Exercises for the lecture on

**Introduction into General Relativity**

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**Sheet 12**

***farewell sheet***

### Problem 1

Quantify the error that you would commit in calculating the gravitational binding energy of a neutron star of 1.5 solar masses and radius 10 km if you used the Newtonian expression for the binding energy rather than the exact expression from GR. Assume for simplicity that the star is homogeneous, i.e. of constant rest-mass density.

## Problem 2

Calculate all orthonormal-frame-components of the curvature tensor for the Reissner-Nordström solution and evaluate the Kretschmann-scalar.

### Problem 3

In Lecture 21 we saw that in order for the Reissner-Nordström solution to have a horizon the charge $q$ (in geometric units) cannot exceed the mass $m$ (in geometric units). Calculate $q/m$ for the Electron and compare. Generalise this to the Kerr-Newman solution (Lecture 22) and calculate $\sqrt{a^2 + q^2}/m$ for the Electron.

## Problem 4

Estimate the ratio $a/m$ of the parameters in the Kerr metric for the values of mass and angular of the Earth, Jupiter and Sun.

## Problem 5

The classical notion of a black-hole presumably becomes meaningless if the Compton wavelength associated to its mass exceeds its Schwarzschild radius. Discuss why that is likely to be true. Derive the bounds implied by that for the mass and the Schwarzschild radius.

## Problem 6

Consider a sphere of radius $R$ in $\mathbb{R}^3$ and suppose it had a homogeneous charge layer of total charge $Q$ and also a homogeneous mass layer of total rest mass $M_0$. A semi-

Newonian expression for the total energy, $E(R)$ (we explicitly indicate the dependence on the radius $R$) would then be given by the sum. of three terms, one for the rest mass (positive), one for the electrostatic energy (positive), and finally one for the gravitational energy (negative):

$$E(R) = M_0 c^2 + \frac{Q^2}{8\pi\varepsilon_0 R} - \frac{GM^2}{2R} \tag{1}$$

Add to this the assumption that the gravitating mass $M$ is not just the rest mass $M_0$ but, in fact, includes *all* participating energies. In this case you must set $M = E(R)/c^2$.

This leads to a quadratic equation for $E(R)$ which you can solve. Show that for the solution $\lim_{R\to 0} E(R)$ exists and calculate its value. Show that this limiting mass has a ratio $m/q = 1$ to the charge (both in geometric units). Has this anything to do with mass/charge relations of existing particles?

Compare this result with what you obtain in a power series expansion in the gravitational constant $G$, thinking that the "effect of gravity" can be taken account of perturbatively. Show that this generally leads to diverging results for the energy at *all* orders.

What general lesson can be drawn from this toy example?