Lecture 2: The Principle of Equivalence and some of its immediate consequences

Today's formulation of "Einsteins Equivalence Principle" (short: EEP) consists of three parts

1) UFF: Universality of free fall (also called the "Weak Equivalence Principle": WEP)

Spatial orbits $\mathbb{R} \ni t \mapsto \vec{x}(t) \in \mathbb{R}^3$ of test particles only depend on initial conditions, but not on physical or chemical properties of test particle.

Violations of UFF are parametrized by Eötvös-Parameter η ("eta"): Let A and B two test masses and $a(A)$, $a(B)$ their respective accelerations in a given gravitational field, then

$$\eta(A,B) := \frac{|a(A) - a(B)|}{[a(A) + a(B)]/2} \qquad (2.1)$$

In Newtonian theory

$$a(A/B) = \left[\frac{m_g}{m_i}\right]_{A/B} g \tag{2.2}$$

where g is the gravitational field strength not depending on A/B. Hence, writing

$$X = \frac{[m_g/m_i]_A}{[m_g/m_i]_B} \tag{2.3}$$

We have

$$\eta(A,B) = 2\,\frac{[m_g/m_i]_A - [m_g/m_i]_B}{[m_g/m_i]_A + [m_g/m_i]_B}$$

$$= 2\,\frac{X-1}{X+1} = 2\,\frac{X-1}{X-1+2}$$

$$= \frac{X-1}{1+\frac{X-1}{2}} = (X-1) - \frac{1}{2}(X-1)^2 + \cdots \tag{2.4}$$

Thus η measures directly deviations from $X=1$, from non-universality of the ratio m_g/m_i.

Torsion balance experiments in the years 1990 - 2008 have set upper bounds on η for certain pairs (A, B) of materials, e.g.

$$\left.\begin{aligned} \eta(\mathrm{Al}, \mathrm{Pt}) &= (-0.3 \pm 0.9) \times 10^{-12} \\ \eta(\mathrm{Be}, \mathrm{Ti}) &= (0.3 \pm 1.8) \times 10^{-13} \end{aligned}\right\} \quad (2.5)$$

2) LLI : Local Lorentz invariance
In freely-falling reference frames the laws of non-gravitational interactions are those of SRT. They do not depend on spatial orientation and velocity (no "preferred-frame effects").

Possible violations are those that would show in a non-isotropy of the 2-way (fourth and back) speed of light in a Michelson-Morley-Experiment. Here the current upper bound is

$$\frac{\Delta c}{c} < 5 \cdot 10^{-17} \qquad (2.6)$$

3) LPI : Local Position Invariance.
As for LLI we have a local validity of SRT, independent of spatio-temporal location (no preferred location effects).

Alternatively

3') UGR : Universality of Gravitational Redshift

UCR : Universality of Clock Rates.

UGR says that the clock-rates are universally affected by gravitational field. In Newtonian limit of small velocities and weak fields this reads

$$\frac{\Delta \nu}{\nu} = \frac{\Delta \phi}{c^2}$$

$\vec{g}$ ↓ $\bullet\ \vec{x}_1 \Rightarrow \phi(x_1)$

$\bullet\ \vec{x}_2 \Rightarrow \phi(\vec{x}_2)$

$$\Leftrightarrow \quad \frac{\nu(x_1) - \nu(x_2)}{[\nu(x_1) + \nu(x_2)]/2} = \frac{\phi(x_1) - \phi(x_2)}{c^2} \qquad (2.7)$$

where $\nu(x_{1,2})$ are the clock rates of clock 1,2 observed either at $\vec{x}_1$ or $\vec{x}_2$ (both!)

Possible violations are parametrised by α ("alpha"), possibly dependent on space, time and type of clock, so that

$$\frac{\Delta \nu}{\nu} = (1+\alpha)\,\frac{\Delta \phi}{c^2} \qquad (2.8)$$

In GR $\alpha = 0$.

In "Gravity-Probe-A" experiment H-Maser clocks on sounding rockets (Scout-Rocket) at 10.000 km altitude were compared with identical clocks on Earth (1976). This means

$$\frac{\Delta \nu}{\nu} \cong 4.5 \times 10^{-10} \qquad (2.9)$$

Note that for small heights Δh above the Earth's surface, we have

$$\frac{\Delta \phi}{c^2} = \frac{g\,\Delta h}{c^2} = \frac{9{,}81\ m\cdot s^{-2}\quad \Delta h}{(299792458)^2\ m^2\cdot s^{-2}}$$

$$= (9{,}1616 \cdot 10^{15}\ m)^{-1}\ \Delta h$$

$$= (0{,}97\ ly)^{-1}\ \Delta h$$

$$\cong 10^{-16} \cdot \Delta h / m \qquad (2.10)$$

Today this "Red-Shift" is measured in laboratories over heights of $\Delta h \gtrsim 30$ cm $\overset{\nabla}{\circ}$. Clock stabilities reach 10^{-18} level (Braunschweig).

For Gravity-Probe-A Experiment, which was lanched on June 18. 1976, duration of experiment/flight 1h 55 min, one could infer upper bound on α of:

$$|\alpha| < 7 \cdot 10^{-5} \qquad (2.11)$$

This has never been improved on until today and is by far the least well tested part of EEP

$\Rightarrow$ Putting EEP as hypothesis, one can "derive" that gravity can be described geometrically. For this, see:

See handout page

Kip Thorne, David Lee, Alan Lightman "Foundations for a Theory of Gravitational Theories". Physical Review D, Vol. 7, No. 12, year 1973, pages 3563-3577.

The non-trivial statement behind the "geometrisation of gravity" is this: All matter, may it be Neutrinos, Elektrons, Photons, Atoms, People, Rocks, Planets, etc. couple to <u>the same</u> geometry, namely that of – the one and only – Spacetime!

Phrasing interactions of physical systems in geometric terms might be nice, but is no deep statement at all. For example, Newton's laws may be cast into the form of geodesic equations on configuration space (using the Jacobi-metric), but then the metric is different for each system. Here, in GR, we have <u>the same</u> metric for all systems. All systems couple to gravity in such a special, universal fashion, that a single metric suffices to describe all of them simultaneously. This is the true meaning behind EEP!

Reasons for considering violations of UFF

Suppose there were a "fifth force" like gravity, that is a scalar, long-ranging field with potential

$$W_{AE} = -H \frac{Q_A Q_E}{r_{AE}} \qquad (2.12)$$

coupling to "charges" $Q \neq M$ (e.g. baryon number). It would give rise to accelerations of body A in, e.g., field of Earth which depends on

$$q_A := Q_A / M \qquad (2.13)$$

The vertical acceleration would be

$$\left.\begin{aligned} \vec{a}_A &= G M_E (1 + q_A q_E h) \frac{\vec{X}_E - \vec{X}_A}{r^3_{AE}} \\ a_B &= G M_E (1 + q_B q_E h) \frac{\vec{X}_E - \vec{X}_B}{r^3_{BE}} \\ \text{where} \quad & q_A = \frac{Q_A}{M_A},\ q_B = \frac{Q_B}{M_B},\ q_E = \frac{Q_E}{M_E} \\ \text{and} \quad & h = H/G. \end{aligned}\right\} \qquad (2.14)$$

Then the Eötvös - parameter is

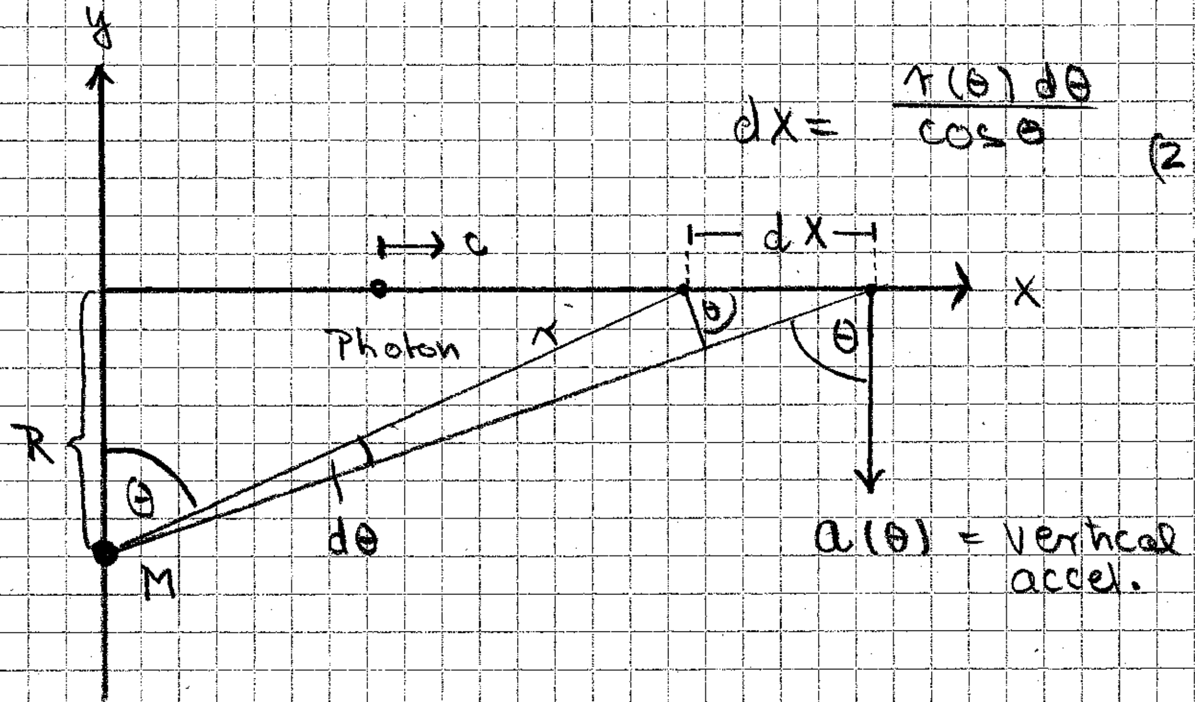
$$\eta(A,B) = \frac{|a_A - a_B|}{\frac{1}{2}(a_A + a_B)}$$

$$= \frac{q_E h (q_A - q_B)}{1 + q_E h \frac{1}{2}(q_A + q_B)} \quad (2.15)$$

This is $\neq 0$ if and only if $q_A \neq q_B$, i.e. if the specific "charges" (for the "fifth force") are no universal constants. This typically happens in models of grand unified theories, e.g. (Kaluza-Klein and String Theory, where Gravity has an Einsteinian Tensor-field plus an additional scalar (so called scalar-tensor theories). There are models of String Theory that are compatible with violations of UFF violations from the 10^{-15}-level onwards. This is - partly - why UFF tests are still pushed: to seek "new" - i.e. beyond Standard Modell - physics.

Simple consequences drawn from EEP

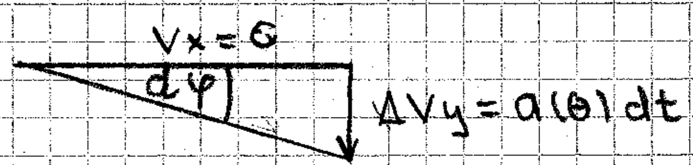
1) Light deflection (Einstein 1911)



$$dX = \frac{r(\theta)\,d\theta}{\cos\theta} \qquad (2.16)$$

$$\left.\begin{aligned} r(\theta) &= \frac{R}{\cos\theta} \\ a(\theta) &= \frac{GM}{r^2}\cos\theta \end{aligned}\right\} \quad a(\theta) = \frac{GM}{R^2}\cos^3\theta \qquad (2.17)$$

Infinitesimal deflection of light ray during time intervall $dt = dX/c$ is

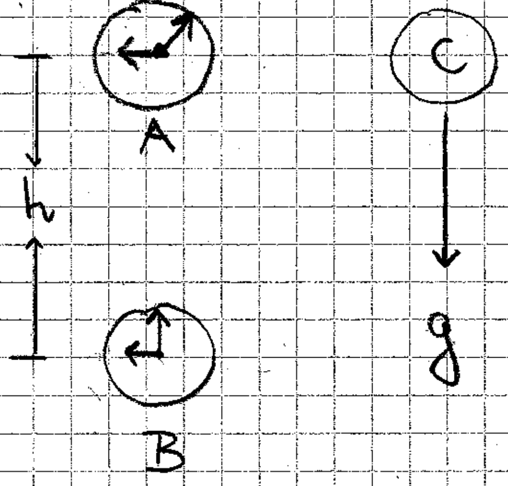


} Here we use EEP !

$$d\varphi = \frac{a(\theta)\,dt}{c} = \frac{GM}{R^2}\cos^3\theta\,\frac{1}{c^2}\,\frac{R}{\cos^2\theta}\,d\theta$$

$$\Delta\varphi = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\varphi = \frac{GM}{Rc^2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{2GM}{Rc^2} = \frac{1}{2}(\Delta\varphi)_{GR}.$$

2) Red-Shift



Two identically constructed clocks, A and B, are at rest in a grav. field, $\vec{g} = -g\vec{e}_z$, at vert. distance h.

A likewise identically constructed clock C falls freely in the gravitational field (which we assume to be homogeneous). For the freely falling clock the laws of SR apply

a) C passes A with velocity V_A. Relative to C A's period is dilated

$$T_A = T_C / (1 - V_A^2/c^2)^{1/2} \qquad (2.19)$$

b.) Likewise for the passage of C at B

$$T_B = T_C / (1 - V_B^2/c^2)^{1/2} \qquad (2.20)$$

c) Eliminating T_C from these two equations gives

$$T_B = T_A \left[\frac{1 - V_A^2/c^2}{1 - V_B^2/c^2} \right]^{1/2} \qquad (2.21)$$

$$\cong T_A \left(1 - \frac{1}{2} \frac{V_A^2 - V_B^2}{c^2} + O\left(\frac{V^4}{c^4}\right) \right)$$

d) Energy conservation holds for C. We do not know its exact form, something like (SR + Newtonian gravity), where m_C = clock's C rest mass:

$$\frac{m_C c^2}{\left(1 - \frac{V_B^2}{c^2}\right)^{1/2}} = \frac{m_C c^2}{\left(1 - \frac{V_A^2}{c^2}\right)^{1/2}} + gh + \cdots \qquad (2.22)$$

But we do know that to leading order in V^2/c^2 it must turn into the Newtonian expression. Hence

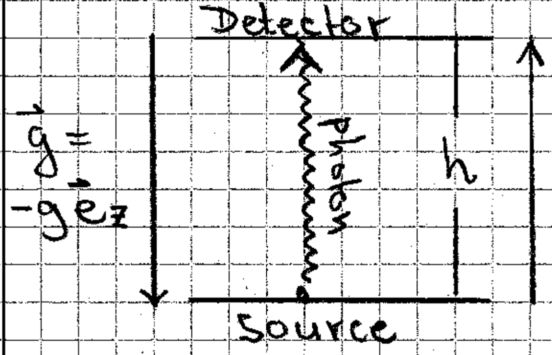
$$\frac{1}{2} m_C V_B^2 = \frac{1}{2} m_C V_A^2 + m_C g h$$

$$\Rightarrow \frac{1}{2} \frac{V_A^2 - V_B^2}{c^2} = -\frac{gh}{c^2}$$

$$\left.\begin{aligned} \Rightarrow T_B &= T_A \left[1 + \frac{gh}{c^2} + \cdots \right] \\ T_A &= T_B \left[1 - \frac{gh}{c^2} + \cdots \right] \end{aligned}\right\} \quad (2.23)$$

Hence, if compared with clock C, the period of clock T_B is longer than that of T_A; or: Clock A ticks faster than clock B

2.1 Red-shift in "photon-language"



$$\vec{a} = g\vec{e}_z \qquad (2.24)$$

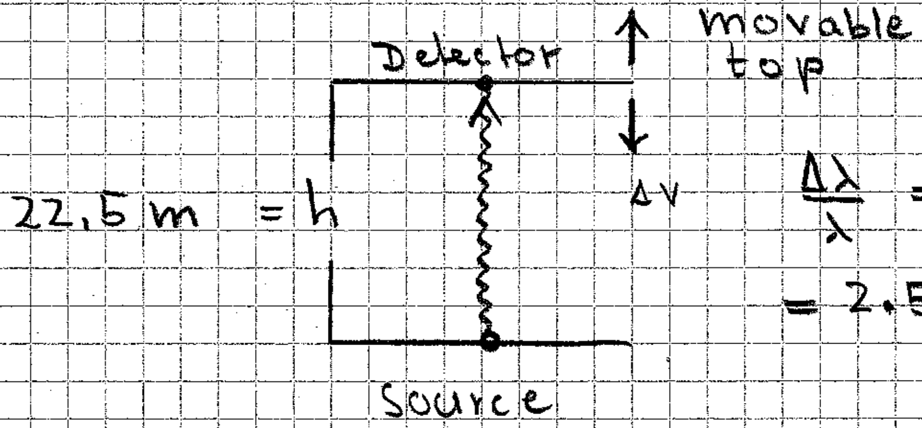
According to EEP, gravity has the same effect on light propagation as acceleration.
In time interval $\Delta t = h/c$ detector has acquired vertical velocity $\Delta v = g\Delta t = gh/c$. Light arrives therefore red-shifted due to Doppler-effect by

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta v}{c} = \frac{gh}{c^2} = \frac{\Delta\phi}{c^2} \qquad (2.25)$$

If you formally ascribe a "gravitational mass of $h\nu/c^2$ to photons of energy $h\nu$ it looses energy $\Delta E = (h\nu/c^2)gh$ in grav. field and arrives at detector with energy $h\nu' = h\nu - \Delta E = h\nu(1 - gh/c^2)$. Hence

$$\frac{\nu - \nu'}{\nu} = \frac{\Delta\nu}{\nu} = gh/c^2. \qquad (2.26)$$

Experiment by Pound & Rebka (1960) and Pound & Snider (1965) on 14.4 keV γ-Fe^{57}-line



$$\frac{\Delta\lambda}{\lambda} = \frac{gh}{c^2} = 2.5 \times 10^{-15} \qquad (2.27)$$

The top with detector can move so as to undo gravitational red-shift by Doppler blue shift. For $h = 22.5$ m the downward velocity of Detector had to be $\Delta V = 2.5$ mm / hour !
Accuracy of frequency measurement was achieved by Mössbauer Effect. Gravitational red-shift could be verified with accuracy < 1%.

Today, red-shift on optical clocks in laboratory has been verified for $h = 0.3$ m.

Chow, Hume, Rosenband, Wineland: "Optical clocks and Relativity" Science (Sept. 2010), Vol 329, p. 1630.