Lecture on the Theory of cold gases (SS 09)

1.- Atom-light interaction

- The two level atom. Optical Bloch equations
- Force exerted by the laser on the atom
- Radiation pressure
- Dipole force

2.- Laser cooling

- The idea of temperature
- Doppler cooling
- Sysiphus cooling
- Subrecoil cooling: VSCPT
- The reabsorption problem

3.- Trapping of neutral atoms

- Magneto-optical trap
- Magnetic traps
- Optical traps

4.- Evaporative cooling

5.- Atom Optics

- Atomic mirrors
- Atomic diffraction
  - Raman-Nath regime
  - Bragg regime
- Atom interferometry
  - Physical implentations
  - Semiclassical analysis of the phases. Sagnac effect.

6.- Ideal degenerate quantum gases

Basic ideas of quantum statistical mechanics

Ideal Bose gas: the Bose-Einstein condensation

- Critical temperature
- Low-dimensional systems. Density of states
- BEC in harmonic traps
- Off-diagonal long-range order

Ideal Fermi gas

- Degenerate Fermi distribution: Fermi energy, Fermi surface
- Zero point pressure

7.- Interparticle interactions

Basics of scattering theory: the s-wave scattering length

Brief introduction to the idea of Feshbach resonances

8.- The weakly-interacting Bose gas I: Basics

Homogeneous Bose gas. Bogoliubov approximation.

Bogoliubov spectrum.

Non-homogeneous Bose gas at zero temperature.

- Condensate wavefunction
- The Gross-Pitaevskii equation
- BEC in a box potential. Idea of healing length

Hydrodynamic equations. The Thomas-Fermi limit

BEC in a harmonic trap

BEC with attractive interactions

9.- The weakly-interacting Bose gas II: Superfluidity

Landau criterion

Two-fluid model. The normal component

Relation between BEC and superfluidity

Quantized vortices

The vortex core. Critical angular velocity

Vortices in harmonic traps

10.- The weakly-interacting Bose gas III: Excitations of a BEC

Bogoliubov-de Gennes equations

General features of the Bogoliubov-de Gennes equations

Quantization of the elementary excitations

Collective oscillations of a harmonically-trapped BEC. Thomas-Fermi limit

Large-amplitude excitations. Self-similar solutions. Expansion of a BEC.

Solitons: Dark solitons, bright solitons, soliton instability

10.- Coherence in BECs and Josephson effect

Double-slit experiment. First-order coherence

Interference between two condensates. Fock versus coherent states

Josephson effect: mean-field analysis

Josephson effect: beyond mean-field. Number-squeezed states

Josephson effect: Bose-Hubbard-Hamiltonian formalism

11.- Bose gases in optical lattices

Band structure. Bloch and Wannier functions.

Tight-binding regime. Dispersion law. Effective mass

Bloch oscillations

Gap solitons

BEC in an optical lattice: interference fringes in time-of-flight. Coherence

Discrete nonlinear Schrödinger equation

Josephson-junction array of BECs

Bose gases in a lattice beyond mean field

Bose-Hubbard Hamiltonian

- Superfluid to Mott-insulator transition
- The Mott-lobe structure. The Mott gap
- Wedding cake structure in harmonic potentials
- Experimental observation of the Mott-insulator

11.- Ultracold Fermi gases I: Basics

Ideal Fermi gas in a harmonic trap

Interacting Fermi gases

BCS Transition

12.- Ultracold Fermi gases II: BCS-BEC crossover and imbalanced mixtures

Two-body collisions in more detail

- Brief review on scattering
- Unitarity
- a>0 : existence of bound state (dimer)
- Pseudopotential
- Feshbach resonances
- BCS-BEC crossover

BEC of dimers

- Dimer-dimer interactions
- BEC of dimers

Unitarity limit

- Universality
- Trapped Fermi gas at unitarity

Superfluidity at the BEC-BCS crossover

Imbalanced Fermi Mixtures

- Clogston-Chandrasekhar limit
- Fulde-Ferrel-Larkin-Ovchinnikov phases
- Imbalanced Fermi gases at unitarity: homogeneous and trapped cases