Lecture 10: Some typical problems

I) Horizon / causality problem

We have seen at the end of Lecture 9 that the angular diameter $\Delta\varphi$ above which regions on the surface of last scattering are causally disconnected in a flat matter-dominated universe is about 1.8°.

$$\Delta\varphi = (1+z_*) \frac{L^{q=\frac{1}{2}}(t_*,0)}{L^{q=\frac{1}{2}}(t_0,t_*)}$$

$$= \frac{1}{\sqrt{1+z_*}-1} \qquad (10.1)$$

which for $z_* = 1100$ gives

$$\Delta\varphi = 0.0031 \mathrel{\hat{=}} 1.8^\circ \qquad (10.2)$$

Had we replaced the calculation of the particle horizon at recombination time t_* with the radiation driven universe, $a(t) \sim t^{1/2}$, so that $q=1$, then

$$\tilde{\Delta\varphi} = (1+z_*) \frac{L^{q=1}(t_*,0)}{L^{q=\frac{1}{2}}(t_0,t_*)} = \frac{2}{3}\Delta\varphi \qquad (10.3)$$

$$\mathrel{\hat{=}} 1.2^\circ$$

making the causally disconnected regions still more numerous. A disk of radius $R\,\Delta\varphi$ on the celestial sphere of radius R has area $\pi\,(R\,\Delta\varphi)^2$. There are approximately

$$n = 4\pi R^2 \,/\, \pi R^2 \Delta\varphi^2$$

$$= \frac{4}{(\Delta\varphi)^2} = 4.1\cdot 10^3 \qquad (10.4)$$

causally disconnected regions on the celestial sphere and the problem is how to explain the fact that they apparently had been in thermodynamic equilibrium in the past (e.g. up to variations of $\Delta T / T \sim 10^{-5}$ they share the same temperature in the microwave background radiation). This is known as the horizon- or causality problem.

This problem will be avoided if the early dynamics is changed to one with slower expansion-rate, like

$$\left.\begin{aligned} a(t) &= a_0\, e^{H(t-t_0)} \\ H &= H_0 = \text{const} \\ q &= -1 \end{aligned}\right\} \text{inflationary Universe} \qquad (10.5)$$

Then $a = 0$ at $t = t_* = -\infty$ and

$$L(t_0, t_1) = c\, a(t_0) \int_{t_1}^{t_0} \frac{dt}{a(t)}$$

$$= \frac{c}{H} \left(e^{H(t_0 - t_1)} - 1 \right) \qquad (10.6)$$

so that

$$L_P(t_0) = \lim_{t_1 \to t_*} L(t_0, t_1)$$

$$= \lim_{t_1 \to -\infty} \frac{c}{H} \left(e^{H(t_0 - t_1)} - 1 \right)$$

$$= \infty \qquad (10.7)$$

<u>In other words: There is no particle horizon at any time.</u>

An exact inflationary dynamics of the kind (10.5) results from the Friedmann equations (6.16) for $\rho = p = k = 0$ and

$$\Lambda = 3H^2/c^2 \qquad (10.8)$$

i.e. $\Omega_{rad} = \Omega_{dust} = \Omega_k = 0$ and

$$\Omega_\Lambda = 1. \qquad (10.9)$$

We will discuss such models later.

We recall the Friedmann equations:

$$\boxed{\begin{aligned} \ddot{a} &= -\frac{4\pi G}{3} a \left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} a \\ \dot{a}^2 &= \frac{8\pi G}{3} \rho a^2 - k c^2 + \frac{\Lambda c^2}{3} a^2 \\ (a^3 \rho)^{\bullet} &+ (a^3)^{\bullet} \frac{p}{c^2} = 0 \end{aligned}} \qquad (6.16)$$

Alternatively, an inflationary dynamics would also result from $\Lambda = k = 0$ and

$$\rho = \frac{3}{8\pi G} H^2 = -\frac{p}{c^2} \qquad (10.10)$$

corresponding to an extreme equation of state

$$\left.\begin{aligned} p &= w \rho c^2 \\ \text{with} \quad w &= -1 \end{aligned}\right\} \qquad (10.11)$$

Certain types of scalar fields ($\rightarrow$ "Inflaton") can realize this equation of state dynamically. Then one of the fundamental questions is how to exit inflation.
And: What is that scalar field?
(Higgs?)

Intermezzo: Scalar inflation

Let us look at a typical scalar field, ϕ, which we take to be real (uncharged) and massive of "mass" m. Its Lagrange-Density is

$$\mathcal{L} = \frac{1}{2} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi - V(\phi) \tag{10.12a}$$

$$\text{where } V(\phi) = \frac{1}{2} m^2 \phi^2 + \dots \tag{10.12b}$$

Note that for the FLRW-metric (5.1f) we have (5.6)

$$g^{\alpha\beta} = \begin{pmatrix} 1 & \vec{0}^T \\ 0 & -a^{-2} \hat{g}^{ab} \end{pmatrix} \tag{10.13}$$

so that, with $\nabla_0 \phi = \frac{1}{c} \dot{\phi}$, $\dot{\phi} = \frac{\partial \phi}{\partial t}$) we have

$$\mathcal{L} = \underbrace{\frac{1}{c^2} \dot{\phi}^2}_{\downarrow} - \underbrace{\left\{ \left(\frac{1}{a(t)}\right)^2 \hat{g}^{ab} \partial_a \phi \, \partial_b \phi + V(\phi) \right\}}_{\downarrow} \tag{10.14}$$

$$= \mathcal{E}_{kin} - \mathcal{E}_{pot} \tag{10.15}$$

Energy-densities

The Euler-Lagrange-Equation corresponding to $\mathcal{L}$ is

$$\nabla_\alpha \left(\frac{\partial \mathcal{L}}{\partial(\nabla_\alpha \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \qquad (10.16)$$

or

$$\nabla_\alpha \, g^{\alpha\beta} \nabla_\beta \phi + V'(\phi) = 0 \qquad (10.17)$$

or

$$\boxed{\Box_g \phi + V'(\phi) = 0} \qquad (10.18)$$

where

$$\Box_g := g^{\alpha\beta} \nabla_\alpha \nabla_\beta \qquad (10.19)$$

is the d'Alembert- or wave-operator on the manifold (M, g).
Recall that in Minkowski-space, where $g_{\alpha\beta} = \mathrm{diag}(1, -1, -1, -1)$ we have, with $x^0 = ct$,

$$\Box = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta \qquad (10.20)$$

Laplace-Op.

Note that in curved space

$$\nabla_\beta \phi = \partial_\beta \phi \tag{10.21}$$

but

$$\nabla_\alpha \nabla_\beta \phi = \partial_\alpha \partial_\beta \phi - \Gamma^{\gamma}_{\alpha\beta} \partial_\gamma \phi \tag{10.22}$$

Hence, in an FLRW - metric

$$\Box \phi = g^{\alpha\beta} \nabla_\alpha \nabla_\beta \phi$$

$$= \nabla_0 \nabla_0 \phi - a^{-2} \hat{g}^{ab} \nabla_a \nabla_b \phi \tag{10.23}$$

Now,

$$\nabla_0 \nabla_0 \phi = \partial_0 \partial_0 \phi - \Gamma^{\alpha}_{00} \partial_\alpha \phi$$

$$= \partial_0 \partial_0 \phi = \frac{1}{c^2} \ddot{\phi} \tag{10.24}$$

Since, according to Lecture 5,

$$\Gamma^{0}_{00} = 0 = \Gamma^{a}_{00} \tag{10.25}$$

(compare equs. (5.8a, d)).

Also

$$\nabla_a \nabla_b \phi = \nabla_a \partial_b \phi$$

$$= \partial_a \partial_b \phi - \Gamma^{\alpha}_{ab} \partial_\alpha \phi$$

$$= \partial_a \partial_b \phi - \Gamma^{c}_{ab} \partial_c \phi - \Gamma^{0}_{ab} \partial_0 \phi \tag{10.26}$$

Again, from Lecture 5, i.e. (5.8.g,e),

$$\Gamma^c_{ab} = \hat{\Gamma}^c_{ab} \qquad (10.27a)$$

$$\Gamma^0_{ab} = a\,a'\,\hat{g}_{ab} \qquad (10.27b)$$

So

$$\nabla_a \nabla_b \phi = \partial_a \partial_b \phi - \hat{\Gamma}^c_{ab}\,\partial_c \phi$$
$$- a\,a'\,\hat{g}_{ab}\,\tfrac{1}{c}\,\dot{\phi}$$
$$= \hat{\nabla}_a \hat{\nabla}_b \phi - a\,a'\,\hat{g}_{ab}\,\tfrac{1}{c}\,\dot{\phi}\,. \qquad (10.28)$$

The wave operator $\Box$ applied to ϕ in FLRW-metric is therefore given by

$$\Box \phi = \frac{1}{c^2}\ddot{\phi} - a^{-2}\,\Delta_{\hat{g}}\,\phi$$
$$+ 3\,(H/c)\,\dot{\phi} \qquad (10.29)$$

Here $\Delta_{\hat{g}} = \hat{g}^{ab}\,\hat{\nabla}_a \hat{\nabla}_b$ is the Laplacian-operator on the 3-dim. Riemannian manifold $(\hat{M}, \hat{g})$ and $H = a'/a$ is the Hubble constant.

The equations of motion for the scalar field then are

$$\frac{1}{c^2}\ddot{\phi} - \left(\frac{1}{a(t)}\right)^2 \Delta_{\hat{g}}\phi$$

$$+3(H/c)\dot{\phi} + V'(\phi) = 0 \qquad (10.30)$$

Suppose ϕ is spatially constant,

$$\phi = \phi(t), \qquad (10.31)$$

then this becomes an ordinary differential equation for $\phi(t)$, given $H(t)$ and $V(\phi)$.

In addition, in models for inflation one makes the slow-roll-assumption that $|\ddot{\phi}|$ is small compared with $3(H/c)\dot{\phi}$ and $V'(\phi)$, in which case

$$3\left(\frac{H}{c}\right)\dot{\phi} + V'(\phi) = 0 \qquad (10.32)$$

"slow-roll"-equation

The energy-momentum tensor for the scalar field is given by

$$T^{\alpha}{}_{\beta} := \frac{\partial \mathcal{L}}{\partial(\nabla_\alpha \phi)} \nabla_\beta \phi - \delta^{\alpha}{}_{\beta} \mathcal{L}$$

$$= g^{\alpha\gamma} \nabla_\gamma \phi \nabla_\beta \phi - \delta^{\alpha}{}_{\beta} \mathcal{L}$$

$$= \nabla^{\alpha} \phi \nabla_\beta \phi - \delta^{\alpha}{}_{\beta} \mathcal{L} \qquad (10.33)$$

For FLRW, $\mathcal{L}$ is

$$\mathcal{L} = \frac{1}{2}\left(\frac{1}{c^2}\dot{\phi}^2 - a^{-2}\hat{g}^{ab}\partial_a\phi\,\partial_b\phi\right)$$

$$- V$$

$$= \frac{1}{2}\left(\frac{1}{c^2}\dot{\phi}^2 - \left(\frac{1}{a}\right)^2 \|d\phi\|^2_{\hat{g}}\right) - V \qquad (10.34)$$

So

$$T^{0}{}_{0} = \nabla^0 \phi \nabla_0 \phi - \mathcal{L}$$

$$= \frac{1}{2c^2}\dot{\phi}^2 + \frac{1}{2a^2}\|d\phi\|^2_{\hat{g}} + V \qquad (10.35)$$

$$T^{1}{}_{1} = \nabla^1 \phi \nabla_1 \phi - \mathcal{L}$$

$$= -\frac{1}{a^2}\hat{g}^{11}(\partial_1\phi)^2 - \frac{1}{2c^2}\dot{\phi}^2$$

$$+ \frac{1}{2a^2}\hat{g}^{ab}(\partial_a\phi)(\partial_b\phi) + V$$

$$= -\frac{1}{2c^2}\dot{\phi}^2 + \frac{1}{6a^2}\|d\phi\|^2_{\hat{g}} + V \qquad (10.36)$$

Where we assumed isotropy

$$\hat{g}^{11}(\partial_1 \phi)^2 = \hat{g}^{22}(\partial_2 \phi)^2$$
$$= \hat{g}^{33}(\partial_3 \phi)^2$$
$$= \frac{1}{3}\hat{g}^{ab}(\partial_a \phi)(\partial_b \phi) \qquad (10.37)$$

Hence, energy-density and pressure of ϕ are given by

$$c^2 \varrho = T^0{}_0$$
$$= \frac{1}{2c^2}\dot{\phi}^2 + \frac{1}{2a^2}\|d\phi\|^2_{\hat{g}} + V \qquad (10.38)$$

$$p = -T^1{}_1 = -T^2{}_2 = -T^3{}_3$$
$$= \frac{1}{2c^2}\dot{\phi}^2 - \frac{1}{6a^2}\|d\phi\|^2_{\hat{g}} - V \qquad (10.39)$$

If ϕ is homogeneous, i.e.

$$d\phi = 0 \qquad (10.40)$$

and

$$V \gg (\dot{\phi}/c)^2 \qquad (10.41)$$

We have, approximately

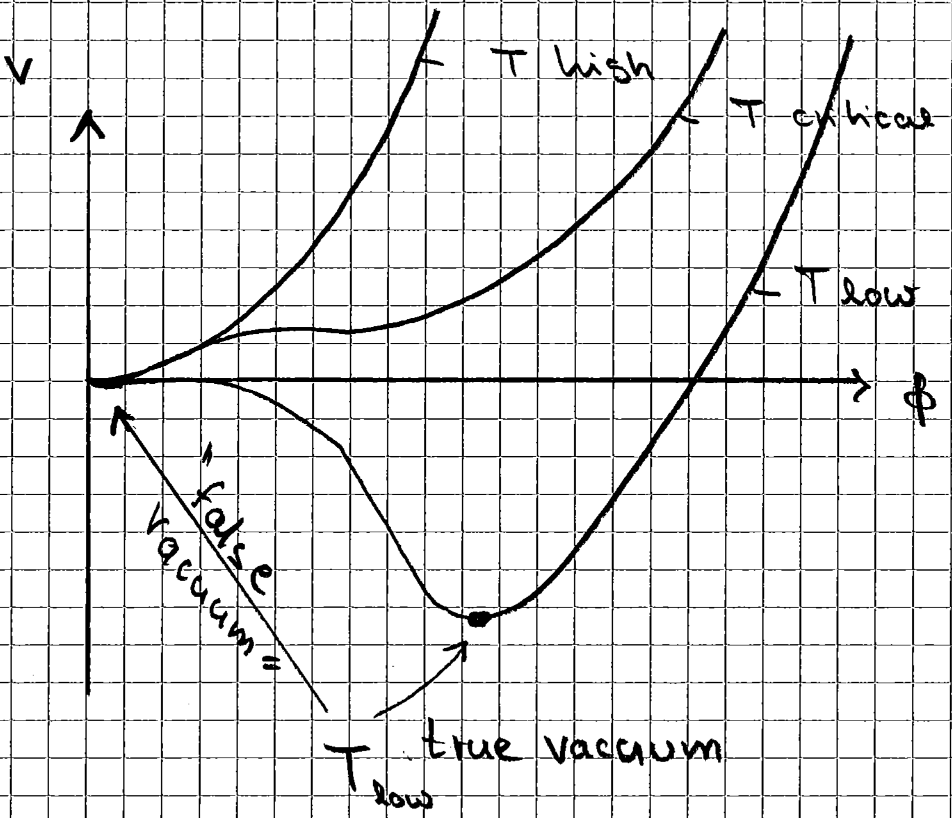
$$c^2 \varrho = -p \cong V \qquad (10.42)$$

i.e. the condition (10-11) that gives a cosmological constant driving inflation.

Often, on a phenomenological basis, temperature-dependent potentials are employed, like

$$V_{eff}(\phi, T) = \lambda \phi^4 - b \phi^3 + a T^2 \phi^2 \qquad 10.43$$

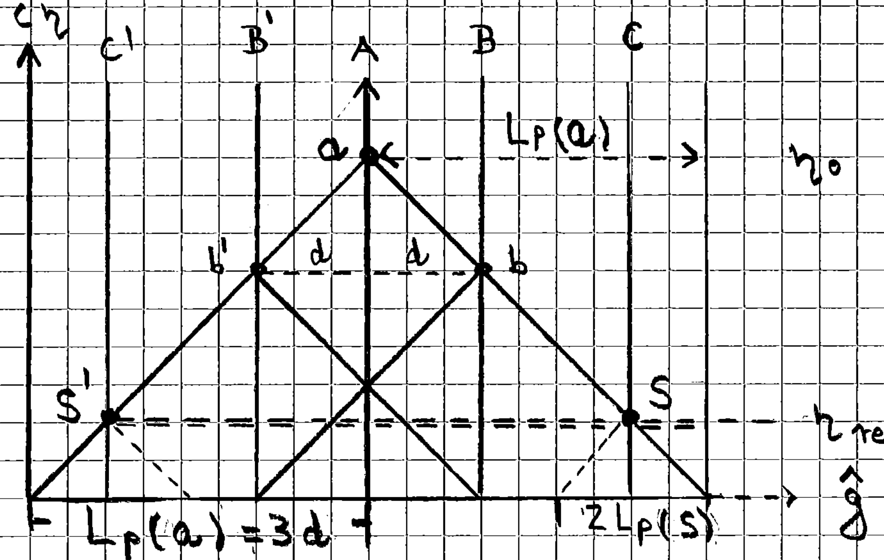
which for high T has a global minimum at $\phi = 0$ (ground state) but then develops another one once T drops below a certain value



– End of Intermezzo –

Back to geometry of Inflation

The conditions discussed so far are sufficient, but not necessary! It is, in fact, not necessary for the inflationary phase to last an infinite amount of time. To see this we look at the following diagram, similar to that on p. (9.8) ($\rightarrow$ conformal diagram):



(10.44)

We are on A at event a. We look at neighbouring objects B and B' and see them at b and b', respectively. B and B' lie in opposite spatial directions just far enough from each other such that their backward lightcones do not intersect. (they are causally disjoint).

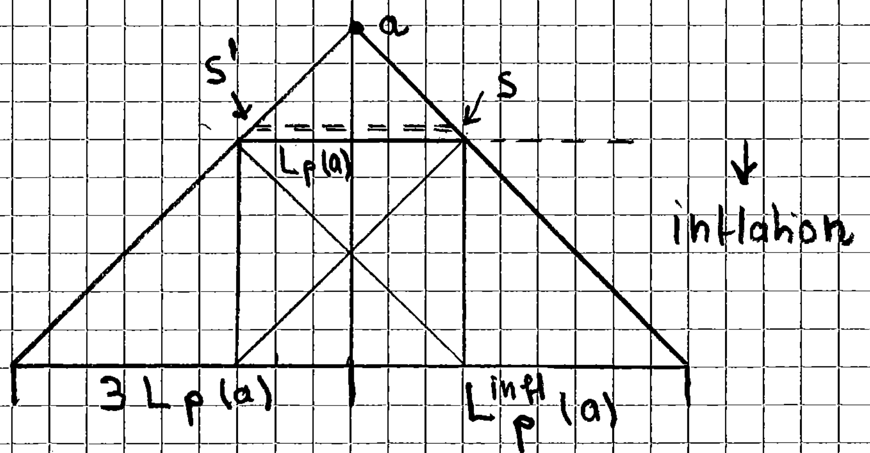
The doubly-dashed line ==== marks the surface of recombination, below which we cannot see.

A "horizon problem" exists if we <u>see</u> points on that surface which are causally disjoint.

Now, our lightcone intersects the recombination surface at S and S'. Hence, a problem only arises if the particle horizon of points on that surface, $L_p(\eta_{rec})$, is smaller than the diameter <u>of</u> the visible universe, which is $\overline{SS'}$. A sufficient condition for that not to occur is that the particle horizon without inflation at a, $L_p(a)$ is less than $\frac{1}{3}$ times that with inflation, or

$$L_p^{(infl)}(a) > 3\, L_p(a) \qquad (10.45)$$

$\Leftrightarrow$ inflation must extend conformal lifetime by at least factor 3.



(10.46)

II) Flatness problem

Currently we have

$$\Omega_m + \Omega_\Lambda \cong 1 \qquad (10.47)$$

i.e. $$\Omega_k \cong 0 \iff k = 0 \qquad (10.48)$$

to a high accuracy. The general (i.e. time dependent) Ω-parameters are

$$\Omega_m(t) := \frac{8\pi G \, \rho_m(t)}{3 H^2(t)} \qquad (10.49a)$$

$$\Omega_\Lambda(t) := \frac{\Lambda c^2}{3 H_0^2} \qquad (10.49b)$$

$$\Omega_k(t) := \frac{-kc^2}{H^2(t)\, a^2(t)} = \frac{-kc^2}{\dot{a}^2(t)} \qquad (10.49c)$$

The problem arises because the value 1 for $\Omega(t) = \Omega_m(t) + \Omega_\Lambda(t)$ is unstable for decelerated expansion, the the Friedmann equation will drive $\Omega(t)$ away from 1

$$\Omega(t) - 1 = -\Omega_k = \frac{kc^2}{\dot{a}^2(t)}$$

$$\Rightarrow \frac{d}{dt}\left(\Omega(t) - 1\right) = -2\,\frac{kc^2}{\dot{a}^3}\,\ddot{a} \qquad (10.50)$$

or

$$\frac{d}{dt}\left(\Omega(t)-1\right) = 2\,k\,c^2 \quad \frac{q(t)}{H(t)} \tag{10.51}$$

with $$q(t) := -\frac{\ddot{a}\,a}{\dot{a}^2} \tag{10.52}$$

$$H(t) := \frac{\dot{a}(t)}{a(t)} \tag{10.53}$$

We assume $H(t) > 0$ and $q(t) > 0$. Then, for $(\Omega(t)-1) > 0$ we have $-\Omega k > 0 \Rightarrow k = +1$ and the right-hand side of (10.51) is positive; If $(\Omega(t)-1) < 0$ we have $-\Omega k < 0 \Rightarrow k = -1$ and the right-hand side is negative. Hence

$$\left.\begin{aligned} \Omega(t) > 1 &\Rightarrow \dot{\Omega}(t) > 0 \\ \Omega(t) < 1 &\Rightarrow \dot{\Omega}(t) < 0 \end{aligned}\right\} \tag{10.54}$$

$\Rightarrow$ $\Omega(t) = 1$ is unstable.

if $H(t) > 0$, $q(t) > 0$.

Conversely, if $H(t) > 0$ but $q(t) < 0$, i.e. accelerated expansion, then

$$\left.\begin{aligned} \Omega(t) > 1 &\Rightarrow \dot{\Omega}(t) < 0 \\ \Omega(t) < 1 &\Rightarrow \dot{\Omega}(t) > 0 \end{aligned}\right\} \quad (10.55)$$

$\Rightarrow$ $\Omega(t) = 1$ is stable

if $H(t) > 0$, $q(t) < 0$.

$\Rightarrow$ Inflation drives us towards small Ω_K and thus may "explain" flatness.

III Origin of Structures

If the universe always evolved by matter- or radiation dominated dynamics, we would face the following problem: Let

$$a(t) \sim t^n,$$

where $n = 1/2$ (rad. dom.)
or $n = 2/3$ (matter dom.)
10.56

A co-expanding object of linear dimension D_0 at time $t = t_0$ has linear extent $D(t_1)$ at time t_1, given by

$$D(t_1) = D_0 \left(\frac{t_1}{t_0}\right)^n \tag{10.57}$$

The Hubble radius (at which objects move with velocity of light away from us at the moment they emitted the light; see Lecture 9, eq.(9.40)) is

$$l_H(t) = \frac{c}{H(t_{max}(t))} \tag{10.58}$$

Here $t_{max}(t)$ is the time at which $t_1 \mapsto l(t, t_1)$ is maximal

From Lecture 9 we know that for constant q (eqn. (9.63)) :

$$t'_{max}(t) = t\,(1+q)^{\left(\frac{1+q}{q}\right)}$$

$$= H^{-1}(t)\,(1+q)^{\frac{1+2q}{q}} \qquad (10.59)$$

where $n = 1/(1+q)$ (10.60)

Note also that in the $a(t) \sim t^n$ models

$$H(t) = \frac{n}{t} = \frac{1}{1+q} \cdot \frac{1}{t} \qquad (10.61)$$

Hence

$$\frac{l_H(t_0)}{l_H(t_1)} = \frac{H(t_{max}(t_1))}{H(t_{max}(t_0))}$$

$$= \frac{t_{max}(t_0)}{t_{max}(t_1)}$$

$$= \left(\frac{t_0}{t_1}\right) \qquad (10.62)$$

On the other hand :

$$\frac{D(t_{max}(t_0))}{D(t_{max}(t_1))} = \left[\frac{t_{max}(t_0)}{t_{max}(t_1)}\right]^n$$

$$= \left(\frac{t_0}{t_1}\right)^n \qquad (10.63)$$

So that

$$\frac{l_H(t_0)}{D(t_{max}(t_0))} \cdot \frac{D(t_{max}(t_1))}{l_H(t_1)} = \left(\frac{t_0}{t_1}\right)^{1-n} \qquad (10.64)$$

or

$$\frac{l_H(t_1)}{D(t_{max}(t_1))} = \frac{l_H(t_0)}{D(t_{max}(t_0))} \cdot \left(\frac{t_1}{t_0}\right)^{1-n} \qquad (10.65)$$

In our cases ($n = 1/2, 2/3$) have $(1-n) > 0$ so the right hand side vanishes as $t_1 \to 0$. Hence

$$\lim_{t_1 \to 0} \left\{ \frac{l_H(t_1)}{D(t_{max}(t_1))} \right\} = 0 \qquad (10.66)$$

⇒ The size of the comoving (co-expanding - contracting) object always exceeds the size of the Hubble horizon. Hence, going back in time the parts of the object lie in causally disconnected regions.

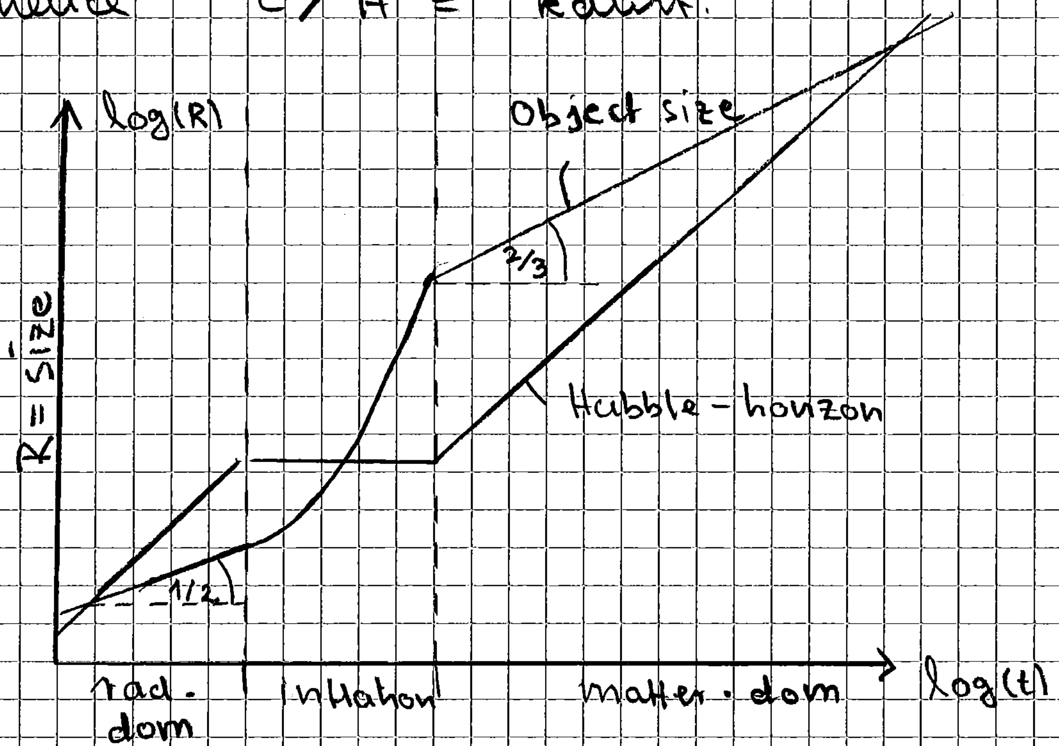
How can this be compatible with early structure formation?
(by early density fluctuations)

This difficulty disappears if we assume an early inflationary phase of exponential expansion, during which

$$D(t_1) = D(t_0)\, e^{-H(t_0 - t_1)} \qquad (10.67)$$

and $H_0 = H_1 = H = \text{const.}$

hence $c / H = \text{konst.}$ (10.68)



(10.69)

In radiation- and matter dominated phase Hubble horizon grows faster than object size in time. During inflation the opposite is true. So, going backwards in time, inflation shrinks object below Hubble-horizon size.

IV The origin of Λ

If Λ is not regarded as part of the l.h.s. of Einstein's equation, i.e. not as a fundamental constant of nature the value of which we just have to accept, then the question as to its origin has to be faced.

A matter-caused Λ could be that of the matter's vacuum-expectation value of $T_{\mu\nu}$:

$$\langle 0 | T_{\mu\nu} | 0 \rangle = \langle T_{\mu\nu} \rangle_{vac}$$

$$= C\, g_{\mu\nu} \qquad (10.70)$$

Since $\langle T_{\mu\nu} \rangle_{vac}$ in SR must be Poincaré-invariant.

Comparison with "zero point" energy of harmonic oscillator in QM

$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2 \qquad (10.71)$$

Heisenberg's uncertainty-rel. is

$$\Delta q \cdot \Delta p \geq \frac{\hbar}{2} \qquad (10.72)$$

Minimal energy compatible with uncertainty

$$E_{min} = \langle H \rangle_{vac} = \frac{1}{2}\omega\hbar \qquad (10.73)$$

Have

$$\langle q^2 \rangle_{vac} = \langle \Delta q^2 \rangle_{vac} = \frac{\hbar}{2m\omega} \qquad (10.74a)$$

$$\langle p^2 \rangle_{vac} = \langle \Delta p^2 \rangle_{vac} = \hbar m\omega/2 \qquad (10.74b)$$

$\Rightarrow$ <u>pure fluctuation phenomena</u>

QED : Field operators are operator-valued distributions

$$\vec{E}_f(x) = \int_{\mathbb{R}^3} d^3y \, \underbrace{\vec{E}}_{\text{Op.}}(\vec{x}+\vec{y}) \, \underbrace{f}_{\text{test-function}}(\vec{y}) \qquad (10.75)$$

Energy-density in vacuum

$$\langle \vec{E}_f^2(x) \rangle_{vac} = 2 \int \frac{d^3k}{(2\pi)^3} \frac{\hbar\omega}{2} |\tilde{f}(\vec{k})|^2 \qquad (10.76)$$

Choose f such

$$\tilde{f}(\vec{k}) = \begin{cases} 1 & \text{for } \|\vec{k}\| < K \\ 0 & \text{for } \|\vec{k}\| \geq K \end{cases} \qquad (10.77)$$

Then

$$\langle \vec{E}_{\varphi}^{2}(x) \rangle_{vac} = c \frac{\hbar K^4}{4\pi} \qquad (10.78)$$

If you interpret this as cosm. constant Λ_{QED}, then

$$\Lambda_{QED} = \frac{8\pi G}{c^4} c \frac{\hbar K^4}{4\pi} = \frac{2 G \hbar}{c^3} K^4$$

$$= 2 \ell_p^2 K^4$$

$$= \frac{2}{\ell_p^2} (\ell_p K)^4 \qquad (10.79)$$

Where

$$\ell_p = \left(\frac{\hbar G}{c^3}\right)^{1/2} = \text{Planck-length}$$

$$= 1.6 \cdot 10^{-35} \text{ m} \qquad (10.80)$$

Similar expressions hold for other quantum fields

Note: The Compton wavelength of a particle of mass m exceeds its Schwarzschild radius R if

$$\lambda_c := \frac{\hbar}{mc} > R = \frac{Gm}{c^2}$$

$$\Leftrightarrow \quad m < \left(\frac{\hbar c}{G}\right)^{1/2} =: m_p \qquad (10.81)$$

$$\Leftrightarrow \quad R < \left(\frac{\hbar G}{c^3}\right)^{1/2} =: l_p \qquad (10.82)$$

$$m_p = 2.176 \times 10^{-8}\ \mathrm{Kg}$$
$$= 1.22 \times 10^{19}\ \mathrm{GeV}/c^2$$
$$= 1.311 \cdot 10^{19}\ u \qquad (10.83)$$

Planck mass

Now

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2} \qquad (10.84)$$

$$\leadsto \quad \Omega_{QFT} = \frac{\Lambda_{QFT}\, c^2}{3H_0^2}$$

$$= \frac{1}{3}\left(\frac{c/H_0}{l_p}\right)^2 (l_p K)^4 \qquad (10.85)$$

Note : $\dfrac{c/H_0}{l_p} = \dfrac{\text{Hubble Radius}}{\text{Planck-Length}}$

$$\cong 10^{61} \quad \ddot{\nabla}\ \ddot{\nabla} \qquad (10.86)$$

If $K \cong 1/l_p$ ($\to$ quantum gravity)

$$\Rightarrow \Omega_\Lambda^{(QG)} \cong 10^{122} \qquad (10.87)$$

"<u>the worst prediction ever</u>"

The real question then is: Why is QFT contribution so little to Ω_Λ, i.e. why is Λ so small in view of Λ_{QFT}?

If we started with a Λ on the left hand side then $\langle T_{\mu\nu}\rangle_{vac}$ plus that would merge to a total

$$\Lambda = \Lambda_{EE} + \Lambda_{QFT} \qquad (10.88)$$

from Einsteins Eq

and one would face a "fine-tuning problem", of why both Λ's coincide up to 10^{-122}?

— All that is not understood at all! —

See, eg. D. Gr and N. Straumann

https://arxiv.org./pdf/astro-ph/0009368.pdf

(in german)