Quantum versus Gravity

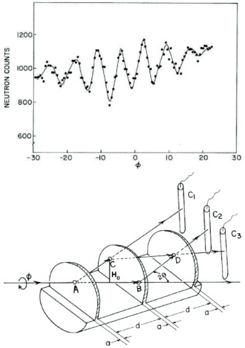
– a conceptual discussion –

Domenico Giulini

ZARM Bremen
ITP and Riemann Centre at University of Hannover

CERS 7
Bremen 16.-18. February 2017

QM & Gravity: Tested so far



Colella Overhauser Werner, PRL 1975

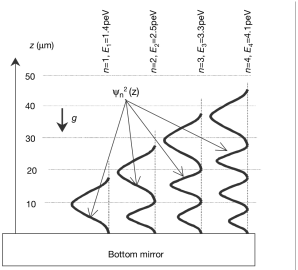


Figure 1 Wavefunctions of the quantum states of neutrons in the potential well formed by the Earth's gravitational field and the horizontal mirror. The probability of finding neutrons at height z, corresponding to the nth quantum state, is proportional to the square of the neutron wavefunction $\psi_n^2(z)$. The vertical axis z provides the length scale for this phenomenon. E_n is the energy of the nth quantum state.

Nesvizhevsky et al., Nature 2002

$$i\hbar\dot{\Psi} = -\frac{\hbar^2}{2m_i}\Delta\Psi + V_{\mathrm{grav}}\Psi$$

$$V_{\mathrm{grav}} = m_g g z$$

Einstein's Equivalence Principle (EEP)

- **Universality of Free Fall (UFF):** "Test bodies" determine path structure on spacetime (not necessarily of Riemannian type). UFF-violations are parametrised by the Eötvös factor

$$\eta(A,B) := 2\,\frac{|a(A)-a(B)|}{|a(A)+a(B)|} \tag{1}$$

- **Local Lorentz Invariance (LLI):** Local non-gravitational experiments exhibit no preferred directions in spacetime, neither timelike nor spacelike. Possible violations of LLI concern, e.g., variations in $\Delta c/c$.
- **Universality of Gravitational Redshift (UGR):** "Standard clocks" are universally affected by the gravitational field. UGR-violations are parametrised by the α-factor

$$\frac{\Delta\nu}{\nu} = (1+\alpha)\frac{\Delta U}{c^2} \tag{2}$$

⇒ Geometrisation of gravity and unification with inertial structure. Gravity is not a force. All matter components "see" the **same** geometry!

Mechanisms for EP-violations

- Suppose next to the gravitational potential

$$V_{AE} = -G\frac{M_A M_E}{r_{AE}} \tag{3}$$

there were another long-ranged potential

$$W_{AE} = -H\frac{Q_A Q_E}{r_{AE}} \tag{4}$$

coupling to charges $Q \not\propto M$.

- This resulted in an effective $1/r$ potential with coupling

$$G_{AE} = G\bigl(1 + h\, q_A q_E\bigr) \tag{5}$$

where $h := H/G$ and $q_A = Q_A/M_A$ is the specific charge.

- Violations of UFF result if the latter is not universal:

$$\eta(A, B) \approx h q_E (q_A - q_B)\,, \tag{6}$$

- Long-ranging scalar fields typically exist in fundamental unifying theories (Kaluza-Klein, Strings). η-factors of $\approx 10^{-15}$ seem compatible with string phenomenology (Damour & Polyakov 1994).

Levels of verification of EEP

- **UFF:** Typical results from torsion-balance experiments by the "Eöt-Wash" group between 1994-2008 are

$$\eta(Al, Pt) = (-0.3 \pm 0.9) \times 10^{-12}, \quad \eta(Be, Ti) = (0.3 \pm 1.8) \times 10^{-13} \qquad (7)$$

  Planned improved levels are $5 \cdot 10^{-16}$ (MICROSCOPE) and 10^{-18} (STEP).
- **LLI:** Currently best Michelson-Morley type experiments give (Herrmann *et al.* 2009)

$$\frac{\Delta c}{c} < 10^{-17} \qquad (8)$$

- **UGR:** Absolute redshift with H-maser clocks in space (1976, $h = 10\,000\,\mathrm{Km}$) and relative redshifts using precision atomic spectroscopy (2007) give

$$\alpha_{\mathrm{abs}} < 2 \times 10^{-4} \qquad \alpha_{\mathrm{rel}} < 4 \times 10^{-6} \qquad (9)$$

  - In Feb. 2010 Müller *et. al.* claimed improvements by 10^4. This is not widely accepted (see below). Long-term expectation in future space missions is to get to 10^{-10} level.
  - In Sept. 2010 Chou *et al.* report measurability of gravitational redshift on Earth for $h = 33\,\mathrm{cm}$ using Al^+-based optical clocks ($\Delta t/t < 10^{-17}$).

UFF – UGR dependence: Energy conservation

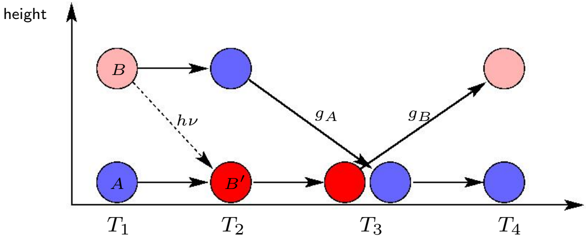
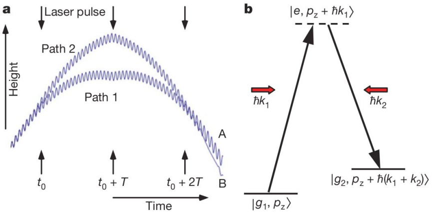


Figure: Gedankenexperiment by NORDTVEDT to show that energy conservation connects violations of UFF and UGR. Considered are two copies of a system that is capable of 3 energy states A, B, and B' (blue, pink, and red), with $E_A < E_B < E_{B'}$. Initially system 2 is in state B and placed a height h above system 1 which is in state A. At time T_1 system 2 makes a transition $B \rightarrow A$ and sends out a photon of energy $h\nu = E_B - E_A$. At time T_2 system 1 absorbs this photon, which is now blue-shifted, and makes a transition $A \rightarrow B'$. At T_3 system 2 has been dropped from height h with acceleration g_A, has hit system 1 inelastically, leaving one system in state A and at rest, and the other system in state B with an upward motion with kinetic energy $E_{\text{kin}} = M_A g_A h + (E_{B'} - E_B)$. The latter motion is decelerated by g_B, which may differ from g_A. At T_4 the system in state B has climbed to the **same** height h by energy conservation. Hence have $E_{\text{kin}} = M_B g_B h$ and therefore $M_A g_A h + M_{B'} c^2 = M_B c^2 + M_B g_B h$, from which we get

$$\frac{\delta\nu}{\nu} = \frac{(M_{B'} - M_A) - (M_B - M_A)}{M_B - M_A} = \frac{g_B h}{c^2}\left[1 + \frac{M_A}{M_B - M_A}\frac{g_B - g_A}{g_B}\right] \quad \text{(10a)}$$

$$\Rightarrow \alpha = \frac{M_A}{M_B - M_A}\frac{g_B - g_A}{g_B} =: \frac{\delta g/g}{\delta M/M} \quad \text{(10b)}$$

An alleged 10^4-improvement of UGR-tests: What is a clock?



(Müller *et al.*, Nature 2010)

Have (using $k := \Delta p/\hbar$)

$$\Delta\phi = k\,T^2 \cdot g^{(\mathrm{Cs})} = k\,T^2 \cdot \frac{m_g^{(\mathrm{Cs})}}{m_i^{(\mathrm{Cs})}} \cdot g^{\mathrm{Earth}}$$
$$= k\,T^2 \cdot \frac{m_g^{(\mathrm{Cs})}}{m_i^{(\mathrm{Cs})}} \cdot \frac{m_i^{(\mathrm{Ref})}}{m_g^{(\mathrm{Ref})}} \cdot g^{(\mathrm{Ref})} = \eta(\mathrm{Cs}, \mathrm{Ref}) \cdot kT^2 \cdot g^{(\mathrm{Ref})} \tag{11}$$

- ▶ Proportional to (1+Eötvös-factor) in UFF-violating theories.
- ℚ How does it depend on α in UGR-violating theories? Müller *et al.* argue for $\propto (1+\alpha)$ by *representation dependent* interpretation of $\Delta\phi$ as a mere redshift.

The "clocks-from-rocks" dispute

- A clock ticking at frequency ω suffers gravitational phase-shift in Kasevich-Chu situation of

$$\begin{aligned}\Delta\phi &= \Delta\omega T \\ &= \omega\frac{\Delta U}{c^2}T \\ &= \omega\frac{g\,\Delta h}{c^2}T \\ &= \omega\frac{g\,\Delta p}{mc^2}\,T^2 \\ &= \left(\frac{\omega}{mc^2/\hbar}\right)\,g\,T^2\,\frac{\Delta p}{\hbar}\,.\end{aligned} \tag{12}$$

This equals (11) if

$$\omega = mc^2/\hbar \tag{13}$$

- Objection!

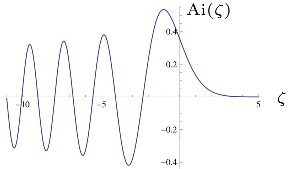
Homogeneous static gravitational field: Bound states

- Time independent Schrödinger equation in linear potential $V(z) = m_g g z$ is equivalent to:

$$\left(\frac{d^2}{d\zeta^2} - \zeta\right)\psi = 0\,, \quad \zeta := \kappa z - \varepsilon \tag{14}$$

where

$$\kappa := \left[\frac{2m_i\, m_g\, g}{\hbar^2}\right]^{\frac{1}{3}}\,, \quad \varepsilon := E \cdot \left[\frac{2m_i}{m_g^2\, g^2\, \hbar^2}\right]^{\frac{1}{3}}\,. \tag{15}$$



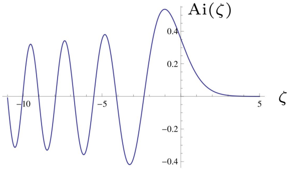
- Complement by hard (horizontal) wall $V(z) = \infty$ for $z \leq 0$ get energy eigenstates from boundary condition $\psi(z=0) = 0$, hence $\varepsilon = -z_n$:

$$E(n) = -z_n \left[\frac{m_g^2}{m_i} \cdot \frac{g^2 \hbar^2}{2}\right]^{\frac{1}{3}}\,. \tag{16}$$

Homogeneous static gravitational field: Free fall

- Classical turning point z_{turn}

$$m_g g\, z_{\text{turn}} = E \Leftrightarrow z_{\text{turn}} = \frac{E}{m_g g} = \frac{\varepsilon}{\kappa} \Leftrightarrow \zeta = 0\,. \tag{17}$$



- Large $(-\zeta)$ - expansion of Airy function gives decomposition of ingoing and outgoing waves with phase delay of

$$\Delta\theta(z) = \frac{4}{3}\Big[\kappa\big(E/m_g g - z\big)\Big]^{3/2} - \pi/2 \tag{18}$$

corresponding to a "Peres time of flight" (Davies 2004)

$$T(z) := \hbar\frac{\partial\Delta\theta}{\partial E} = 2\,\frac{\hbar\kappa^{\frac{3}{2}}}{m_g g}\sqrt{z_{\text{turn}} - z} = 2\sqrt{\frac{m_i}{m_g}}\cdot\sqrt{2\cdot\frac{z_{\text{turn}} - z}{g}} \tag{19}$$

- For other than linear potential we will *not* get *classical* return time.

UFF in QM

Proposition. Consider a particle of inertial mass m in spatially homogeneous force field $\vec{F}(t)$. The classical trajectories solve

$$\ddot{\vec{\xi}}(t) = \vec{F}(t)/m \tag{20}$$

Let $\vec{\xi}(t)$ be a solution with $\vec{\xi}(0) = \vec{0}$ and some initial velocity. The map $\Phi : \mathbb{R}^4 \to \mathbb{R}^4$ defines a *rigid freely-falling frame*:

$$\Phi(t, \vec{x}) = \big(t, \vec{x} + \vec{\xi}(t)\big)\,. \tag{21}$$

Then ψ solves the forced Schrödinger equation

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\Delta - \vec{F}(t)\cdot\vec{x}\right)\psi \tag{22}$$

iff

$$\psi = \big(\exp(i\alpha)\,\psi'\big)\circ\Phi^{-1}\,, \tag{23}$$

where ψ' solves the free Schrödinger equation and

$$\alpha(t,\vec{x}) = \frac{m}{\hbar}\left\{\dot{\vec{\xi}}(t)\cdot\big(\vec{x}+\vec{\xi}(t)\big) - \frac{1}{2}\int^t dt'\,\|\dot{\vec{\xi}}(t')\|^2\right\}\,. \tag{24}$$

S & KG: Inertial motion

- Galilei symmetry is a suitable $1/c \to 0$ limit (contraction) of Poincaré symmetry. Likewise, the Schrödinger equation for ψ is a suitable $1/c \to 0$ limit of the Klein-Gordon equation for ϕ if we set

$$\phi(t,\vec{x}) = \exp\{-imc^2\, t/\hbar\}\, \psi(t,\vec{x})\,. \tag{25}$$

- The Klein-Gordon field transforms as scalar

$$\phi'(t',\vec{x}\,') = \phi(t,\vec{x})\,. \tag{26}$$

Hence (25) implies

$$\psi'(t',\vec{x}\,') = \exp\{-imc^2\,(t-t')/\hbar\}\, \psi(t,\vec{x})\,. \tag{27}$$

- Using

$$t = \frac{t' + \vec{x}\,'\cdot\vec{v}/c^2}{\sqrt{1-v^2/c^2}} = t' + c^{-2}\big(\vec{x}\,'\cdot\vec{v} + t'v^2/2\big) + \mathcal{O}(1/c^4)\,, \tag{28}$$

the $1/c \to 0$ limit of Poincaré symmetry by proper representations turns into Galilei symmetry by non-trivial ray representations

$$\psi'(t',\vec{x}\,') = \exp\{-im(\vec{x}\,'\cdot\vec{v} + t'v^2/2)/\hbar\}\ \psi(t,\vec{x})\,. \tag{29}$$

S & KG: Rigid accelerations

- In Minkowski space, rigid motions in x-direction and of arbitrary acceleration of a body parametrised by ξ are given by family of timelike lines $\tau \mapsto \big(ct(\tau,\xi)\,,\,x(\tau,\xi)\big)$, where

$$ct(\tau,\xi) = c\int^{\tau} d\tau'\cosh\chi(\tau') + \xi\,\sinh\chi(\tau)\,, \tag{30a}$$

$$x(\tau,\xi) = c\int^{\tau} d\tau'\sinh\chi(\tau') + \xi\,\cosh\chi(\tau)\,. \tag{30b}$$

  Here τ is eigentime of body element $\xi = 0$ and $\chi(\tau) = \tanh^{-1}(v/c)$ is rapidity of all body elements at τ.

- The Minkowski metric in co-moving coordinates (τ,ξ) reads $(g := c\dot{\chi})$

$$ds^2 = c^2\,dt^2 - d\vec{x}^2 = \left(1 + \frac{g(\tau)\,\xi}{c^2}\right)c^2\,d\tau^2 - d\vec{\xi}^2\,. \tag{31}$$

- Klein-Gordon equation in co-moving coordinates

$$\Big\{\Box_g + m^2\Big\}\phi = \Big\{(-\det g)^{-1/2}\;\partial_a\big[(-\det g)^{1/2}\;g^{ab}\partial_b\big] + m^2\Big\}\,\phi = 0\,. \tag{32}$$

S & KG: Rigid accelerations

- In analogy to (25) write

$$\phi(t,\vec{x}) = \exp\{-imc^2\,\tau/\hbar\}\,\psi(t,\vec{x}) \tag{33}$$

and take $1/c^2 \to 0$ limit; get

$$i\hbar\partial_\tau\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\vec{\xi}^2} + mg(\tau)\xi\right)\psi\,. \tag{34}$$

This corresponds to particle in homogeneous but time-dependent gravitational field pointing in negative ξ-direction.

- Note that again ϕ transformed as scalar (compare (26))

$$\phi^{\mathrm{inert}}(t,\vec{x}) = \phi^{\mathrm{acc}}(\tau,\vec{\xi}) \tag{35}$$

but that again this is not true for ψ, where (compare (25))

$$\begin{aligned}\phi^{\mathrm{inert}}(t,\vec{x}) &= \exp\{-imc^2\,t/\hbar\}\,\psi^{\mathrm{inert}}(t,\vec{x})\\ \phi^{\mathrm{acc}}(\tau,\vec{\xi}) &= \exp\{-imc^2\,\tau/\hbar\}\,\psi^{\mathrm{acc}}(\tau,\vec{\xi})\,,\end{aligned} \tag{36}$$

- Hence (compare (27))

$$\psi^{\mathrm{acc}}(\tau,\vec{\xi}) = \exp\{-imc^2\,(t-\tau)/\hbar\}\;\psi^{\mathrm{inert}}(t,\vec{x})\,. \tag{37}$$

Schrödinger-Newton equation

- Consider Einstein – Klein-Gordon system

$$R_{ab} - \tfrac{1}{2} g_{ab} R = \tfrac{8\pi G}{c^4}\; T^{KG}_{ab}(\phi)\,, \qquad (\Box_g + m^2)\phi = 0 \tag{38}$$

- Make WKB-like ansatz

$$\phi(\vec{x}, t) = \exp\left(\frac{ic^2}{\hbar} S(\vec{x}, t)\right) \sum_{n=0}^{\infty} \left(\frac{\sqrt{\hbar}}{c}\right)^n a_n(\vec{x}, t), \tag{39}$$

and perform $1/c$ expansion (D.G. & A. Großardt 2012).

- Obtain

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\Delta + mV\right)\,\psi \tag{40}$$

where

$$\Delta V = 4\pi G\big(\rho + m|\psi|^2\big)\,. \tag{41}$$

- Ignoring self-coupling, this just generalises previous results and conforms with expectations.

Schrödinger-Newton equation

- Without external sources get **"Schrödinger-Newton equation"** (Diosi 1984, Penrose 1998):

$$\boxed{i\hbar\,\partial_t\psi(t,\vec{x}) = \left(-\frac{\hbar^2}{2m}\Delta - Gm^2\int\frac{|\psi(t,\vec{y})|^2}{\|\vec{x}-\vec{y}\|}\,d^3y\right)\psi(t,\vec{x})} \qquad (42)$$

- It can be derived from the action

$$\begin{aligned}\mathcal{S}[\psi,\psi^*] = \int dt\Bigg\{&\frac{i\hbar}{2}\int d^3x\Big(\psi^*(t,\vec{x})\dot{\psi}(t,\vec{x}) - \psi(t,\vec{x})\dot{\psi}^*(t,\vec{x})\Big)\\ &-\frac{\hbar^2}{2m}\int d^3x\,(\vec{\nabla}\psi(t,\vec{x}))\cdot(\vec{\nabla}\psi^*(t,\vec{x}))\\ &+\frac{Gm^2}{2}\iint d^3x\,d^3y\,\frac{|\psi(t,\vec{x})|^2\,|\psi(t,\vec{y})|^2}{\|\vec{x}-\vec{y}\|}\Bigg\}\,. \end{aligned} \qquad (43)$$

SNE: Dimensionless form

- Introducing a length-scale ℓ we can use dimensionless coordinates

$$\vec{x}' := \vec{x}/\ell\,, \qquad t' := t \cdot \frac{\hbar}{2m\ell}\,, \qquad \psi' = \ell^{3/2}\psi \tag{44}$$

and rewrite the SNE as

$$i\,\partial_{t'}\psi'(t',\vec{x}') = \left(-\Delta' - K\int \frac{|\psi'(t',\vec{y}')|^2}{\|\vec{x}'-\vec{y}'\|}\,d^3y'\right)\psi'(t',\vec{x}')\,, \tag{45}$$

with dimensionless coupling constant

$$K := 2\cdot\frac{Gm^3\ell}{\hbar^2} = 2\cdot\left(\frac{\ell}{\ell_P}\right)\left(\frac{m}{m_P}\right)^3 \approx 6\cdot\left(\frac{\ell}{100\,\mathrm{nm}}\right)\left(\frac{m}{10^{10}\,\mathrm{u}}\right)^3 \tag{46}$$

- Here we used Planck-length and Planck-mass

$$\ell_P := \sqrt{\frac{\hbar G}{c^3}} = 1.6\times 10^{-26}\,\mathrm{nm}\,, \quad m_P := \sqrt{\frac{\hbar c}{G}} = 1.3\times 10^{19}\,\mathrm{u}\,. \tag{47}$$

Symmetries and scaling properties of SNE

- The SNE has the same symmetries as ordinary Schrödinger equation: Full inhomogeneous Galilei group, including parity and time reversal, and global $U(1)$ phase transformations.
- Also it has the following scaling covariance: Let

$$S_\lambda[\psi](t,\vec{x}) := \lambda^{9/2}\psi(\lambda^5 t\,,\,\lambda^3\vec{x})\,, \tag{48}$$

then $S_\lambda[\psi]$ satisfies the SNE for mass parameter λm iff ψ satisfies SNE for mass parameter m

Collapse: Naive estimate

- Free Gaussian

$$\Psi_{\text{free}}(r,t) = \left(\pi a^2\right)^{-3/4} \left(1 + \frac{i\,\hbar\,t}{m\,a^2}\right)^{-3/2} \exp\left(-\frac{r^2}{2a^2\,\left(1 + \frac{i\,\hbar\,t}{m\,a^2}\right)}\right). \tag{49}$$

- Radial probability density, $\rho(r,t) = 4\pi\, r^2\, |\Psi_{\text{free}}(r,t)|^2$, has a global maximum at

$$r_p = a\sqrt{1 + \frac{\hbar^2 t^2}{m^2 a^4}} \quad \Rightarrow \quad \ddot{r}_p = \frac{\hbar^2}{m^2\, r_p^3}\,. \tag{50}$$

- At time $t = 0$ (say) this outward acceleration due to dispersion, $\ddot{r}_p = \frac{\hbar^2}{m^2\,a^3}$, equals gravitational inward acceleration $\frac{G\,m}{r^2}$ at time $t = 0$ if (compare (46))

$$m^3 a = m_P^3 \ell_p. \tag{51}$$

- For $a = 500\,\mathrm{nm}$ this yields a naive estimate for the threshold mass for collapse of about $4 \times 10^9\mathrm{u}$.

Stationary states: Analytical existence and numerical values

- Note that outward acceleration due to dispersion is $\propto r^{-3}$ and inward acceleration due to gravity $\propto r^{-2}$. Hence there will be no collapse to a δ-singularity.
- An analytic proof for the existence of a stable ground state has been given by E. Lieb in 1977 in the context of the Choquard equation for one-component plasmas, which is, however, formally identical.
- Tod et al. investigated bound states numerically and found the (unique) stable ground state at

$$E_0 = -0.163 \frac{G^2 m^5}{\hbar^2} = -0.163 \cdot mc^2 \cdot \left(\frac{m}{m_P}\right)^4 \tag{52}$$

Stationary states: Rough estimates

- A rough energy-estimate for the ground state is obtained, as usual, by setting

$$E \approx \frac{\hbar^2}{2ma^2} - \frac{Gm^2}{2a}\,. \tag{53}$$

- Minimising in a then gives rough estimates for ground state

$$a_0 = \frac{2\hbar^2}{Gm^3} = 2\ell_P \cdot \left(\frac{m_p}{m}\right)^3\,, \qquad E_0 = -\frac{1}{8}\frac{G^2m^5}{\hbar^2} \tag{54}$$

- Sanity check for applicability of Newtonian gravity (weak field approximation) is that diameter of mass distribution is much larger than its Schwarzschild radius

$$a_0 = \frac{2\hbar^2}{Gm^3} \gg \frac{2Gm}{c^2} \quad \Leftrightarrow \quad \left(\frac{m}{m_p}\right)^4 \ll 1 \tag{55}$$

General SNE

- SNE is of form

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\Delta + \left(\phi\star|\psi|^2(t,\vec{x})\right)\right)\psi(t,\vec{x}) \tag{56}$$

where

$$\phi\star|\psi|^2(t,\vec{x}) = -Gm^2\int\frac{|\psi(t,\vec{x})|^2}{\|\vec{x}-\vec{y}\|}\,d^3y \tag{57}$$

i.e.

$$\phi(\vec{x}) = -\frac{Gm^2}{r}\,. \tag{58}$$

- Equation (56) is still valid with modified ϕ for separated centre-of-mass wave-function. For example, for homogeneous spherically-symmetric matter distribution get

$$\phi(r) = \begin{cases} -\frac{Gm^2}{R}\left(\frac{3}{2}-\frac{r^2}{2R^2}\right) & \text{for } r < R \\ -\frac{Gm^2}{r} & \text{for } r \geq R \end{cases} \tag{59}$$

- **This equation can be derived for the centre-of-mass wavefunction of an N-particle system obeying the original n-particle SNE of Diósi (1984).**

The N-particle SNE

Principle: *Each particle is under the influence of the Newtonian gravitational potential that is sourced by an active gravitational mass-density to which each particle contributes proportional to its probability density in position space as given by the marginal distribution of the total wave function.*

- Hence

$$\rho(t;\vec{x}) = \sum_{j=0}^{N} m_j P_j(t;\vec{x}) = \sum_{j=0}^{N} m_j \int |\Psi_N(t;\vec{y}_1,\cdots,\vec{y}_N)|^2\, \delta^{(3)}(\vec{y}_j-\vec{x})\, d^{3N}y \tag{60}$$

giving rise to the gravitational potential

$$\begin{aligned} U_G(t;\vec{y}_1,\cdots,\vec{y}_N) &= -G\sum_{i=0}^{N}\int \frac{m_i\rho(t;\vec{x})}{\|\vec{y}_i-\vec{x}\|}\, d^3x \\ &= -G\sum_{i=0}^{N}\sum_{j=0}^{N}\int \frac{m_i m_j P_j(t;\vec{x})}{\|\vec{y}_i-\vec{x}\|}\, d^3x \end{aligned} \tag{61}$$

- Note that the mutual gravitational interaction is not local and includes self interaction, in contrast to what we usually assume in electrodynamics. It is this difference that implies modifications of the dynamics for the centre-of-mass wavefunction. These modifiations are like for the 1-particle SNE if the width of the wave function is large compared to the support of the matter distribution (D.G. & A. Großardt 2014).

Separation

- Using instead of $\{\vec{x}_i \mid i = 0, 1, \cdots N\}$ centre-of-mass $\vec{c}$ and relative co-ordinates $\{\vec{r}_\alpha \mid \alpha = 1, \cdots N\}$ (thereby distinguishing the 0-th particle),

$$\vec{c} := \frac{1}{M}\sum_{a=0}^{N} m_a\,\vec{x}_a = \frac{m_0}{M}\vec{x}_0 + \sum_{\beta=1}^{N}\frac{m_\beta}{M}\vec{x}_\beta\,, \tag{62a}$$

$$\vec{r}_\alpha := \vec{x}_\alpha - \vec{c} = -\frac{m_0}{M}\,\vec{x}_0 + \sum_{\beta=1}^{N}\left(\delta_{\alpha\beta} - \frac{m_\beta}{M}\right)\vec{x}_\beta \tag{62b}$$

- Get in large N limit with $\Psi(\vec{x}_0, \cdots \vec{x}_N) = \psi(\vec{c})\chi(\vec{r}_1, \cdots \vec{r}_N)$

$$U_G(t;\vec{c},\vec{r}_1,\cdots,\vec{r}_N) = -G\sum_{\alpha=1}^{N} m_\alpha \int d^3\vec{c}\,' \int d^3\vec{r}\,' \frac{|\psi(t;\vec{c}\,')|^2\rho_c(\vec{r}\,')}{\|\vec{c}-\vec{c}\,'+\vec{r}_\alpha-\vec{r}\,'\|}\,, \tag{63}$$

where

$$\rho_c(t;\vec{r}) := \sum_{\beta=1}^{N} m_\beta \left\{\int \prod_{\substack{\gamma=1\\ \gamma\neq\beta}}^{N} d^3\vec{r}_\gamma\right\} |\chi(t;\vec{r}_1,\cdots,\vec{r}_{\beta-1},\vec{r},\vec{r}_{\beta+1},\cdots,\vec{r}_N)|^2\,. \tag{64}$$

Approximation

- For a separation into centre-of-mass and relative motion we wish to get rid of $\vec{r}_\alpha$-dependence in (63).
- This can, e.g., be achieved by assuming the width of the c.o.m wave function to be much larger than diameter of mass districution. Then,

$$\begin{aligned} U_G = &- G \sum_{\alpha=1}^{N} m_\alpha \int d^3\vec{c}' \int d^3\vec{r}' \frac{|\psi(t;\vec{c}')|^2 \rho_c(\vec{r}')}{\|\vec{c} - \vec{c}' + \vec{r}_\alpha - \vec{r}'\|} \\ &\approx -GM \int d^3\vec{c}' \int d^3\vec{r}' \frac{|\psi(t;\vec{c}')|^2 \rho_c(\vec{r}')}{\|\vec{c} - \vec{c}' - \vec{r}'\|} = U_G(t;\vec{c}) \end{aligned} \tag{65}$$

- Alternatively one may apply a Born-Oppenheimer approximation that consists of replacing U_G with its expectation-value in the state χ for the relative motion:

$$\begin{aligned} U_G = &- G \sum_{\alpha=1}^{N} m_\alpha \int d^3\vec{c}' \int d^3\vec{r}' \frac{|\psi(t;\vec{c}')|^2 \rho_c(\vec{r}')}{\|\vec{c} - \vec{c}' + \vec{r}_\alpha - \vec{r}'\|} \\ \approx &- G \int d^3\vec{c}' \int d^3\vec{r}' \int d^3\vec{r}'' \frac{|\psi(t;\vec{c}')|^2 \rho_c(\vec{r}') \rho_c(\vec{r}'')}{\|\vec{c} - \vec{c}' - \vec{r}' + \vec{r}''\|} \\ = &U_G(t;\vec{c}) \end{aligned} \tag{66}$$

$\Rightarrow$ Both cases result in SNE for c.o.m in the form (56) with $\phi = U_G(t;\vec{c})$.

Consequences

- For wide c.o.m–wave functions SNE leads to inhibitions of qm-dispersion, as discussed before. Typical collapse times for widths of $500\,\mathrm{nm}$ and masses about $10^10\,\mathrm{amu}$ are of the order of hours. However, by scaling law (48), this reduces by factor 10^5 for tenfold mass and 10^{-3} fold width.
- For narrow c.o.m.–wavefunctions in Born-Oppenheimer scheme one obtains an effective self-interaction in c.o.m. SNE of

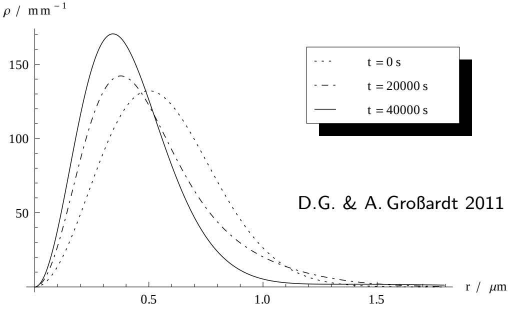
$$U_G(t;\vec{c}) \approx I_{\rho_c}(\vec{0}) + \tfrac{1}{2} I''_{\rho_c}(\vec{0}) \cdot \Big(\vec{c}\otimes\vec{c} - 2\,\vec{c}\otimes\langle\vec{c}\rangle + \langle\vec{c}\otimes\vec{c}\rangle\Big)\,. \qquad (67)$$

where $I_{\rho_c}(\vec{b})$ is the gravitational interaction energy between ρ_c and $T_{\vec{d}}\rho_c$.
- In one dimension and with external harmonic potential this gives rise to modified Schrödinger evolution:

$$i\hbar\partial_t\psi(t;c) = \left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial c^2} + \tfrac{1}{2}M\omega_c^2c^2 + \tfrac{1}{2}M\omega_{\mathrm{SN}}^2\big(c-\langle c\rangle\big)^2\right)\psi(t;c)\,. \qquad (68)$$

As a consequence covariance ellipse of the Gaussian state rotates at frequency $\omega_q := (\omega_c^2+\omega_{\mathrm{SN}}^2)^{(1/2)}$ whereas the centre of the ellipse orbits the origin in phase with frequency ω_c. This asynchrony is a genuine effect of self-gravity. It has been suggested that it may be observable via the output spectra of optomechanical systems (Yang.et al. 2013).

The time-dependent SN-Equation



- Time evolution of rotationally symmetric Gauß packet of initial width $500\,\mathrm{nm}$. Collapse sets in for masses $m > 4 \times 10^9\,\mathrm{u}$, but collapse times are of many hours (recall scaling laws, though).
- This is a 10^6 correction to earlier simulations by Carlip and Salzman (2006).

Summary

- There is no obvious way to translate EP = UFF+LLI+UGR to non-classical systems.
- Statements concerning *Quantum Tests of the Equivalence Principle* need qualification.
- How does the Schrödinger function couple to all components of the gravitational field; e.g., a gravitational wave? Give *first-principles* derivation!
- What if gravity stays classical?
- How, then, do systems in non-classical states gravitate?
- There is an army of arguments against fundamental semi-classical gravity; but how conclusive are they really?
- Interesting consequences from gravity-induced non-linearities in the Schrödinger equation of many particle systems can be derived concerning the centre-of-mass motion.
- The suggestion that these may be tested in the laboratory or in satellites is not considered ridiculous by experimentalists.

THANKS!

Summary

- There is no obvious way to translate EP = UFF+LLI+UGR to non-classical systems.
- Statements concerning *Quantum Tests of the Equivalence Principle* need qualification.
- How does the Schrödinger function couple to all components of the gravitational field; e.g., a gravitational wave? Give *first-principles* derivation!
- What if gravity stays classical?
- How, then, do systems in non-classical states gravitate?
- There is an army of arguments against fundamental semi-classical gravity; but how conclusive are they really?
- Interesting consequences from gravity-induced non-linearities in the Schrödinger equation of many particle systems can be derived concerning the centre-of-mass motion.
- The suggestion that these may be tested in the laboratory or in satellites is not considered ridiculous by experimentalists.

THANKS!

Schrödinger 1927

Der Energieimpulssatz der Materiewellen;
von E. Schrödinger

. . . . . .

Fragt man sich nun, ob diese in sich geschlossene Feldtheorie — von der vorläufigen Nichtberücksichtigung des Elektronendralles abgesehen — der Wirklichkeit entspricht in der Art, wie man das früher von dergleichen Theorien erhofft hatte, so ist die Frage zu *verneinen*. Die durchgerechneten Beispiele, vor allem das H-Atom, zeigen nämlich, daß man in die Wellengleichung (1) *nicht* diejenigen Potentiale einzusetzen hat, welche sich aus den Potentialgleichungen (15′) mit dem Viererstrom (9) ergeben. Vielmehr hat man bekanntlich beim H-Atom in (1) für die φ_a die *vorgegebenen* Potentiale des Kerns und eventueller „äußerer" elektromagnetischer Felder einzutragen und die Gleichung nach ψ aufzulösen. Aus (9) berechnet sich dann die von diesem ψ „erzeugte" Stromverteilung, aus ihr nach (15′) die von ihr erzeugten Potentiale. Diese ergeben dann, zu den vorgegebenen Potentialen hinzugefügt, diejenigen Potentiale, mit denen das Atom als ganzes nach außen wirkt. Man

. . . . . .

Gerade die *Geschlossenheit* der Feldgleichungen erscheint somit in eigenartiger Weise durchbrochen. Man kann das heute wohl noch nicht ganz verstehen, hat es aber mit folgenden zwei Dingen in Zusammenhang zu bringen.

. . . . . .

Ob die Lösung der Schwierigkeit wirklich nur in der von einigen Seiten[2]) vorgeschlagenen bloß *statistischen* Auffassung der Feldtheorie zu suchen ist, müssen wir wohl vorläufig dahingestellt sein lassen. Mir persönlich erscheint diese Auffassung heute nicht mehr[1]) endgültig befriedigend, selbst wenn sie sich praktisch brauchbar erweist. Sie scheint mir einen allzu prinzipiellen Verzicht auf das Verständnis des Einzelvorgangs zu bedeuten.

- Schrödinger “closes” the set of Schrödinger-Maxwell equations by letting ψ source the electromagnetic potentials to which ψ couples, thereby introducing non-linearities, similar to radiation-reaction in the classical theory.
- He asserts that “computations” for the H-atom lead to discrepancies which refute such a self-coupling.
- He wonders why in Quantum Mechanics the closedness of the system of field equations is violated in such a peculiar fashion (“in eigenartiger Weise durchbrochen”) and comments of possible impact of probability interpretation on classical concepts of local exchange of energy and momentum.