Christmas exercises for the lecture on Special Topics in GR & Relativistic Cosmology

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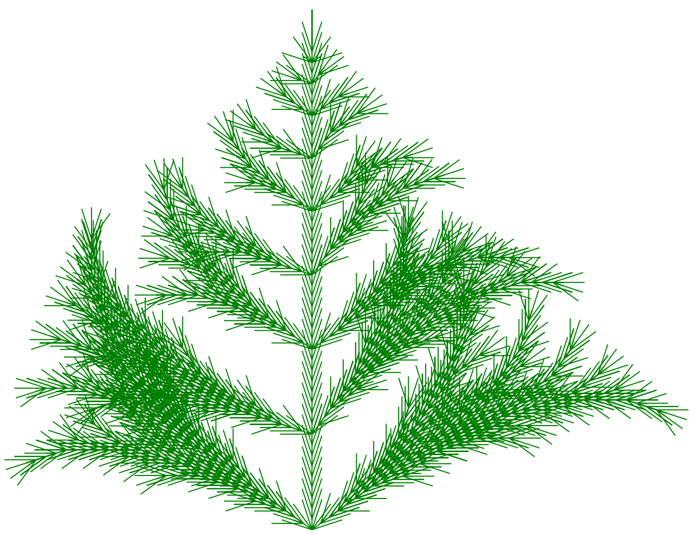

stay healthy!!!

Sheet 5

Problem 1

The current temperature of the CMB is given by 2.72548 ± 0.00057 K. Calculate the redshift of the epoch at which the maximum of the Planck distribution corresponds to a frequency for which $h\nu$ equals the ionisation-energy of hydrogen. Why is this redshift much larger than $z_{re} = 1100$ that is usually quoted for the surface of last scattering? (Hint: For total ionisation it would be sufficient if the density of photons above $13.6\,eV$ exceeds the baryon density.)

Problem 2

At the end of Lecture 9-2 we calculated the angular separation $\Delta\varphi$ above which points on the surface of last scattering are causally disconnected. We obtained

$$\Delta\varphi = (1 + z_{re})\,\frac{L(t_{re}, 0)}{L(t_0, t_{re})}\,, \tag{1}$$

and calculated $L(t_{re}, 0)$ as well as $L(t_0, t_{re})$ assuming a matter-dominated flat universe. Repeat this calculation by calculating $L(t_{re}, 0)$ for a radiation-dominated universe.

Problem 3

Consider FLRW models with $\Omega_\Lambda = 0$ and $\Omega_k < 0$. The last condition means that $k = 1$, so that the spatial Riemannian manifold $(\hat{M}, \hat{g})$ is of constant *positive* sectional curvature. According to a theorem in differential topology it must be compact if it is assumed to be complete (as we shall do). If, in addition, we assume it to be simply connected it must be the 3-sphere S^3 (as a result of the Poincaré conjecture, which is now known to be true).

3.1) Show (without any calculation) that such models always re-collapse; i.e., a "Big-Bang" is always followed by a "Big-Crunch" in finite cosmological time. Let $t = 0$ be the cosmological time at which initially $a = 0$.

3.2) Integrate the Friedmann equations analytically under the additional assumption that $\Omega_{rad} = 0$ and calculate the *lifetime of the universe*; that is, the time t_* from the "Big-Bang" to the "Big-Crunch".

3.3) Assume light-flash at $t = 0$ at some point, say $\chi = 0$, in space. Calculate the radial coordinate distance $\chi(t)$ of the light front of that flash at time $t > 0$. At what time will the light front have swept through the entire (compact!) universe? (For a 3-sphere, χ is a polar angle that ranges from 0 to π.) So how long would an observer have to wait (assuming he/she had existed from the Big-Bang on) until he/she has seen the entire universe?

3.4) Repeat steps 3.2) and 3.3) by now assuming Ω_{dust} rather than Ω_{rad} to vanish.