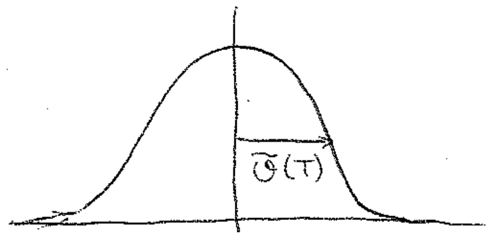
• Laser Cooling

* In this lecture we will learn how one can use the mechanical effects of light on atoms to cool an atomic sample. We will discuss different laser cooling methods (in different degree of detail, and in general rather briefly). The goal here is to show how one can use different effects related with the atom-laser interaction (ranging from Doppler effect to dark-states) to induce cooling.

* But first of all let's recall the idea of temperature (T). Let's consider an atomic sample in free space (no potential energy). In thermal equilibrium the velocity distribution of the sample follows a Maxwell-Boltzmann distribution, i.e. a Gaussian function of the form:

$$f(v) = \frac{1}{\sqrt{2\pi}\,\tilde{v}(T)} e^{-v^2/2\tilde{v}(T)^2}$$



where $\tilde{v} \equiv \sqrt{k_B T/m}$

Therefore the temperature T of the sample can be directly read out from the velocity spreading of the system.

* Therefore we may understand a cooling mechanism as a process that reduces the velocity spreading of the atoms.

* In the following we will introduce the ideas of Doppler cooling, Sisyphus cooling and subrecoil cooling. Our discussion will be necessarily very short. For more details you may have a look to the book "Laser cooling and trapping" of H. J. Metcalf and P. van der Straten.

* Doppler cooling

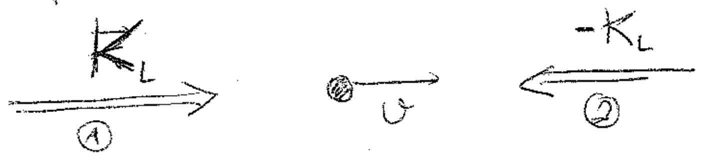
* The intuitive idea of Doppler cooling is very simple to understand. Suppose that you have an atom moving with velocity v along the x axis. Suppose that the atom is affected by 2 counterpropagating lasers in the direction x and $-x$, respectively (Note: these lasers must be weak enough to consider the mechanical forces of each laser independently; I come back to this point later)

* Remember the Doppler effect. An atom moving with velocity $\vec{v}$ sees an effective frequency

$$\omega_L' = \omega_L \underbrace{- \vec{K}_L \cdot \vec{v}}_{\text{Doppler shift}}$$

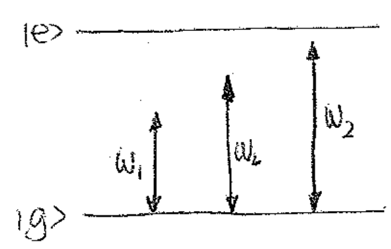
(Note: This effect is important in broad ranges of physics, from astronomy to the velocity-meters used by the police.)

· Let's see now what happens with the atom affected by 2 counterpropagating lasers



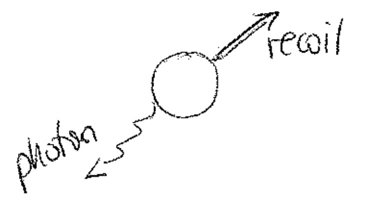
Then
$$\omega_1 = \omega_L - K_L v$$
$$\omega_2 = \omega_L + K_L v$$

* Let's assume red detuning $\delta = \omega_L - \omega_A < 0$



therefore the atoms absorbs with higher probability the photon ②, i.e. the counterpropagating photon. The atomic velocity is then reduced by $\vec{v} \longrightarrow \vec{v} - \hbar K_L/m \, \vec{e}_x$

* A subsequent spontaneous emission is emitted in a random direction, inducing a recoil (due to momentum conservation) in a random direction



$$\vec{v} \longrightarrow \vec{v} + \hbar \vec{k}_{rec}/m$$

* Then the combination of absorption and (randomly-oriented) spontaneous emission changes the velocity in the form

$$\vec{v} \longrightarrow \vec{v} - \frac{\hbar}{m}(k_L \vec{e}_x - \vec{k}_{rec})$$

* After many cycles of absorbtion and spontaneous emission there's a net absorbed momentum in the counterpropagating direction, whereas in average the spontaneous emission effect is canceled.

Hence the atoms experience a net force opposed to the velocity, i.e. a friction. It's clear that this will reduce the velocity spreading of the sample and hence that this is a cooling mechanism. This mechanism is the so-called Doppler cooling and it was proposed in 1975 by Hänsch and Schawlow.

* Now that we have understood the intuitive physical picture of this friction force, let's have a look to the details of this friction force.

* Let's consider first the case of a plane running wave. The dipole force is in that case zero because Ω_1 is not space dependent. We have then only the radiation pressure. As mentioned already in p. 10 the only difference when the atom moves is that ~~one~~ must consider the Doppler effect. Then

$$\delta \longrightarrow \delta + k_L v$$

← we consider first the laser as counterpropagating against the atom velocity (i.e. the laser 2)

Hence: $$\vec{F}_R^{(2)}(v) = \frac{-\hbar k_L \Gamma}{2}\left[\frac{\Omega_1^2/2}{\left(\frac{\Omega_1^2}{2}+\frac{\Gamma^2}{4}\right)+(\delta+k_L v)^2}\right] \underset{\text{for small velocities } (k_L v \ll \Gamma)}{=}$$

$$\simeq \frac{-\hbar k_L \Gamma}{2}\left[\frac{\Omega_1^2}{\frac{\Omega_1^2}{2}+\frac{\Gamma^2}{4}+\delta^2}\right]\left[1 - \frac{2k_L v\delta}{\frac{\Omega_1^2}{2}+\frac{\Gamma^2}{4}+\delta^2}\right]$$

$$= F_R(v=0) + \hbar k_L^2 \Gamma \left[\frac{\delta\,\frac{\Omega_1^2}{2}}{\left(\delta^2+\frac{\Gamma^2}{4}+\frac{\Omega^2}{2}\right)^2}\right] v$$

$$= F_R(v=0) + \underbrace{\eta v}_{\text{friction force if } \eta<0, \text{ i.e. if } \delta<0 \text{ (red detuning)}}$$

(as we mentioned already before)

* For the laser co-propagating with the atom

$$F_R^{(1)}(v) = \frac{\hbar k_L \Gamma}{2}\left[\frac{\Omega_1^2/2}{\left(\frac{\Omega_1^2}{2}+\frac{\Gamma^2}{4}\right)+(\delta-k_L v)^2}\right] = -F_R(v=0)+\eta v$$

* For sufficiently weak lasers we may consider that the resulting force is the sum of the forces provided by each running wave independently. Hence

$$\vec{F}(v) = F_R^{(1)}(v) + F_R^{(2)}(v) = 2\eta v$$

This provides hence a net friction force with friction coefficient

$$2\eta = \hbar k_L^2 \Gamma \left(\frac{\delta\,\Omega_1^2}{\left(\delta^2+\frac{\Gamma^2}{4}+\frac{\Omega^2}{2}\right)^2}\right)$$

* The approximation of assuming the net force as the sum of the forces associated to each running-wave independently is valid only if the lasers are weak enough $\Omega_1 \ll \Gamma$, such that each running wave is far from saturation (Note: if this is not the case the mechanism gets more complicated and one can even reach a situation where damping requires blue detuning, i.e. $\delta > 0$)

* Up to now we have just considered cooling in one dimension. But the mechanism can be easily generalized to 2D and 3D by considering counterpropagating beams along x, y and z.
* The friction experienced by the atom is actually rather large, i.e. the light acts as a very viscous medium. Let's put some numbers:

$$m\dot{v} = -|\eta| v \rightarrow v = e^{-|\eta| t/m} v_0 = e^{-t/\tau} v_0$$

let $\Omega \ll \Gamma$, $S_0 = \frac{\Omega_1^2/2}{\delta^2 + \Gamma^2/4} \simeq 1/10$, and $\delta = -\Gamma/2$

Then $\tau \simeq 100\,\mu s$

The velocity hence is very quickly reduced. Such an extremely viscous medium has been (quite properly) called

<u>optical molasses</u>.

* With the Doppler cooling technique one cannot reach arbitrarily low temperatures. The reason lies in the fact that in addition to the mean value of the force we must consider as well the fluctuations of the atomic momentum due both to the spontaneous emission process and also to the randomness of light absorption. The latter is the result of the fact that the direction from which a given photon is absorbed is uncorrelated with the others. ~~The contribution to the~~ These fluctuations lead to a growth of the variance of the momentum with time $\Delta P^2 = 2 D_P t$ where D_P is the so-called <u>diffusion coefficient</u>.

* The contribution to D_p coming from the randomness of spontaneous emission is easy to estimate. Each spontaneous emission event is accompanied by a momentum kick $\hbar k_L$ of the atom in a random direction. The number of spontaneous emissions per interval Δt is $\Delta n = \Gamma \rho_{ee}^{st} \Delta t$ where (from p. 6) $\rho_{ee}^{st} = \frac{1}{2}\left(\frac{s}{1+s}\right)$.

The resulting momentum spread is hence

$$\Delta p^2 = (\hbar k_L)^2 \Delta n = 2\left[\frac{\hbar^2 k_L^2 \Gamma}{2}\left(\frac{s}{1+s}\right)\right]\Delta t$$

The diffusion coefficient associated with the randomness of spontaneous emission is hence

$$D_p^{sp} = \frac{\hbar^2 k_L^2}{2}\Gamma\left(\frac{s}{1+s}\right) \underset{s\ll 1}{\simeq} \frac{\hbar^2 k_L^2 \Gamma}{2} s$$

* The diffusion coming from the randomness of light absorption is more difficult to obtain, ~~and~~ we will not do it here. The result is actually that $D_p^{abs} \simeq \frac{\hbar^2 k_L^2 \Gamma}{2} s$ also.

Hence the total diffusion in 1D molasses is

$$D_p \simeq \hbar^2 k_L^2 \Gamma s$$

In 3D molasses it is 3 times as large $D_p \simeq 3\hbar^2 k_L^2 \Gamma s$

* Then one one side we have cooling $\left(\frac{dp^2}{dt} = 2p\frac{dp}{dt} = 2pm\frac{dv}{dt} = -2pm|\eta| v = -2|\eta| p^2\right)$ and on the other hand we have heating $\left(\frac{dp^2}{dt} = 2D_p\right)$

Hence $$\frac{dp^2}{dt} = \underbrace{-2|\eta| p^2}_{\text{cooling}} + \underbrace{2D_p}_{\text{heating}}$$

* The equilibrium occurs when heating and cooling compensate

$$\frac{dP^2}{dt} = 0 \longrightarrow \left(\frac{P^2}{2m}\right)_{eq} = \frac{D_p}{2m|\eta|}$$

* In a 3D gas we can associate (as I discussed in p. 14) a temperature to a given P^2, such that

$$\frac{P^2}{2m} = \frac{3}{2} k_B T \qquad \text{(Maxwell-Boltzmann)}$$

Hence we get an equilibrium temperature

$$k_B T = \frac{D_p}{3m|\eta|} = \frac{\hbar\Gamma}{4}\left[\frac{2|\delta|}{\Gamma} + \frac{\Gamma}{2|\delta|}\right]$$

The minimal temperature achievable is that when $\delta = -\Gamma/2$ and it is:

$$\boxed{k_B T_{DOPPLER} = \frac{\hbar\Gamma}{2}} \longrightarrow \text{Doppler temperature}$$

* The Doppler temperature sets the limit for Doppler cooling.

For e.g. Rubidium $T_{DOPPLER} \simeq 140\,\mu K$

Sodium $\simeq 240\,\mu K$

Hydrogen $\simeq 2.4\, mK$

* We will see now that this is not at all the maximum we can do with laser cooling!

* <u>Sisyphus cooling</u>

* When Phillips' group performed experiments on Sodium (in 1988) in optical molasses, they expected to find a minimal temperature of $T_{DOPPLER}$ ($\simeq 240\mu K$ for Sodium). What they found was a much lower temperature (below $50\mu K$), and in addition not for $\delta = -\Gamma/2$ but $|\delta|$ of several times Γ. What was happening there?

* The answer to this question is the so-called Sisyphus cooling (which was explained by Dalibard and Cohen-Tannoudji in 1989). This cooling mechanism concerns atoms with a degenerated ground state (i.e. $J_g \neq 0$). Here we will consider the simplest example, namely the one-dimensional motion of atoms with a degenerated ground state with $J_g = 1/2$ (i.e. 2 state $|g_{\pm 1/2}\rangle$) and an excited state with $J_e = 3/2$ (i.e. 4 states $|e_{\pm 3/2}\rangle, |e_{\pm 1/2}\rangle$)

* As before we assume that laser light is exerted on the atoms. The laser field is composed by a superposition of 2 plane waves counterpropagating along the z axis. They have orthogonal polarizations (respectively along $\vec{e}_x$ and $\vec{e}_y$) and possess the same amplitude $\mathcal{E}_0$:

$$\vec{E}_1 = \mathcal{E}_0 \vec{e}_x \cos(k_L z - \omega t)$$

(Note: the phases between the lasers are chosen for simplicity of the calculation.)

$$\vec{E}_2 = -\mathcal{E}_0 \vec{e}_y \sin(k_L z - \omega t)$$

The total field $\vec{E} = \vec{E}_1 + \vec{E}_2$ can be re-written in the form

$$\vec{E}(z,t) = \vec{E}^{(+)}(z)\, e^{-i\omega t} + \text{c.c.}$$

where $\vec{E}^{(+)}(z) = \mathcal{E}_0 \vec{\epsilon}(z)/\sqrt{2}$

and $\vec{\epsilon}(z) = \cos kz\, \vec{\epsilon}_- + i \sin kz\, \vec{\epsilon}_+$

$$\vec{\epsilon}_+ \equiv \frac{1}{\sqrt{2}}(\vec{e}_x + i\vec{e}_y) \Rightarrow \sigma_+\text{-polarisation}$$

$$\vec{\epsilon}_- \equiv \frac{1}{\sqrt{2}}(\vec{e}_x - i\vec{e}_y) \Rightarrow \sigma_-\text{-polarisation}$$

* This is indeed an interesting situation. The polarization varies along z periodically
    * For $z = 0, \frac{\lambda}{2}, \lambda \ldots$ it's polarized σ_-
    * For $z = \frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots$ it's polarized σ_+

and in between it has elliptical polarization (being linear (π-polarized) for $z = \lambda/8, 3\lambda/8, \ldots$).

* Let's see now how this laser field affects the atoms. As we already discussed in p. 2 the atom-laser interaction is of the form

$$\hat{H}_{AL} = -\hat{\vec{D}}^{+} \cdot \vec{E}(z,t)\, e^{-i\omega t} + h.c. \qquad \text{(electric dipole approximation)}$$

As mentioned above $\vec{E}(z,t)$ is a spatially-dependent combination of $\vec{\epsilon}_{q=\pm}$ polarization.

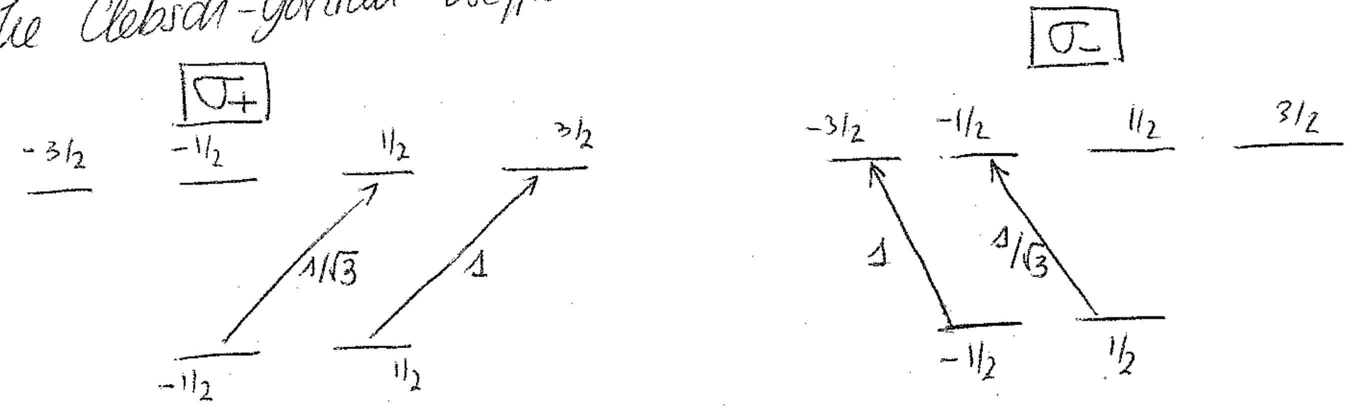
[Remember that $\vec{D}^{+} = \vec{d}\,|e\rangle\langle g| \Rightarrow$ where $\vec{d} = -e\langle e|\vec{r}|g\rangle$ (p. 2)]

Then $$\vec{\epsilon}_q \cdot \vec{D}^{+}|m_g\rangle = \underbrace{\langle J_g\, 1\, m_g\, q\,|\,J_e\, m_e\rangle}_{\text{This is the Clebsch-Gordan coefficient.}}\,|m_e\rangle$$

* Associated with each transition $|m_g\rangle \to |m_e\rangle$ and a polarization $\vec{\epsilon}_q$ there's a Clebsch-Gordan coefficient. I recall you an important property of the Clebsch-Gordan coefficients:
    * $m_g + q = m_e \longrightarrow$ i.e.
        * $\sigma_+ : q = +1 \longrightarrow m_e = m_g + 1$
        * $\sigma_- : q = -1 \longrightarrow m_e = m_g - 1$

    (Selection rules)

The Clebsch-Gordan coefficients aren't the same for all transitions:



Then:

$$\hat{H}_{AL} = \frac{\hbar\Omega_0\sqrt{2}}{2}\cos(kz)\left[|e_{-3/2}\rangle\langle g_{-1/2}| + \frac{1}{\sqrt{3}}|e_{-1/2}\rangle\langle g_{1/2}|\right]e^{-i\omega t}$$

$$-\frac{i\hbar\Omega_0\sqrt{2}}{2}\sin(kz)\left[|e_{+3/2}\rangle\langle g_{1/2}| + \frac{1}{\sqrt{3}}|e_{+1/2}\rangle\langle g_{-1/2}|\right]e^{-i\omega t}$$

$$+ \text{H.C.}$$

where Ω is the Rabi frequency associated to each wave for the transition with Clebsch-Gordan 1:

$$\Omega = -\langle e_{3/2}|\hat{D}\cdot\vec{\epsilon}_+|g_{1/2}\rangle\frac{\mathcal{E}_0}{\hbar}$$

* The Rabi frequency associated to the $\sigma_\pm$ wave is then

$$\left.\begin{array}{l}\Omega_+(z) = \sqrt{2}\,\Omega_0\sin kz\\ \Omega_-(z) = \sqrt{2}\,\Omega_0\cos kz\end{array}\right\} \longrightarrow \Omega_\pm^2(z) = \Omega_0^2\{1\mp\cos(2kz)\}$$

* We are interested now in the dipolar potential (U_D) that these lasers exert on the ground states $|g_{\pm1/2}\rangle$. Remember our discussion of p. 11. For $|\delta|\gg\Gamma$ and $s\ll1$ the dipole potential associated with a Rabi freq. Ω_1 was $U_D(\vec{R}) = \frac{\hbar}{4\delta}\Omega_1^2(\vec{R})$.

* The dipole potential for the state $|g_{+1/2}\rangle$ is the result of both the coupling to $|e_{-1/2}\rangle$ (given by $\frac{\hbar}{2}\Omega_-(z)\frac{1}{\sqrt{3}}|e_{-1/2}\rangle\langle g_{1/2}|$) and the coupling to $|e_{3/2}\rangle$ (given by $\frac{\hbar}{2}(-i\Omega_+(z))\,|e_{3/2}\rangle\langle g_{1/2}|$)

Clebsch-Gordan coefficient (in the square)

Hence:

$$U_{1/2}(z) = \frac{\hbar}{4\delta}\left\{\Omega_+^2(z) + \frac{1}{3}\Omega_-^2(z)\right\}$$

For the same reason

$$U_{-1/2}(z) = \frac{\hbar}{4\delta}\left\{\Omega_-^2(z) + \frac{1}{3}\Omega_+^2(z)\right\}$$

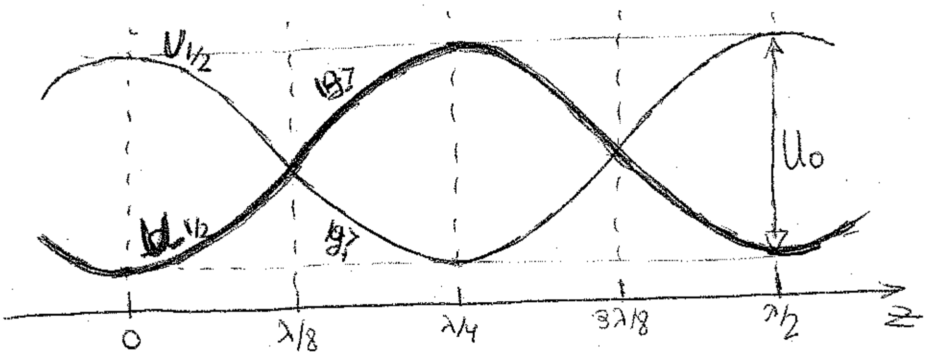
* Using $\Omega_\pm^2(z) = \Omega_0^2 \{1 \mp \cos 2kz\}$

and defining $U_0 = \hbar\Omega_0^2/3|\delta|$ (we assume $\delta<0$)

we obtain:

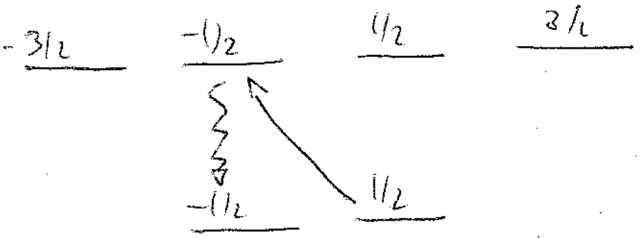
$$\boxed{U_{\pm 1/2}(z) = -\frac{U_0}{2}\left(2 \mp \cos(2kz)\right)}$$

* Therefore the dipole potential for $+1/2$ and $-1/2$ varies in space in a different way, i.e. the two dressed states $|g\pm\rangle$ behave as this:



* This is the first ingredient of the Sysiphus cooling. The second ingredient is related with spontaneous emission.

Let's consider at the point z an atom in the state $|g+1/2\rangle$. This atom may be transferred to the state $|g-1/2\rangle$ by absorbing a σ_- photon, and then emitting a linearly polarized photon spontaneously

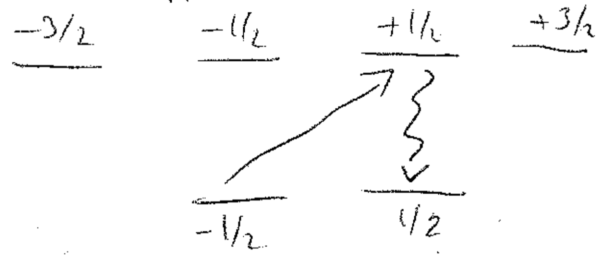


The transference rate from $+1/2$ to $-1/2$ is hence proportional to $|\Omega_-(z)|^2 \propto \cos^2(kz)$, which is hence maximal for $z=0, \lambda/2 \cdots$ where the light is purely σ_-, and it's zero for $z=\lambda/4, 3\lambda/4$ where the light is purely σ_+. One may actually write the transfer rate $\gamma_{+\to-}(z) = \gamma\cos^2(kz)$

where $\gamma = 2\Gamma \frac{s_0}{9}$, $s_0 \simeq \frac{\Omega_0^2}{2\delta^2}$

(Note: we don't enter into all details here, more details can be found in e.g. the book of P. Meystre "Atom Optics".)

* The opposite is true for the transfer from $-1/2$ into $+1/2$ which occurs via the absorption of a σ_+ photon. Hence

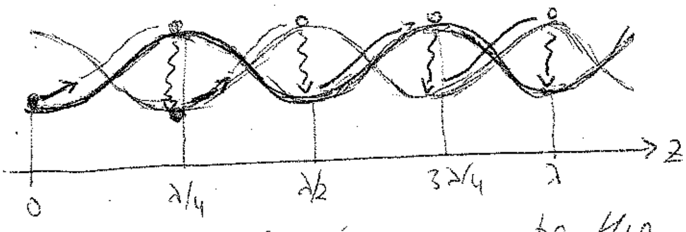


$$\gamma_{-\to+}(z) = \gamma \sin^2(kz)$$

and the transfer rate is zero for $z = 0, \lambda/2 \ldots$ and maximal for $z = \lambda/4, 3\lambda/4 \ldots$

* We have now the 2^{nd} ingredient of the Sisyphus cooling. The transference rates between the ground state levels are also space dependent, and namely with the same periodicity as the dipole potential.

* Now that we have the ingredients, let's see how the cooling mechanism works. Let's consider an atom in $z=0$ in $|g_{-1/2}\rangle$ which is moving to the right. The atom is at the bottom of a valley, and has to climb the hill to move to the right. When climbing the hill and specially at the top it has a maximal probability to decay to $|g_{1/2}\rangle$. If it does so, then the atom finds itself again in the valley,



and in order to move to the right it must climb a hill again, and at $\lambda/2$ it has a maximal probability to decay into $|g_{-1/2}\rangle$, and so on.

* As a result of climbing all these hills, the kinetic energy of the atom is reduced. The energy lost per cycle: climbing + transfer is $\frac{dE_c}{dt} \propto -\gamma U_0$

* One could intuitively understand that the cooling is stopped when the atom lacks the energy to escape from a valley. This occurs roughly when the temperature reaches a value

$$k_B T \sim U_0 \propto \frac{\hbar \Omega_0^2}{|\delta|}$$

(Note: a rigorous calculation provides the result $\frac{1}{8}\frac{\hbar\Omega_0^2}{|\delta|}$)

* It's clear that the temperature that one may reach in this way may be much lower than the $T_{Doppler}$ (which was exclusively fixed by Γ).

* The previous discussion might convey the wrong impression that arbitrarily low temperatures can be achieved simply by reducing the depth of the dipole potential. This is however not so. In the previous discussion we actually forgot the recoil provided to the atom during the absorbtion + spontaneous emission cycle. (Note: we implicitely assumed that the energy lost while climbing the potential barrier was large compared to the recoil energy). This approximation obviously fails when $U_0 \approx \frac{\hbar^2 k_L^2}{2m}$ (≡ This is the so called recoil energy (E_{rec}))

* Consequently the actual limit of Sisyphus cooling is of several tens of recoil temperatures $T_{rec} = E_{rec}/k_B$.

* As long as the spontaneous emission is present one cannot beat the limit $\approx T_{rec}$. (Note: for Cesium e.g. (with $M = 132$ u.a.m.) one has $T_{rec} \sim 1\,\mu K$)

But we will see now that one can engineer other techniques to even beat this limit!

* Subrecoil cooling : VSCPT

• From our previous discussion we have seen that spontaneous emission is both necessary (it's the way to dissipate energy) and bad (due to the recoil) for laser cooling.

• The best would be a method to

- on one side use the dissipative character of the spontaneous emission,
- on the other side switch-off spontaneous emission once cool.

In order to do that we will use the key idea of <u>dark-states</u>, which we will introduce in this section. We will discuss here in particular the idea of Velocity Selective Coherent Population Trapping (or shorter <u>VSCPT</u>).

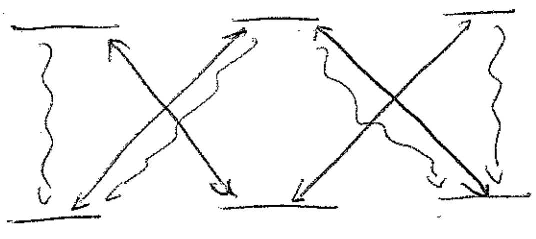
* As for the Sisyphus cooling we consider degenerated ground and excited states. This time $J_g = 1$ and $J_e = 1$. We consider two counterpropagating lasers with σ_+ and σ_- polarization:

$$\vec{E}(z,t) = \frac{1}{2}\mathcal{E}\,\vec{\epsilon}(z)\,e^{-i\omega t} + c.c.$$

where the field polarization is now $\quad \xrightarrow{\sigma_+} \quad \xleftarrow{\sigma_-}$

$$\vec{\epsilon}(z) = e^{ikz}\vec{\epsilon}_+ + e^{-ikz}\vec{\epsilon}_-$$

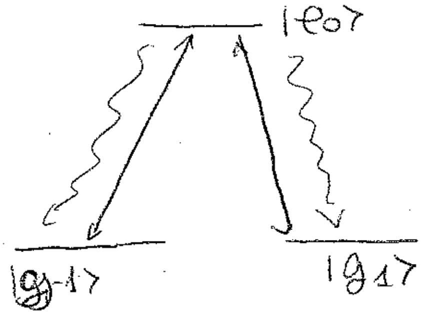
For such a laser configuration we may split the $J_g = 1 \leftrightarrow J_e = 1$ transitions into 2 uncoupled systems



a V system and a Λ system.

The key point here is that the $|m_g=0\rangle \leftrightarrow |m_e=0\rangle$ transition is forbidden by selection rules (there's no spontaneous decay from $|m_e=0\rangle$ into $|m_g=0\rangle$. Hence population can be pumped from the V-system into the Λ-system but not viceversa. As a consequence we may safely forget the V-part and focus only in the Λ system.

* So we have effectively a 3-level system. The atom-laser Hamiltonian is then



$$\hat{H}_{AL} = \frac{\hbar\Omega}{2\sqrt{2}}\left[|e_0\rangle\langle g_1|e^{-ikz} - |e_0\rangle\langle g_{-1}|e^{ikz}\right]e^{-i\omega t} + H.c.$$

where $\Omega = -d\mathcal{E}/\hbar$, and the extra $1/\sqrt{2}$ and the minus sign come from the Clebsch-Gordan coeffs.

* In the following it will be very useful to make the discussion in the momentum representation

$$e^{\pm ikz} = \int dp\ |p\rangle\langle p \mp \hbar k|$$

(this is nothing else than saying that the photon absorbed gives a momentum to the atom)

Then:

$$\hat{H}_{AL} = \frac{\hbar\Omega}{2\sqrt{2}}\int dp\left[|e_0, p\rangle\langle g_1, p+\hbar k| - |e_0, p\rangle\langle g_{-1}, p-\hbar k|\right]e^{-i\omega t} + H.c.$$

So now we have pretty clear both the internal-state physics and the mechanical effects of the light in the center of mass.

* It's clear from the form of $\hat{H}_{AL}$ that processes of absorption and spontaneous emission maintain the manifolds

$$\mathcal{M}(p) = \{|e_0, p\rangle, |g_1, p+\hbar k\rangle, |g_{-1}, p-\hbar k\rangle\}$$

closed. Of course spontaneous emission can couple between different manifold. This is of course crucial and we will come back to this point in a moment.

* Let's now focus in one of these manifolds, $\mathcal{M}(p)$.

$$H_{AL}|e_0, p\rangle = \frac{\hbar\Omega}{2\sqrt{2}}\left[|g_1, p+\hbar k\rangle - |g_{-1}, p-\hbar k\rangle\right]e^{-i\omega t}$$

* Clearly the laser couples just $|e_0, p\rangle$ with a particular coherent superposition

$$|\psi_c(p)\rangle = \frac{1}{\sqrt{2}}\left[|g_1, p+\hbar k\rangle - |g_{-1}, p-\hbar k\rangle\right]$$

such that

$$H_{AL}|e_0,p\rangle = \frac{\hbar\Omega}{2}|\psi_c(p)\rangle e^{-i\omega t}$$

$$H_{AL}|\psi_c(p)\rangle = \frac{\hbar\Omega}{2}|e_0,p\rangle e^{i\omega t}$$

but $H_{AL}|\psi_{nc}(p)\rangle = 0$ with $|\psi_{nc}(p)\rangle = \frac{1}{\sqrt{2}}\left[|g_1, p+\hbar k\rangle + |g_{-1}, p-\hbar k\rangle\right]$

The state $|\psi_{nc}(p)\rangle$ is not coupled to the excited state by the laser field, being an example of dark state.
(Note: The dark state is induced by quantum interference. The laser is there but quantum interference inhibits absorbtion)

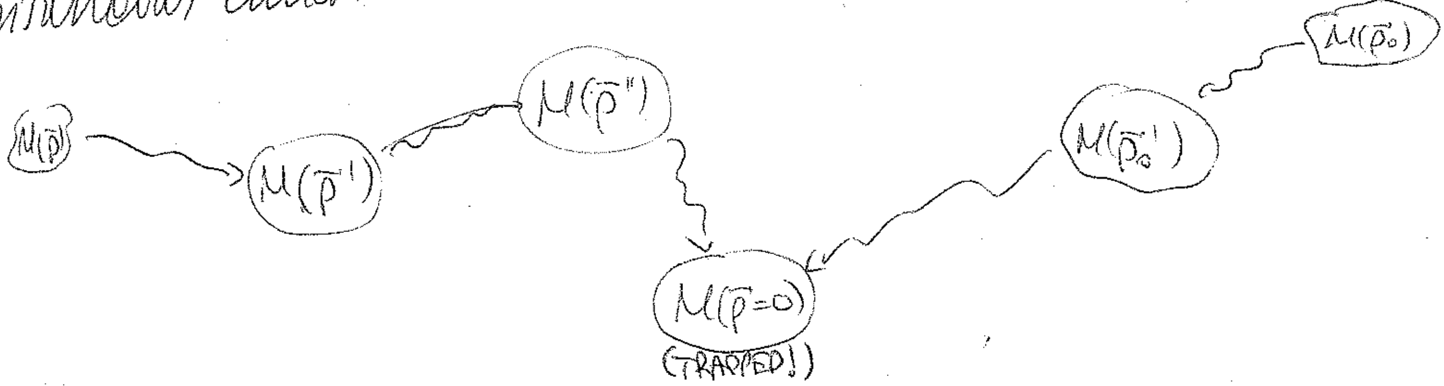
* If spontaneous emission results in a transition from $|e_0,p\rangle$ into $|\psi_{nc}(p)\rangle$ (and this is possible because spontaneous emission is not affected by the quantum interference discussed above), it would appear that the atom pumped into $|\psi_{nc}(p)\rangle$ would remain there forever (this effect is called population trapping).

* However this is not true, and this point is actually crucial to understand VSCPT. The reason is that we forgot to consider the kinetic energy $\hat{P}^2/2M$.

The states $|e_0,p\rangle$, $|g_1,p+\hbar k\rangle$ and $|g_{-1},p-\hbar k\rangle$ are eigenstates of $\hat{P}^2/2m$, but not $|\psi_c(p)\rangle$ and $|\psi_{nc}(p)\rangle$. There's a coupling $\langle\psi_c(p)|\frac{p^2}{2m}|\psi_{nc}(p)\rangle = \frac{\hbar k p}{M}$

and hence the atom can scape from $|\Psi_{nc}(p)\rangle$, unless $p=0$ (and this is a crucial point!). Only an atom falling into $|\Psi_{nc}(p=0)\rangle$ will be fully dark, and hence fully decoupled and not seeing any extra spontaneous emission.

* And now we can easily understand how VSCPT works. Consider an atom initially in the manifold $M(\vec{p})$. An spontaneous emission makes it decay into another manifold $M(\vec{p}\,')$ where $\vec{p}\,' = \vec{p} + \hbar\vec{k}_{rec} \pm \hbar\vec{k}_L$ where $\vec{k}_{rec}$ is $k_L \vec{u}_{rec}$ where $\vec{u}_{rec}$ is arbitrarily random. If $\vec{p}\,' \neq 0$ there's allways a finite probability that the atom scapes from the dark state and then goes via spontaneous emission to other manifold $M(\vec{p}\,'')$ and so on.

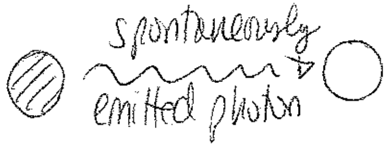


One can estimate the probability (per unit time) that the atom leaves the state $|\Psi_{nc}(p)\rangle$:

$$\Gamma'' = 4\left(\frac{kp}{M}\right)^2 \frac{\Gamma}{\Omega^2}$$

Clearly the life-time of $|\Psi_{nc}(p)\rangle$ is progressively longer the lower is p. Because the only state with an infinite lifetime is $|\Psi_{nc}(p=0)\rangle$, subsequent fluorescence cycles eventually lead to an accumulation of atoms in that state. Hence the final distribution of atoms is given (in 1D cooling) by two arbitrarily narrow peaks at $p = \pm \hbar k_L$. Note that, as we already commented, the temperature is given by the width of the peaks. Hence one may reach an extremelly low T in this way, much lower than the recoil limit.

* Unfortunately even these techniques are limited, and indeed one cannot reach quantum degeneracy (more about quantum degeneracy later in this course!) only with the help of laser cooling. With laser cooling one may reach temperatures $\sim \mu K$ (which temperature exactly depends of course on the particular atomic species considered). However one cannot reach the phase space densities (which as we will see later in this courses relates to $n T^{-3/2}$, with n the atomic density) sufficiently large to achieve quantum degeneracy. The reason is the so-called photon-reabsorbtion. If the density of the sample becomes too large, light scattered by one atom is reabsorbed by others.



Note that dark-state techniques work because quantum interference prevent photon absorbtion of a laser photon. However quantum interference doesn't apply for the spontaneously emitted photon by other atom. Reabsorbtion is then a killer mechanism for laser cooling, since for resonant light, the optical thickness of an atomic sample is such that light simply can't escape, and hence the sample can't be cooled further.

* In order to reach quantum degeneracy one needs something else, the so-called evaporative cooling. More about it later.