• TRAPPING OF NEUTRAL ATOMS

* In this lecture we will learn how to trap neutral atoms

Note: I will not discuss the trapping and cooling of ions, because the techniques and some ideas are somehow different, and also because in the rest of the course I will mainly focus on neutral atoms)

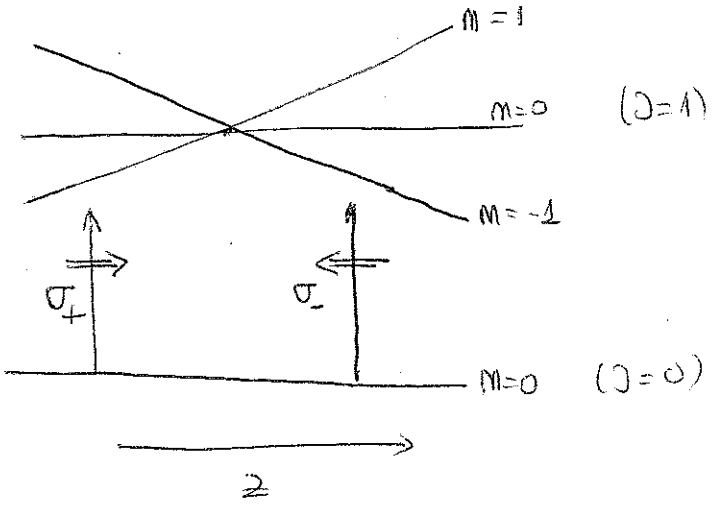
We will see that there are different ways of trapping neutral atoms, which we can divide in 2 main types: dissipative traps (magneto-optical traps) and conservative traps (magnetic or optical dipole traps).

* The magneto-optical trap

* In the previous lecture we have learned how to use the radiation pressure to cool atoms. We will see now how to use the radiation pressure to trap atoms by means of the so-called magneto-optical trap (MOT).

* Let's discuss first a one-dimensional situation.

We consider a ground state with $J=0$ and an excited state with $J=1$. The atoms are in an inhomogeneous magnetic field, such that $B=0$ at $z=0$, $B>0$ for $z>0$ and $B<0$ for $z<0$ (we consider the magnetic field linear in z) (Note: such a field may be created by a pair of anti-Helmholtz coils). The atoms are affected by 2 counterpropagating lasers of the same intensity and same frequency but with polarization σ_+ and σ_- respectively. We assume red detuning ($\delta<0$).



* The intuitive idea of the MOT is easy to grasp. Atoms moving say to the right at $z>0$ are feeling more (more resonant) the σ_- laser and hence are pushed to the left. Same for $z<0$. Hence both at $z>0$ and $z<0$

there's a force pushing towards the origin, and hence a trapping around $z=0$.

* let's see now this in some more detail.

The $\sigma_\pm$ laser induces a transition between $m_g=0$ and $m_e=\pm 1$ with an spatially dependent detuning

$$\delta_\pm = \delta \mp k_L v \mp \beta z$$

($\beta = \mu(\Delta B/\Delta z)$ where $\pm\beta z$ is the Zeeman shift associated to $m_e=\pm 1$)

As in our discussion of the Doppler cooling if the lasers are sufficiently weak ($\Omega \ll \Gamma$) we may assume that the resulting radiation pressure (p. 17) is the sum of the radiation pressures of the σ_+ and the σ_- lasers:

$$F \cong F_{\sigma_+} + F_{\sigma_-} \cong \hbar k_L \frac{\Gamma}{2} \Omega_1^2 \left\{ \frac{1}{(\delta - k_L v - \beta z)^2 + (\Gamma/2)^2} - \frac{1}{(\delta + k_L v + \beta z)^2 + (\Gamma/2)^2} \right\}$$

For small velocities and small Zeeman shifts ($k_L v + \beta z \ll \Gamma$) we can Taylor expand to obtain

$$F \simeq 4\hbar k_L \frac{\Gamma}{2} \Omega_1^2 \frac{\delta}{[\delta^2 + \Gamma^2/4]^2} (k_L \cdot v + \beta z)$$

This is a friction and a restoring force only if $\delta = -|\delta| < 0$

$$= -\eta v - M \omega_{trap}^2 z$$

where $\eta = 4\hbar k_L^2 \Omega_1^2 \frac{|\delta| \Gamma/2}{[\delta^2 + \Gamma^2/4]^2}$ ⟶ Friction coefficient

$$\omega_{trap}^2 = 4\hbar k \frac{\Gamma}{2} \Omega^2 \frac{|\delta| \beta}{M [\delta^2 + \Gamma^2/4]^2}$$ ⟶ Harmonic oscillator frequency

Hence we have the combination of a friction (as that we have seen in the optical molasses) and a restoring force, i.e. a trapping.

* This mechanism can be easily generalized to 3D by 3 counterpropagating beams and 3 pairs of coils.

* Magnetic traps

* The MOT is an example of dissipative trap. In addition to the trapping force we have friction. At low temperatures the spontaneous emission (as we already should know well) induces heating.

* In this section we will discuss the case of magnetic traps, which are conservative traps based in the Zeeman effect.

The energy of an atomic level of angular momentum $\vec{F}$ and magnetic quantum number m_F (along the direction z of a magnetic field $\vec{B}$) is:

$$E(m_F) = g\mu_B m_F B$$

(μ_B: Bohr's magneton; g: gyromagnetic ratio)

States such that $g m_F > 0$ are the so-called weak-field seeking states, because they're attracted towards minima of the magnetic field.

On the contrary strong-field seekers will tend towards regions of larger B.

* Now we reach to an important statement:

"In a region devoid of charges and currents, the strength of a quasistatic electric or magnetic field can have local minima but no local maxima."

As a result, only weak-field seeking states can be trapped in quasi-static magnetic traps.

* The proof of this statement is rather simple. Let's consider the field $\vec{B}(\vec{r})$. We assume a suspected maximum at $\vec{r} = 0$.

For $\vec{r}$ close to 0: $\vec{B}(\vec{r}) = \vec{B}(0) + \delta\vec{B}$

and $\vec{B}^2(\vec{r}) = \vec{B}^2(0) + 2\vec{B}(0)\cdot\delta\vec{B} + (\delta\vec{B})^2$

* If $\vec{r}=0$ were a maximum we must have $\vec{B}(0)\cdot\delta\vec{B}<0$ for sure, because the other 2 terms are positive.

Without loss of generality we consider $\vec{B}(0)=B_z(0)\vec{e}_z$, with $B_z(0)>0$

hence $\delta B_z(\vec{r})<0$ for $\vec{r}$ near 0. We will show that this can't be.

* For a quasistatic field $\vec{\nabla}\vec{B}=0$
$\vec{B}$ field in a region devoid of currents $\Rightarrow \vec{\nabla}\times\vec{B}\cong 0$

We apply Green's theorem

$$\int_V (\phi\nabla^2\psi-\psi\nabla^2\phi)=\int_{S(V)}(\phi\vec{\nabla}\psi-\psi\vec{\nabla}\phi)\cdot d\vec{s}$$

where $\phi=1/r \longrightarrow \nabla^2\phi=4\pi\delta(r)$

$\searrow \vec{\nabla}\phi=-\vec{r}/r^3$

$\psi=\delta B_z(\vec{r})\longrightarrow$ for a quasistatic field $\vec{\nabla}(\delta B_z)=0 \rightarrow \nabla^2(\delta B_z)=0$

$V\Rightarrow$ sphere of radius R centered at the origin $r=0$

Hence $$-4\pi\underbrace{\delta B_z(0)}_{0}=\int_{S(V)}\delta B_z(\vec{r})\frac{\vec{r}}{r^3}\cdot d\vec{s}=\int_0^{\pi}d\theta\sin\theta\int_0^{2\pi}d\phi\;\delta B_z(\theta\phi)$$

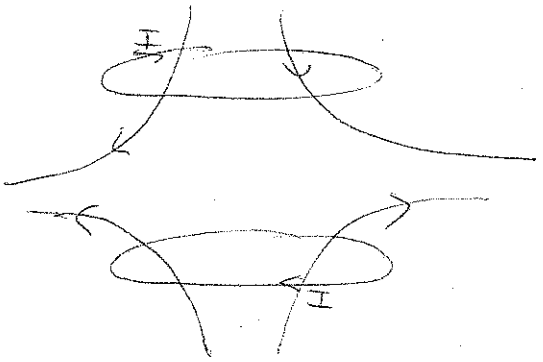
Hence either $\delta B_z(\vec{r})=0$ or $\delta B_z(\vec{r})$ cannot have the same sign in all directions

* On the contrary a minimum of $\vec{B}(\vec{r})$ at $\vec{r}=0$ demands only that $2\vec{B}(0)\cdot\delta\vec{B}+(\delta\vec{B})^2>0$

for all $\vec{r}$ near 0. And this may be simply achieved by considering $\vec{B}(0)=0$. This is the case of the so called

Quadrupole traps

* One may take two coils in anti-Helmholtz configuration.



These coils create a magnetic field of the form

$$\vec{B}(x,y,z) = B'(x\,\vec{e}_x + y\,\vec{e}_y - 2z\,\vec{e}_z)$$

$$|\vec{B}| = B'(x^2+y^2+4z^2)^{1/2}$$

This is a so-called quadrupole trap, and $|\vec{B}|$ is clearly a minimum at $\vec{r}=0$. If one pumps the atoms in a weak-field seeking state (using optical pumping, based in absorption + spont. em. cycles) one may then achieve a trap which is easy to implement and rather deep.

However this kind of traps has a very serious problem. We assume that the magnetic moment of atoms follows the spatially dependent magnetic field adiabatically. This is only true if the magnetic field varies slowly compared with the transition frequency $\omega_B = g\mu_B m_F B/\hbar$; this implies that:

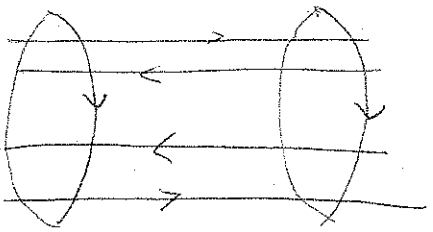
$$\frac{v\cdot\vec{\nabla}B}{B} \ll \frac{g\mu_B m_F B}{\hbar}$$

However at the trap center $B=0$, and hence near the trap center the atoms may undergo non-adiabatic spin flips, so-called Majorana spin flips. Since the spin flip corresponds to the atoms undergoing a transition from a weak-field seeking to a strong-field seeking state (which isn't trapped), this flips correspond to a loss mechanism. In other words there's a hole in the middle of the trap! There're ways of plugging this hole (e.g. putting an intense blue detuned laser at the trap center) but we won't review them here.

* But, as mentioned before we don't need necessarily $\vec{B}(0)=0$ to get a minimum. We may have a finite value. This is the case of the so-called Ioffe-Pritchard trap. In its original form one has

* Four straight current-carrying bars that form a linear quadrupole field

$$\vec{B}_a(\vec{r}) = b' \begin{pmatrix} x \\ -y \\ 0 \end{pmatrix}$$



* Two end pinch coils

$$\vec{B}_b(\vec{r}) = B_0 \vec{e}_z + \frac{b''}{4} \begin{pmatrix} -2xz \\ -2yz \\ 2z^2 - x^2 - y^2 \end{pmatrix}$$

These coils are such that B_0 admits a minimum ($b''>0$) along the $x=y=0$ axis at $z=0$.

The total field $\vec{B}(\vec{r}) = \vec{B}_a(\vec{r}) + \vec{B}_b(\vec{r})$ can be Taylor expanded around $\vec{r}=0$. One then obtains

$$|\vec{B}(\vec{r})| \simeq B_0 + \left(\frac{b'^2}{2B_0} - \frac{b''}{4}\right)(x^2+y^2) + \frac{b''}{2} z^2$$

As long as $2b'^2 > B_0 b''$ then one gets an harmonic potential with a minimum at $\vec{r}=0$ (always for weak-field seekers)

$$U = \mu \left[\frac{B_\rho''}{2} (x^2+y^2) + \frac{b''}{2} z^2 \right]$$

with $B_\rho'' = \frac{b'^2}{2B_0} - \frac{b''}{4}$

* these traps are very efficient in producing tight traps.

(Note: In actual experiments it's used the so called cloverleaf design which replaces the 4 bars by a pair of four coils surrounding each pinch coil).

* Optical traps

* In page (11) we already mentioned that the dipole potential may be employed to trap atoms. For red detuning ($\delta < 0$) atoms are attracted to the regions of maximal laser intensity. (Note: in principle one can also trap atoms with blue detuned lasers, with the advantage that the trapping is produced in regions of low intensity, hence minimizing the dissipative effects of spontaneous emission; however relatively more complicated arrangements are necessary with blue detuned lasers, and hence here we will just discuss briefly the red-detuned case.)

* The simplest way to trap atoms optically is by focusing a red detuned gaussian laser beam (Chu and Ashkin, 1986). The intensity distribution of a focused gaussian beam is of the form:

$$I(\rho, z) = \frac{2}{\pi} \frac{P_0}{w^2(z)} e^{-2\rho^2/w^2(z)}$$

where P_0 is the total laser power, $\rho = \sqrt{x^2+y^2}$, and $w(z)$ is the beam waist

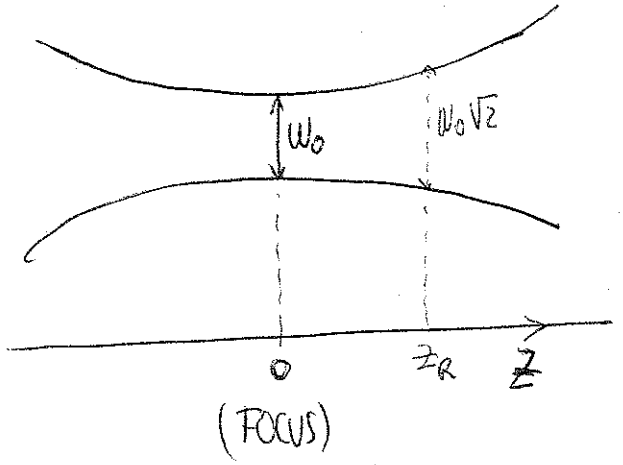
$$w(z) = w_0 \sqrt{1 + (z/z_R)^2}$$

with $z_R = \frac{w_0^2 \pi}{\lambda} \equiv$ Rayleigh range

$w_0 =$ minimal waist

(Note: the minimal waist $w_0 \propto \lambda$ = laser wavelength) → and actually $w_0 > \lambda$

the beam waist looks like this:



Note: Clearly this shape is produced by diffraction, which of course is a consequence of uncertainty principle. The more one focus, the faster it open after the focus.

* Remember (p.11) that for sufficiently large detuning $|\delta| \gg \Gamma$ and low intensity ($s \ll 1$) the dipole potential is of the form (for $\delta < 0$)

$U_D(\vec{r}) = -\frac{\hbar}{4|\delta|}\Omega_1^2(\vec{r})$. Remember also that $\Omega_1^2(\vec{r})$ is directly proportional to the intensity. Hence we may write the resulting potential at the beam waist as:

$$U_D(\vec{r}) = \frac{-U_0}{(1+(z/z_R)^2)} \exp\left[\frac{-2(\rho^2/w_0^2)}{(1+(z/z_R)^2)}\right]$$

Where $U_0 = U_D(\vec{r}=0)$ is the trap depth.

* When the thermal energy $k_B T$ of the atoms is small compared to U_0, the spread of the atoms in the trap is radially small compared to the waist w_0, and axially small compared to the Rayleigh range z_R. In this case we may perform a Taylor expansion:

$$U_D(\vec{r}) \cong -U_0\left[1 - \left(\frac{z}{z_R}\right)^2 - 2\frac{\rho^2}{w_0^2}\right]$$

$$= -U_0 + \frac{1}{2}m\left(\frac{2U_0}{m z_R^2}\right)z^2 + \frac{1}{2}\left(\frac{4U_0}{m w_0^2}\right)\rho^2$$

i.e. the optical potential can be approximated by a cylindrically symmetric harmonic oscillator with <u>radial frequency</u>

$$\omega_\rho = \sqrt{\frac{4U_0}{m w_0^2}}$$

and <u>axial frequency</u>

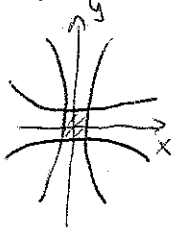
$$\omega_z = \sqrt{\frac{2U_0}{m z_R^2}}$$

* Note that $\frac{w_\rho}{w_z} = \sqrt{2}\pi \frac{w_0}{\lambda} > 1$

Hence the potential in the radial direction is typically significantly steeper than that in the axial direction. This leads to cigar-shaped traps (more elongated along the laser axis).

* Another widely employed arrangement allows for a tight confinement in all directions. This is the so-called crossed dipole trap, where as the name suggests one crosses two focussed beams in their foci at 90° degrees. Interference effects can be neglected, when the polarizations of the 2 beam are orthogonal to each other. Let's assume that the two lasers are exactly the same. Then, assuming the laser along x and y respectively, we get:

$$I(x,y,z) = I_0 \left[\frac{1}{(1+(x/R)^2)} \exp\left[\frac{-(y^2+z^2)/w_0^2}{1+(x/R)^2} \right] + \frac{1}{(1+(y/R)^2)} \exp\left[\frac{-(x^2+z^2)/w_0^2}{1+(y/R)^2} \right] \right]$$



where now we denote
R ≡ Rayleigh range

Around $x=y=z=0$ we get

$$U_D(x,y,z) = -U_0 + \frac{1}{2}m\left[\frac{2U_0}{m}\left(\frac{1}{R^2}+\frac{2}{w_0^2}\right)\right](x^2+y^2) + \frac{m}{2}\left[\frac{8U_0}{mw_0^2}\right]z^2$$

Then we get

$$w_{x,y} = \left[\frac{2U_0}{m}\left(\frac{1}{R^2}+\frac{2}{w_0^2}\right)\right]^{1/2}$$

$$w_z = \left[\frac{8U_0}{mw_0^2}\right]^{1/2}$$

} Now we get a tight confinement in both directions.

* Still the trap is not completely isotropic, since the tightest confinement is in the z-direction. The laser beams are therefore usually set in the horizontal plane, so that the strongest dipole force works in the direction of gravity.

(Note: for deep dipole traps gravity is usually no problem, although when e.g. doing evaporative cooling (see later) it may play a significant role).

• Evaporative Cooling

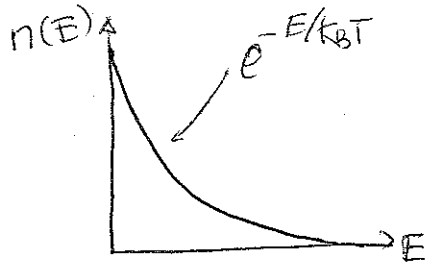
* Now that we have understood how to trap atoms, we are in a good position to discuss a different sort of cooling, the so-called evaporative cooling. This type of cooling is rather different than the laser cooling mechanisms discussed before. Contrary to the laser cooling techniques, this cooling is not a single-atom process but rather a cooling achieved statistically for the whole ensemble. Hence the approach is rather different.

* Evaporative cooling is actually a very familiar effect. It's the mechanism that cools down a cup of coffee! As time goes on, the molecules escape coffee cools down as the most energetic (warmest) molecules escape from the cup. The remaining molecules rethermalize at a lower temperature.

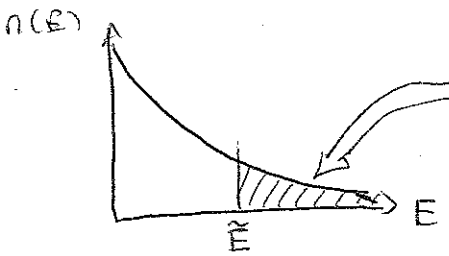
The evaporative cooling of atomic samples works in much the same way, except that the cup of coffee is replaced by an atomic trap.

* Schematically we can get an intuitive idea:

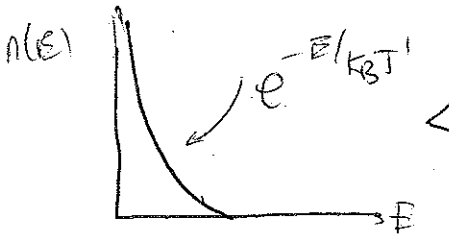
Before cooling



Before cooling the particles follow a Maxwell-Boltzmann distribution at a given temperature T



We evaporate particles with energy $E > \tilde{E}$. This obviously reduces the number of particles but crucially also the <u>average energy per particle</u>



Rethermalization leads to a lower temperature T' for the remainder of the sample. This is because the average energy per particle in thermal equilibrium is proportional to the temperature.

* In the following we will discuss some further details about the idea of evaporative cooling. Our discussion will be necessarily rather brief (for more detailed discussion see the book of Metcalf and van der Straaten, "laser cooling and trapping").

We consider in the following the case of an isotropic harmonic trap of frequency ω (evaporative cooling is typically done in magnetic traps using radio-frequency techniques as we discuss below, although evaporative cooling may be also performed in optical traps). Let's consider N particles at a temperature T (we consider this temperature sufficiently large such that we can treat the problem classically).

The total energy of the gas (kinetic + potential) is then

$$E = 3 N k_B T$$

and the phase-space distribution function is:

$$d^6\rho \equiv \underbrace{\frac{N}{(2\pi)^3} \frac{1}{\bar{r}^3 \bar{p}^3} e^{-\frac{r^2}{2\bar{r}^2}} e^{-\frac{p^2}{2\bar{p}^2}}}_{\text{phase space density } D(\vec{r},\vec{p})} d^3r\, d^3p = \frac{1}{h^3} D(\vec{r},\vec{p})\, d^3r\, d^3p$$

where $\bar{p} = \sqrt{m k_B T} = m\bar{v}$

$\bar{r} = \bar{v}/\omega$

At the center of the trap and for $\vec{p} = 0$ we get

$$D(0,0) = N \left(\frac{\hbar\omega}{k_B T}\right)^3$$

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* Note: The classical treatment is valid if the phase space density $D(0,0) \ll 1 \longrightarrow$ i.e. if $N \ll \left(\frac{k_B T}{\hbar\omega}\right)^3$. In turn this implies $n_0 \lambda_T^3 \ll 1$, where $\lambda_T = h/\sqrt{2\pi m k_B T}$ is the thermal de Broglie wavelength, and $n_0 = \frac{N}{(2\pi)^{3/2}\bar{r}^3}$. More about λ_T later in this course.

* The idea of phase space density is crucial, since it's actually this figure the important value and not the temperature itself. The objective (as we will discuss in more detail in future lectures) is to reach $D \to 1$ (or larger). In that case as we will see one enters into the so-called quantum-degeneracy regime, which is the ultimative goal of these techniques.

* OK, so now we have understood some details about thermal distributions. In page (42) we said that the evaporative cooling relies in a rethermalization process after evaporation. This rethermalization is possible through elastic collisions among the trapped particles, which we consider as exclusively binary collisions. For cold atoms ($T < 1\,mK$) only collisions in s-wave (isotropic) are important (more about scattering theory later in these lectures). At low energies the cross-section for collisions (σ) is a constant independent of the kinetic energy of the colliding particles, in particular $\sigma = 8\pi a^2$, where a is the s-wave scattering length.

* The average collisional rate (γ) is given by

$$\gamma = \sqrt{\frac{2}{\pi}}\, n_0 \sigma \bar{v} = \left(\frac{m\omega^3 \sigma}{2\pi^2 k_B}\right) \frac{N}{T}$$

(Note: γdt is the average probability that a given particle undergoes a collision with one of the others.)

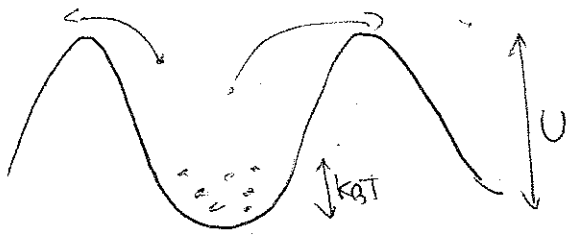
* E.g. $N = 10^8$ Rb atoms

$\frac{\omega}{2\pi} = 100\ Hz$

$T = 200\,\mu K \to n_0 = 6\times10^{11}\,cm^{-3}$, $\bar{v} = 14\,cm/s$, $\bar{r} = 0.2\,mm$ } $D(0,0) = 10^{-6}$

$a = 5.1\,nm \to \sigma = 6.5\times10^{-16}\,m^2$

} $\gamma = 40$ collisions/second



Let's suppose that a given time we eliminate all dN particles with energy larger than say

$$U = \eta\, k_B T$$

where $\eta \gg 1$ (then $dN \ll N$). We will see now with a simple estimation that this coefficient η cannot be whatever.

One can then evaluate easily the energy lost

$$dE = dN\,(U + k_2 k_B T) = dN\, k_B T\,(\eta + k)$$

where $k_2 k_B T$ represents the average energy of an evaporated particle in excess of the evaporation threshold U. The coefficient k_2 is of the order of 1.

* Let's suppose that the rest of the particles re-thermalize via binary collisions. They will do it into a new temperature such that:

$$E - dE = 3\,(N - dN)\,k_B\,(T - dT)$$

Taylor expanding ($dN \ll N$, $dT \ll T$):

$$E - dN\, k_B T\,(\eta + k) = 3N k_B T - 3 k_B T\, dN - 3N k_B\, dT$$

Hence $$\frac{dT}{T} = \left(\frac{\eta + k - 3}{3}\right)\frac{dN}{N}$$

The temperature decreases only if $dT > 0$, hence $\boxed{\eta + k > 3}$

Since $k \sim 1 \longrightarrow \eta > 2$.

So the reduction of the temperature doesn't happen for whatever cut. The evaporation threshold U must be sufficiently large compared to $k_B T$.

* Even more. The rethermalization is based on binary collisions. We have to maintain this rethermalization efficient during the cooling process, and for this we need that the collisional rate doesn't decrease:

$$\gamma = \left(\frac{m\omega^3\sigma}{2\pi^2 k_B}\right)\frac{N}{T} = \chi\frac{N}{T} \qquad \chi \equiv \text{constant}$$

After an evaporation we have

$$\gamma + d\gamma = \chi\left(\frac{N-dN}{T-dT}\right) \cong \chi\frac{N}{T}\left(1 - \frac{dN}{N} + \frac{dT}{T}\right)$$

$$\frac{d\gamma}{\gamma} = \frac{dT}{T} - \frac{dN}{N} \geq 0$$ ← the rate doesn't decrease.

Hence $\frac{dT}{T} \geq \frac{dN}{N} \rightarrow \frac{\eta+k-3}{3} \geq 1 \rightarrow \boxed{\eta + k \geq 6}$

This is the so-called runaway condition

So runaway evaporation demands $\eta \gtrsim 5$.

* One detail more. Let's see what happens with the phase space density: $D = \left(\frac{\hbar\omega}{k_B}\right)^3 \frac{N}{T^3}$

after the evaporation

$$D + dD = \left(\frac{\hbar\omega}{k_B}\right)^3 \frac{N-dN}{(T-dT)^3} \cong \left(\frac{\hbar\omega}{k_B}\right)^3 \frac{N}{T^3}\left(1 - \frac{dN}{N} + 3\frac{dT}{T}\right)$$

Then $\frac{dD}{D} = 3\frac{dT}{T} - \frac{dN}{N} = \frac{dN}{N}(\eta + k - 4)$

Then the phase-space density increases (which is our goal) if $\eta + k > 4 \rightarrow \boxed{\eta \gtrsim 3}$

* So if one is in the run-away evaporation ($\eta > 5$) then both the phase space density and the temperature are reduced.

* Let's put some numbers.

Let's consider that we have the case $\eta + \kappa = 6$ (so right at the runaway evaporation). A gain of a factor ξ in the phase space density D is obtain with a reduction of a factor $\sqrt{\xi}$ of the atom number. (Note: remember that for $\eta + \kappa = 6 \Rightarrow \frac{dD}{D} = 2\frac{dN}{N} = 2\frac{dT}{T}$)

For example a gain $\xi = 10^6$ in phase space density demands a reduction of a factor 1000 of both N and T.

* If one proceeds in discrete evaporation steps (each step implies evaporation + rethermalization) one may choose $dN \ll N$ at each step such that the above linear estimation remains valid.

Let's consider $dN = 0.1\,N \xrightarrow{\eta+\kappa=6} dD = 0.2\,D$.

Hence at each evaporative step the phase space density is multiplied by a factor 1.2. Suppose that one wants to gain a factor 10^6 in phase space density. Then one needs roughly $N = \ln(10^6)/\ln(1.2) \simeq 75$ steps. For every step we need rethermalization, which typically involve of the order of 3 elastic collisions per atom. Since every collision occurs in a time scaler $1/\gamma$, then the full cooling takes a time $75 \times 3 \times 1/\gamma$, so of the order of $\simeq 200/\gamma \sim$ <u>5 seconds</u> (remember the estimation in page (44)). This is the typical experimental time for evaporation

(Note: 10^6 is the typical enhancement factor for D that one needs to go from a laser cooled sample (with $D \sim 10^{-6}$) into quantum degeneracy (which, as we will see, demands $D \geq 1$).

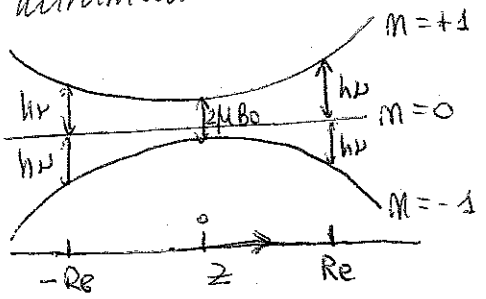
* A more accurate description of evaporative cooling typically demands the use of numerical methods, in particular molecular dynamics (based in the Boltzman equation), which allows to study the evaporation dynamics. But these methods are beyond the scope of this course.

* We will very briefly discuss now <u>how evaporative cooling is actually performed</u>.

Typically evaporative cooling is done in a magnetic trap (although optical traps may be employed as well). A very easy way of performing evaporation is by means of a so-called <u>rf-knife</u>. Let's see how this works.

* Let's consider atoms in a state $m=1$, which is trapped by a potential $U(\vec{r}) = |\mu|\,|B(\vec{r})|$. We assume e.g. a Ioffe-Pritchard-like trap with a minimum $B(\vec{r}=0) = B_0$. Suppose now that we apply a radiofrequency wave with a frequency ω such that

$$h\nu > \mu B_0$$



From the figure it's clear that the radiofrequency is resonant with the $m=\pm1 \leftrightarrow 0$ transition at a distance Re.

*

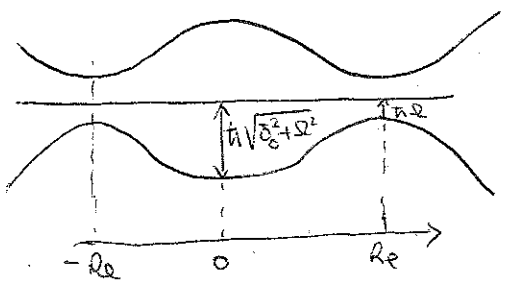
* Assuming a spherically symmetric trap (which implies $2b'^2 = 3B_0 b''$ in our discussion of the Ioffe-Pritchard trap in p. (37)) the radiofrequency will be resonant at a radius

$$R_e = \sqrt{\frac{2h(\nu-\nu_0)}{m\omega^2}}$$

where $\nu_0 = \mu B_0/h$ (Larmor frequency) and ω is the trap frequency.

* It's easier to see what happens if we move to the <u>dressed state picture</u> (remember e.g. our discussion of p. (12)):

If the atom moves sufficiently slowly such that it follows adiabatically the dressed state potential, then it's clear that atoms with a sufficiently large energy are evaporated $\longrightarrow$ this is the <u>rf-knife</u>.



with $\delta_0 = h(\nu-\nu_0)$

(<u>Note</u>: the rf-cooling is an evaporation which is actually more precisely selective than energy selective. In 1D one would say that kinetic energies $E > \hbar\sqrt{\delta_0^2+\Omega^2}$ are typically evaporated. However in 3D it's more subtle due to the appearance of circular trajectories, for which the same knife is just effective for double so high energies. But we won't enter on these subtleties).

* Just a couple of final comments concerning evaporative cooling.

• In the previous discussion we have implicitely assumed that the collisional rate γ fulfills $\gamma \ll \omega$. This means that between 2 collisions the atom oscillates many times in the trap. This is the so-called collisionless regime. In this regime a particle with an energy larger than the cut U reaches the knife position before colliding again.

The opposite regime is the collisional regime or hydrodynamic regime. In that regime $\gamma \gg \omega$, so the atom collides many times before oscillating once. In that regime the evaporation is strongly reduced. This is clear, because although many collisions produce atoms a priori evaporables ($E > U$), these collisions are in fact inefficient because due to the many collisions the atom reaches the knife with an energy below U, and it isn't evaporated.

So, the moral is: large γ is ok, but too large is bad!

* In the previous discussion we have assumed a spherical trap. Typically the traps are not spherically symmetric (they may have even bizarre shapes due to e.g. gravitational sags). This has two consequences. One one hand the evaporation surface is not a sphere but an ellipsoid (or even more strange). On the other hand, one may reach for very anisotropic traps, the hydrodynamic regime in one direction but not in the others.

* All these details are of course important in the practical implementation of evaporative cooling, but we won't have a look to them in further detail. Evaporative cooling is a crucial technique to reach quantum degeneracy. We will see in further lectures what happens with the atoms then.