* If the atom is not at rest but in motion, all remains similar except that the atom due to the Doppler effect sees a laser frequency $\omega_L - \vec{k}_L \cdot \vec{v}$, where $\vec{v}$ is the atom velocity. This will be crucial when discussing the Doppler cooling

* For large laser intensity $\frac{s}{1+s} \simeq 1$, and $F_R \simeq \hbar k_L \frac{\Gamma}{2}$.
For a typical atom like Rubidium this represents an acceleration
$a = \hbar k_L \Gamma / 2m \simeq 10^5 \; m/s^2 \simeq 1.1 \times 10^4 g$ (!!)

• <u>DIPOLAR FORCE</u>

* This force depends on the intensity gradient

$$\vec{F}_D(\vec{R}) = - \frac{\hbar s(\vec{R})}{1+s(\vec{R})} \delta \frac{\vec{\nabla}\Omega_1(\vec{R})}{\Omega_1(\vec{R})}$$

(<u>Note</u>: this force can be also easily understood from photon absorption. If the intensity profile is not spatially homogeneous, this means that the laser is a superposition of more than one plane wave. The atom may absorb a photon $\vec{k}_1$ and emit (stimulated emission) a photon in $\vec{k}_2$. The atom hence gets a net kick $\hbar(\vec{k}_1 - \vec{k}_2)$ which is the basis of the dipole force.)

* Since $s(\vec{R}) = \frac{\Omega_1^2(\vec{R})/2}{\delta^2 + \Gamma^2/4} \longrightarrow \vec{\nabla} s = 2s \frac{\vec{\nabla}\Omega_1}{\Omega_1}$

Hence

$$\vec{F}_D(\vec{R}) = -\frac{\hbar\delta}{2} \frac{\vec{\nabla} s(\vec{R})}{1+s(\vec{R})} = -\vec{\nabla}\left[\frac{\hbar\delta}{2} \ln\left[1+s(\vec{R})\right]\right]$$

* Hence $\vec{F}_D(\vec{R})$ is a conservative force with an associated potential

$$\boxed{U_D(\vec{R}) = \frac{\hbar\delta}{2} \ln\left[1+s(\vec{R})\right]} \longrightarrow \text{Dipolar potential}$$

* The sign of the potential depends on the sign of δ
  * Red detuning ($\delta<0$) $\longrightarrow$ attraction to regions of higher intensity (one may use this to build an atom trap as we will mention later)
  * Blue detuning ($\delta>0$) $\longrightarrow$ repulsion from regions of higher intensity (one may use this to build an atom mirror as we will also see later).

* If $|\delta| >> \Gamma$, and weak intensity such that $s<<1$ then $\ln[1+s] \simeq s \simeq \frac{\Omega_1^2}{2\delta^2}$, hence

$$\boxed{U_D(\vec{R}) = \frac{\hbar\,\Omega_1^2(\vec{R})}{4\delta}}$$

* Look that if we increase both the laser intensity (i.e. Ω_1^2) and the detuning we may keep the dipolar potential constant, but the spontaneous emission (proportional to $\Gamma\rho_{ee}^{st} \overset{s<<1}{\simeq} \Gamma\frac{s}{2} \simeq \Gamma\frac{\Omega_1^2}{4\delta^2} = \left(\frac{\Gamma}{\delta}\right)\left(\frac{\Omega_1^2}{4\delta}\right)$) will be ~~smaller~~ less and less important.
* In this way for sufficiently large $\delta>>\Gamma$ we may forget the spontaneous emission, and realize a purely conservative potential.

(Note: For typical experiments the dipole potential $|U_0|/k_B \sim mK$, i.e. the atoms must be rather cool already. We will see the idea of Temperature and cooling later)

* We can easily recover the dipole force from the so-called <u>dressed-state picture</u>. In this way we will get an even clearer picture of $\vec{F}_D$.

Let's come back to the Hamiltonian

$$\hat{H} = -\hbar\delta\, |e\rangle\langle e| + \frac{\hbar}{2}\Omega_1(\vec{R})\left\{ |e\rangle\langle g|\, e^{-i\phi(\vec{R})} + |g\rangle\langle e|\, e^{i\phi(\vec{R})} \right\}$$

(where we have moved to the rotating frame with the laser frequency ω_L. This is why (when compared to page 3) we have the detuning δ, and all $e^{\pm i\omega_L t}$ disappear).

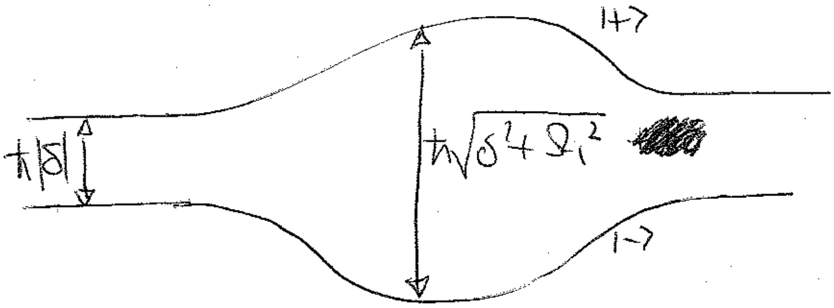
* In a matrix form

$$\hat{H} = \hbar \begin{pmatrix} -\delta & \frac{\Omega_1(\vec{R})}{2} e^{i\Phi(\vec{R})} \\ \frac{\Omega_1(\vec{R})}{2} e^{-i\Phi(\vec{R})} & 0 \end{pmatrix}$$

which has eigenvalues

$$E_\pm(\vec{R}) = -\hbar\frac{\delta}{2} \pm \frac{\hbar}{2}\sqrt{\delta^2+\Omega_1^2}$$

The corresponding eigenstates $|\pm\rangle$ are the <u>so-called dressed states</u>



(For $\delta>0$, $|\pm\rangle = |g,e\rangle$ when $\Omega_1=0$
$\delta<0$, $|\pm\rangle = |e,g\rangle$ when $\Omega_1=0$)

$$S \simeq \frac{\Omega_1^2}{2\delta^2}; \quad \frac{\vec{\nabla}S}{2S} = \frac{\vec{\nabla}\Omega_1}{\Omega_1}$$

Note that for $\delta \gg \Gamma$, $S \ll 1$,

$$-\vec{\nabla}E_\pm(\vec{R}) = \mp\frac{\hbar}{2}\frac{\Omega_1\vec{\nabla}\Omega_1}{\sqrt{\delta^2+\Omega_1^2}} = \mp\frac{\hbar\Omega_1^2}{2\sqrt{\delta^2+\Omega_1^2}}\frac{\vec{\nabla}\Omega_1}{\Omega_1} =$$

$$= \frac{-\hbar\delta}{\sqrt{1+2S}}\frac{\vec{\nabla}S}{2} \underset{S\ll 1}{\simeq} \frac{-\hbar\delta}{2}\frac{\vec{\nabla}S}{1+S} \longrightarrow \text{i.e. the dipolar force.}$$

* Note that for $|\Omega_1| \ll \delta$ and $\delta > 0$, the ground state $|g\rangle$ evolves inside the laser region ($\Omega_1(\vec{R}) \neq 0$) into $|+\rangle$ and hence feels a repulsive potential.
If $\delta < 0$ (red detuning) the ground state $|g\rangle$ evolves into $|-\rangle$ and feels an attractive potential.

* By playing with the density profile of the laser intensity one may tailor different types of potentials for the atoms. For example, by employing 2 counterpropagating lasers, we may form a standing wave. Hence $\Omega_1^2(\vec{R}) = \Omega_0^2 \cos^2 QX$
and for $|\Omega_1| \ll \delta$, $\delta > 0$ $\rightarrow$ $E_\pm(\vec{R}) \cong V_0 \cos^2 QX$

We get a periodic potential for the atoms, similar to that of electrons in a crystal. This "light crystal" receives the name of optical lattice. We will come back to the physics of cold atoms in optical lattices later in this set of lectures.