The Cosmological Constant

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Some History

1882 Pierre Simon de Laplace (Traité de Mécanique Céleste) asks: Will Gravity be absorbed by matter? He considers

$$F = Gmm' \exp(-\lambda r)/r^2 \,.$$

1897 Hugo Seeliger asks: How should one change the Newtonian law of universal attraction, so as to apply to an infinite Universe of finite mass-density ? (cf. Olbers' paradox). He considers

$$\Delta\phi - \lambda\phi = 4\pi G\rho$$

and calculates the perihelion precession caused by λ-term. He fits λ to Mercury data and finds that precession for other planets are then predicted far too large (as compared to Newcomb's data).

1917 A. Einstein: "Cosmological Investigations in General Relativity"

> "I have again committed a crime in gravitation theory, which puts me close to be locked away in a mental hospital."
>
> *Einstein to Paul Ehrenfest, 1917*

Einstein's Field Equations with CC

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda \;=\; 8\pi G\, T_{\mu\nu}$$

'Newtonian limit'

$$\begin{aligned} \Delta\phi &= 4\pi G[\rho + (\rho_\Lambda + 3p_\Lambda)] \\ \rho_\Lambda &= -p_\Lambda = \Lambda/8\pi G \end{aligned}$$

- Allows spatially closed, static Universe of constant matter density $\rho = \Lambda/4\pi G$ corresponding to a 3-sphere of radius $a = 1/\sqrt{\Lambda}$.
- Avoids standard Newtonian problems:
  1) divergences,
  2) evaporation objection ("Verödungseinwand"),
  3) small velocities of stars.
- Apparently subjects to Einstein's 'Mach-doctrine':
  No matter ($T_{\mu\nu} = 0$) $\Rightarrow$ no inertial structure ($g_{\mu\nu} = 0$). But this was already refuted in 1917 by De Sitter's matter-free solution.

The standard model in cosmology

In GR a cosmological model consists of the following two inputs:

1. A solution (M, g) to the Einstein-Matter-Equations

$$ds^2 = g_{\mu\nu}(t, \vec{x})\, dx^\mu\, dx^\nu\,.$$

2. A timelike geodesic vector field X on (M, g), which represents the mean flow of matter ($\rightarrow$ 'Hubble flow'). It serves to define the notion of 'local rest-frames'.

Characteristic requirement for the standard model is isotropy in <u>*every*</u> local rest-frame. A theorem in differential geometry assures now that the metric can be written in the *Friedmann-Lemaître* form:

$$\boxed{ds^2 = -dt^2 + a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\,\Omega^2\right\}}$$

$$k = 0, +1, -1 \qquad X = \partial/\partial t$$

The Friedmann equations

$$\left[\frac{\dot{a}}{a}\right]^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)a + \frac{\Lambda}{3}$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

Set $p = w\rho$ ($w = 0$ dust, $w = 1/3$ photons), then each two of these imply the third. The last eq. integrates to

$$\rho a^{3(1+w)} = \text{const.}$$

Define Hubble parameter $H_0 := \dot{a}/a|_{t=t_0=\mathrm{Now}}$ and critical density $\rho_{\mathrm{krit}} := 3H_0^2/8\pi G$. Then first Friedmann eq. translates to the:

'cosmological triangle'

$$\Omega_M := \frac{\rho_0}{\rho_{\mathrm{krit}}} \qquad \Omega_\Lambda := \frac{\Lambda}{3H_0^3} \qquad \Omega_K := \frac{-k}{a_0^2 H_0^2}$$

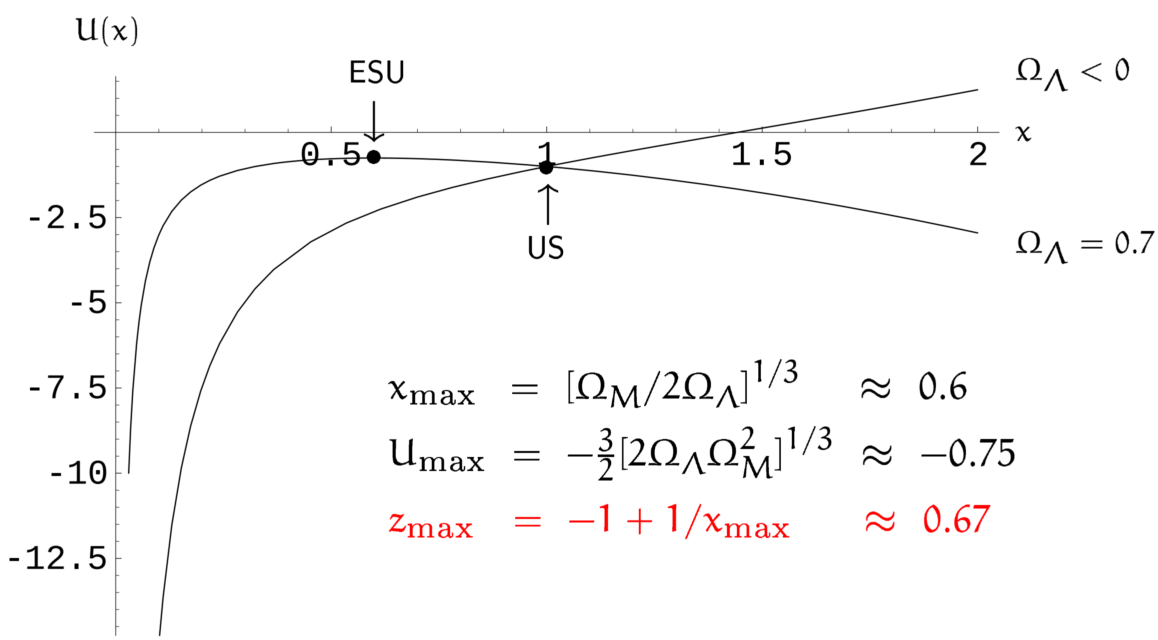
$$\Omega_M + \Omega_\Lambda + \Omega_K = 1$$

Mechanical analogy

Friedmann's equations are equivalent the the following 'mechanical' problem, where $\tau = H_0 t$ and $x(\tau) = a(t)/a_0$:

$$\left(\frac{dx}{d\tau}\right)^2 + \underbrace{\left(-x^{-n}\Omega_M - x^2\Omega_\Lambda\right)}_{U(x)} = \underbrace{1 - \Omega_M - \Omega_\Lambda}_{E}$$

where e.g. $n = 1$ for dust and $n = 2$ for photons. For $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ (flat universe) have



$$\begin{aligned} x_{max} &= [\Omega_M/2\Omega_\Lambda]^{1/3} &\approx 0.6 \\ U_{max} &= -\tfrac{3}{2}[2\Omega_\Lambda\Omega_M^2]^{1/3} &\approx -0.75 \\ z_{max} &= -1 + 1/x_{max} &\approx 0.67 \end{aligned}$$

Einstein abandons the CC

"If there is no quasi-static world, then let's forget about the cosmological term !"

Einstein to Hermann Weyl, 1923

"By means of these [Friedmann's] equations one may show instability of this solution. [...] For this reason alone I am inclined to dispute any physical significance of my former solution, independent of Hubble's experimental findings."

Einstein, 1931

"The cosmological term would have never been introduced had the Hubble expansion been known at the time GR was conceived. Retrospectively it now appears even less justified, since it now lost its only original justification, which is to lead to a natural solution of the cosmological problem."

Einstein, 1954

Useful relations

- For parameter values $0.1 \le \Omega_M \le 1$, $|\Omega_\Lambda| \le 1$ the 'world's age' comes out approximately

age of the world

$$t_0 \approx \frac{2}{3H_0} \cdot [0.3 + 0.7 \cdot \Omega_M - 0.3 \cdot \Omega_\Lambda]^{-1/3} \approx 13.5\,\text{by}$$

- From the temperature of the microwave background follows the radiation part Ω_M^{rad}, from which the redshift z_{eq} of the instant where $\Omega_M^{\text{rad}} = \Omega_M^{\text{dust}}$ can be calculated:

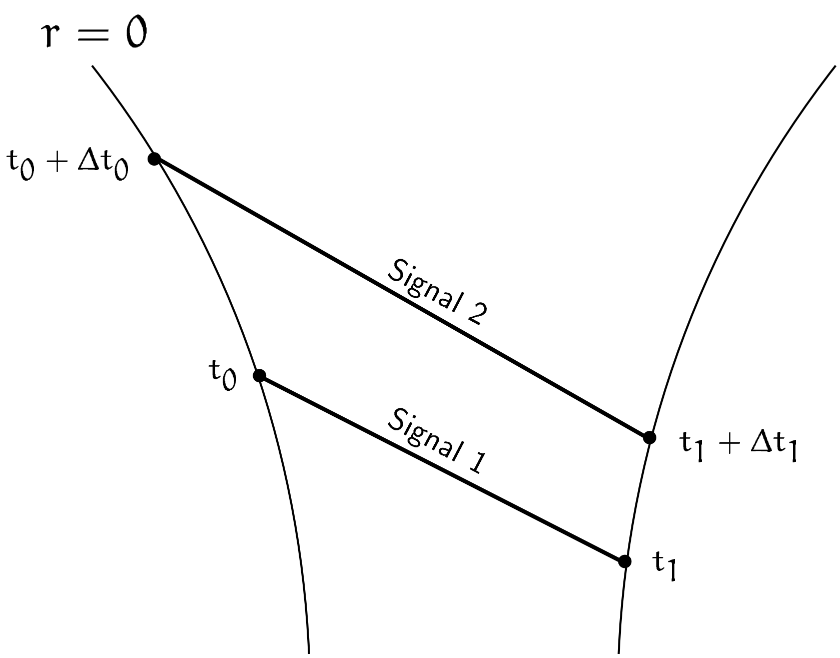
End of radiation dominance

$$z_{\text{eq}} \approx 23\,900 \cdot \Omega_M \cdot H_0^2[100\,\text{Km/s} \cdot \text{Mpc}] \approx 3717$$

Red numbers refer to $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 72\,\text{Km/s} \cdot \text{Mpc}$.

Observable quantities: redshift

Distance and relative velocity are not directly measurable quantities in relativistic cosmology. They are replaced by redshift and luminosity distance. These physically rely on the existence of systems with fixed emission frequencies (spectral lined) and fixed absolute luminosities, i.e. 'standard candles' (Cepheids, SN1a).



$$\frac{\Delta t_0}{\Delta t_1} = \frac{a(t_0)}{a(t_1)}$$

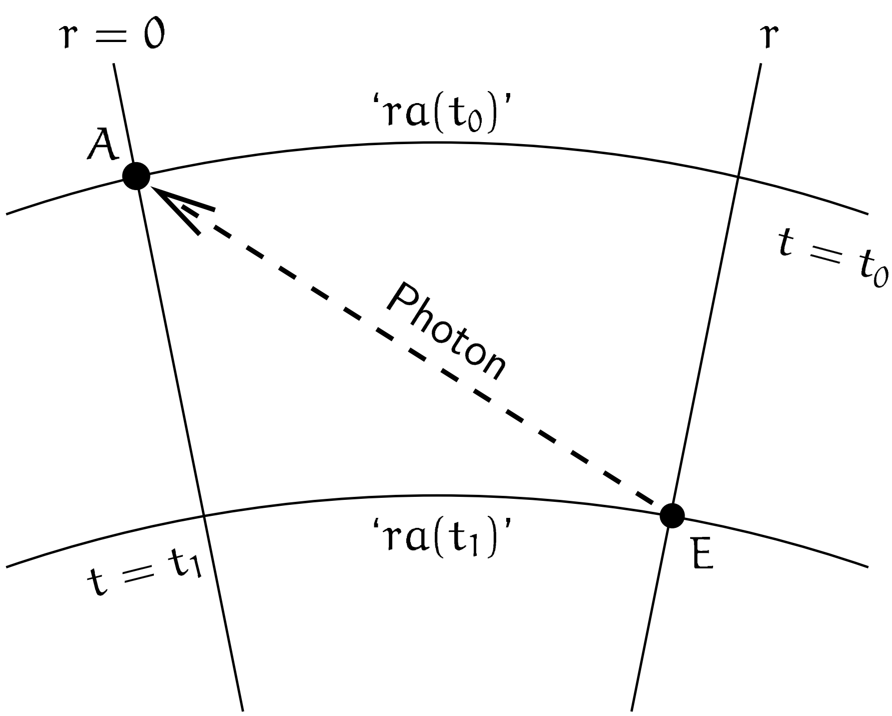
Redshift

$$z := \frac{\nu_1 - \nu_0}{\nu_0} = \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1$$

Luminosity distance

Given a source of absolute luminosity L in radial coordinate distance r. Let ℓ be the apparent luminosity received at the origin $r = 0$ at coordinate time t_0.

$$D_L = \sqrt{\frac{L}{4\pi\ell}} = r a(t_0) \frac{a(t_0)}{a(t_1)}$$



The Hubble plot

By means of the law of light propagation (lightlike geodesics) in the background metric, one can display light-travel-time, coordinate-distance, and luminosity-distance as function of redshift z:

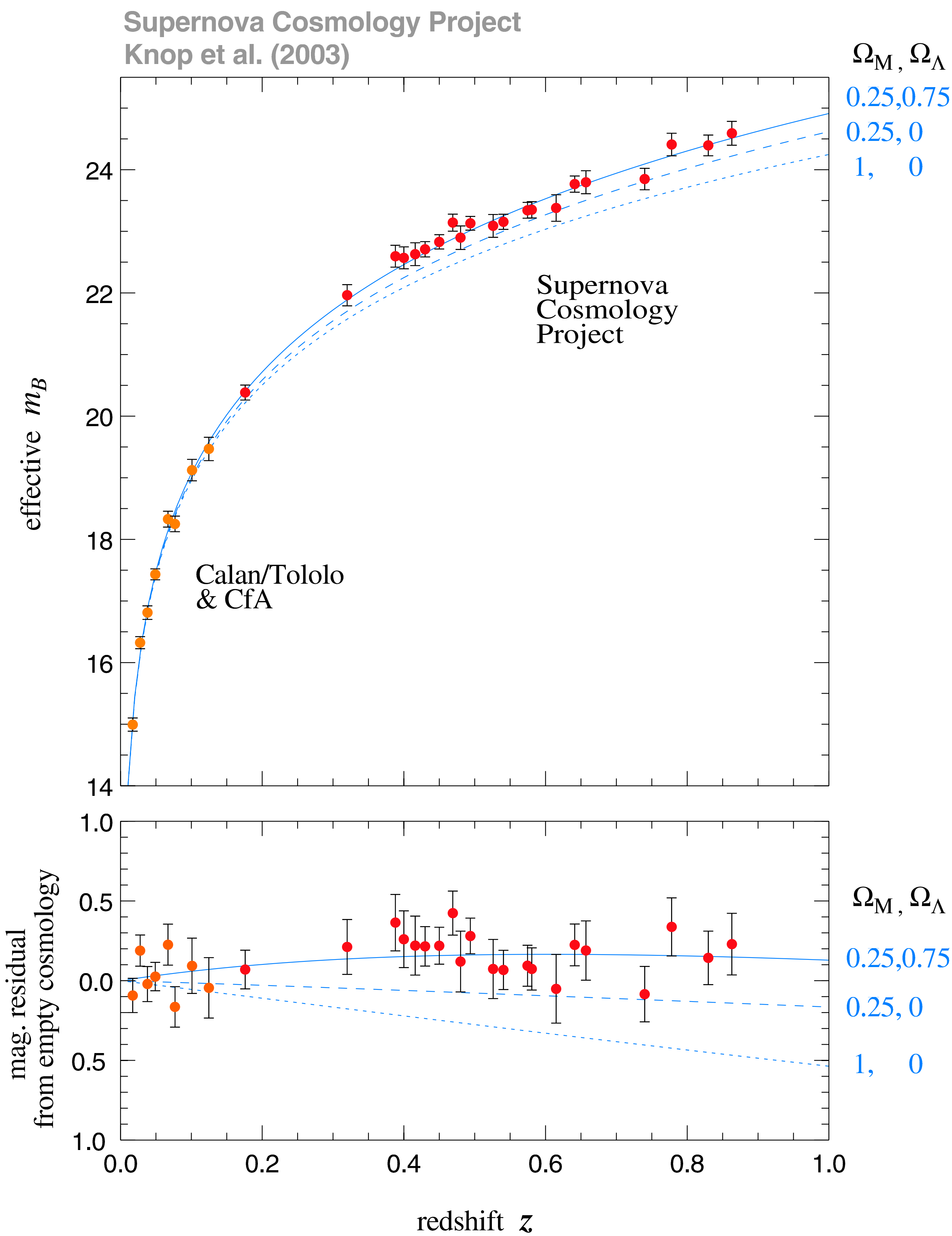
Hubble relation

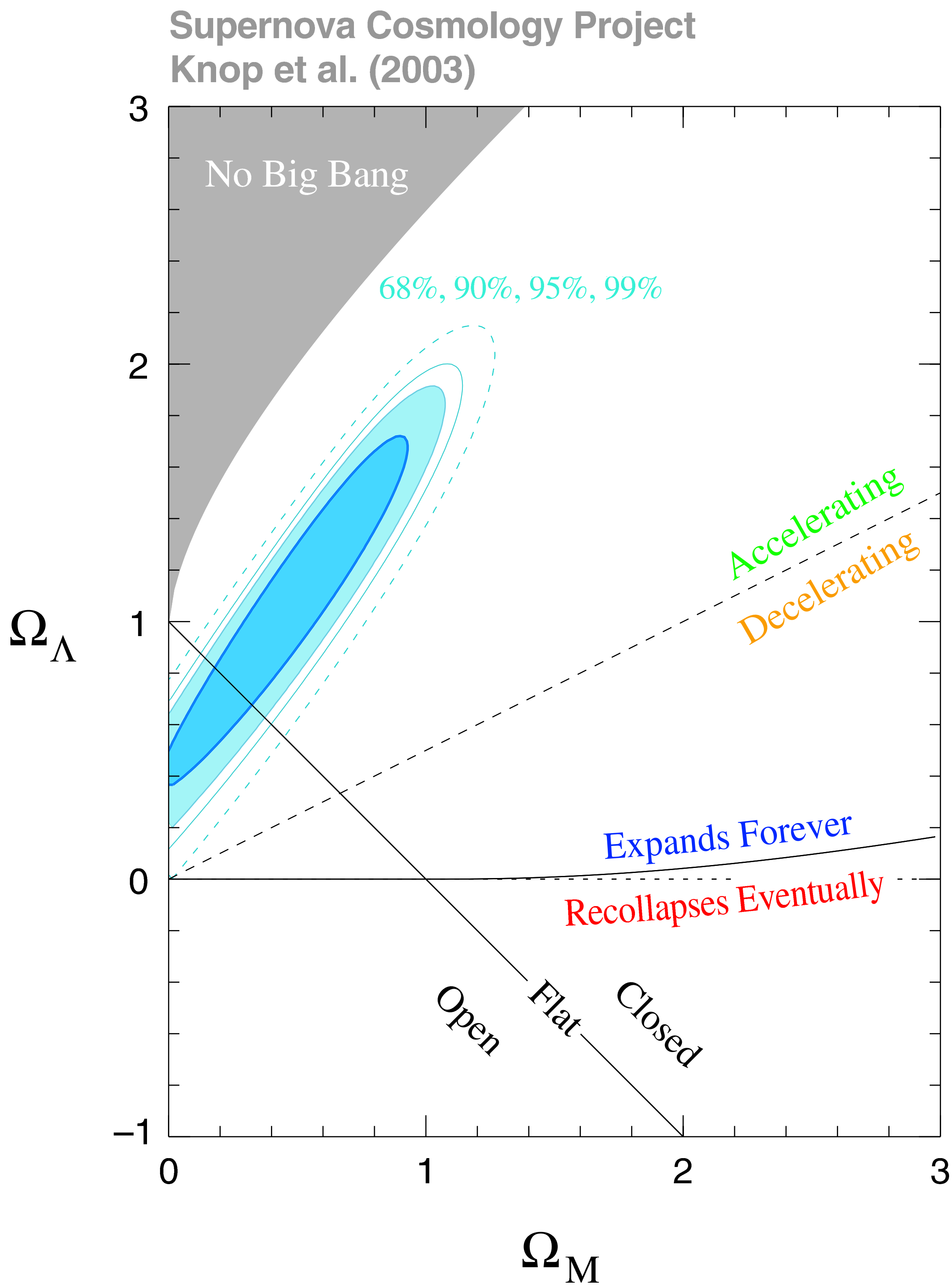
$$D_L(z) = H_0^{-1}\left(z + \tfrac{1}{2}(1 - q_0)z^2\right) + O(z^3)$$

$$q_0 = -\frac{\ddot{a}_0/a_0}{H_0^2} \qquad \text{('deceleration parameter')}$$

from 2nd Friedmann equation

$$q_0 = \tfrac{1}{2}\Omega_M - \Omega_\Lambda$$


Supernova Cosmology Project
Knop et al. (2003)
Ω_M , Ω_Λ
0.25,0.75
0.25, 0
1, 0
effective m_B
24
22
20
18
16
14
Supernova
Cosmology
Project
Calan/Tololo
& CfA
1.0
0.5
0.0
0.5
1.0
mag. residual
from empty cosmology
0.0
0.2
0.4
0.6
0.8
1.0
redshift z


Supernova Cosmology Project
Knop et al. (2003)
No Big Bang
68%, 90%, 95%, 99%
Accelerating
Decelerating
Expands Forever
Recollapses Eventually
Open
Flat
Closed
Ω_Λ
Ω_M
3
2
1
0
−1
0
1
2
3

Microwave background

- The cosmological microwave background (CMB) allows a view on the hypersurface $z_{\mathrm{ls}} \approx 1100$, the so-called 'surface of last scattering'. Temperature anisotropies in CMB reveal information on the density fluctuations of the baryon-photon-plasma at that time, in particular on the angular distance θ_{max} of the first maximum in the power spectrum of $\Delta T/T$.

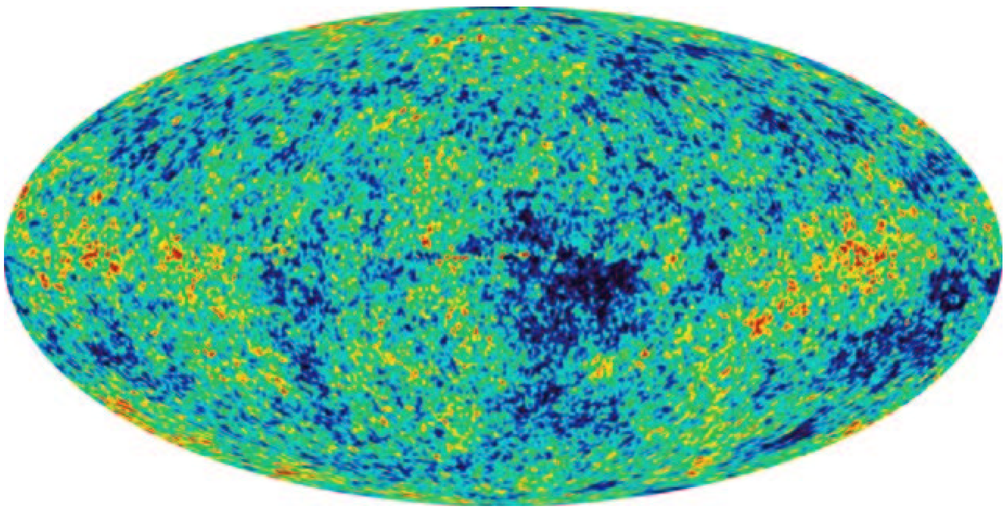
first 'acoustic' peak

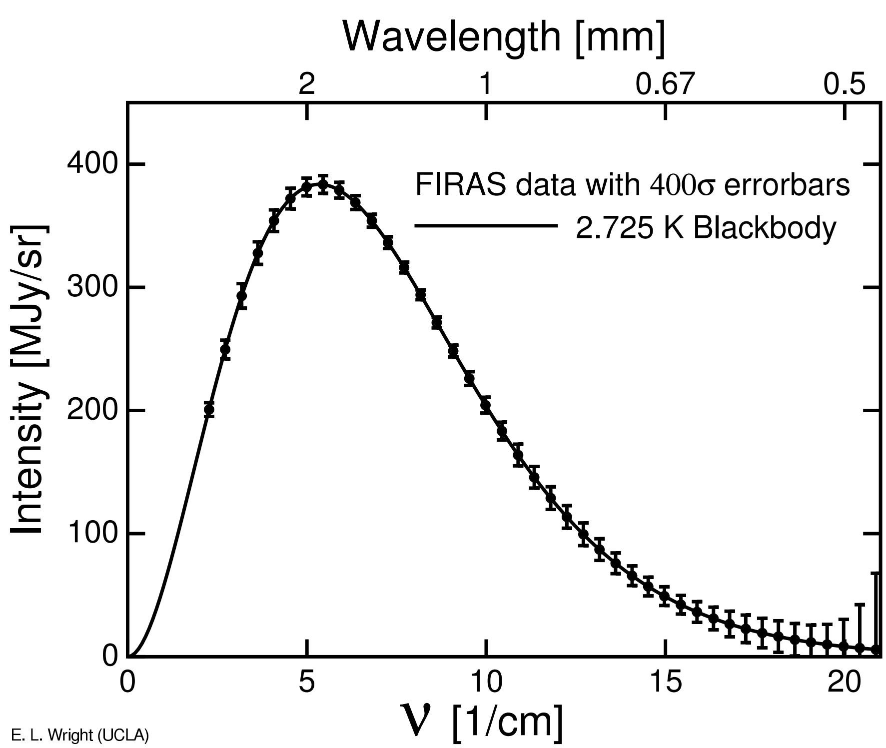
$$\begin{aligned} \theta_{\mathrm{max}} &\approx 0.62^{\circ} \; \left(1 - \tfrac{1}{2}\Omega_K + \tfrac{1}{14}\Omega_\Lambda\right) \\ \ell_{\mathrm{max}} &\approx 220 \quad \left(1 + \tfrac{1}{2}\Omega_K - \tfrac{1}{14}\Omega_\Lambda\right) \end{aligned}$$

- This essentially measures the combination $\Omega_M + \Omega_\Lambda$, which is complementary to the information of the Hubble plot:

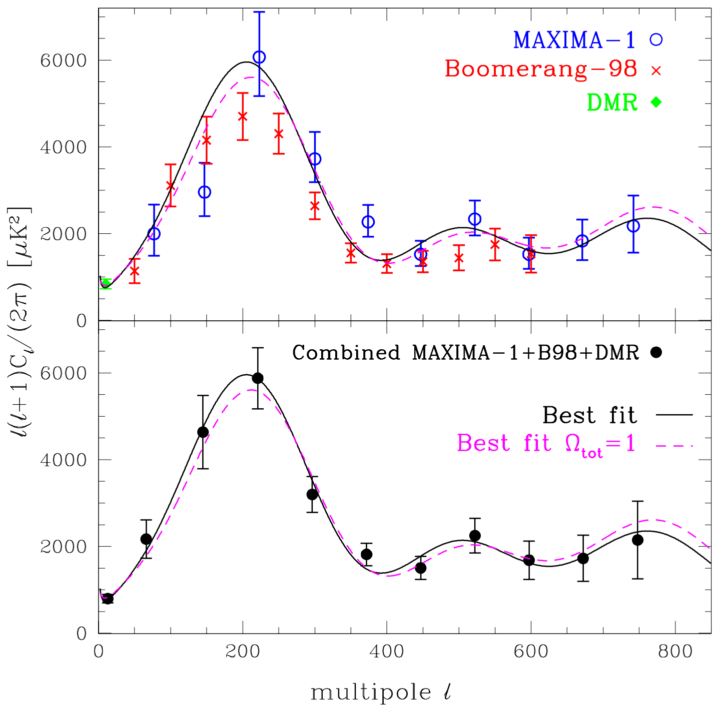
$$q_0 = \tfrac{1}{2}\Omega_M - \Omega_\Lambda$$

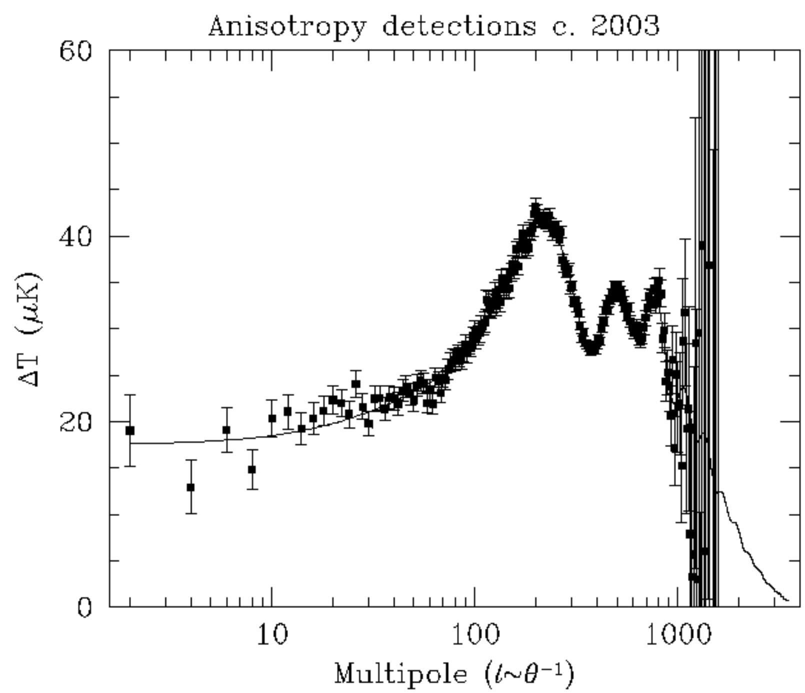
CMB sky and spectrum



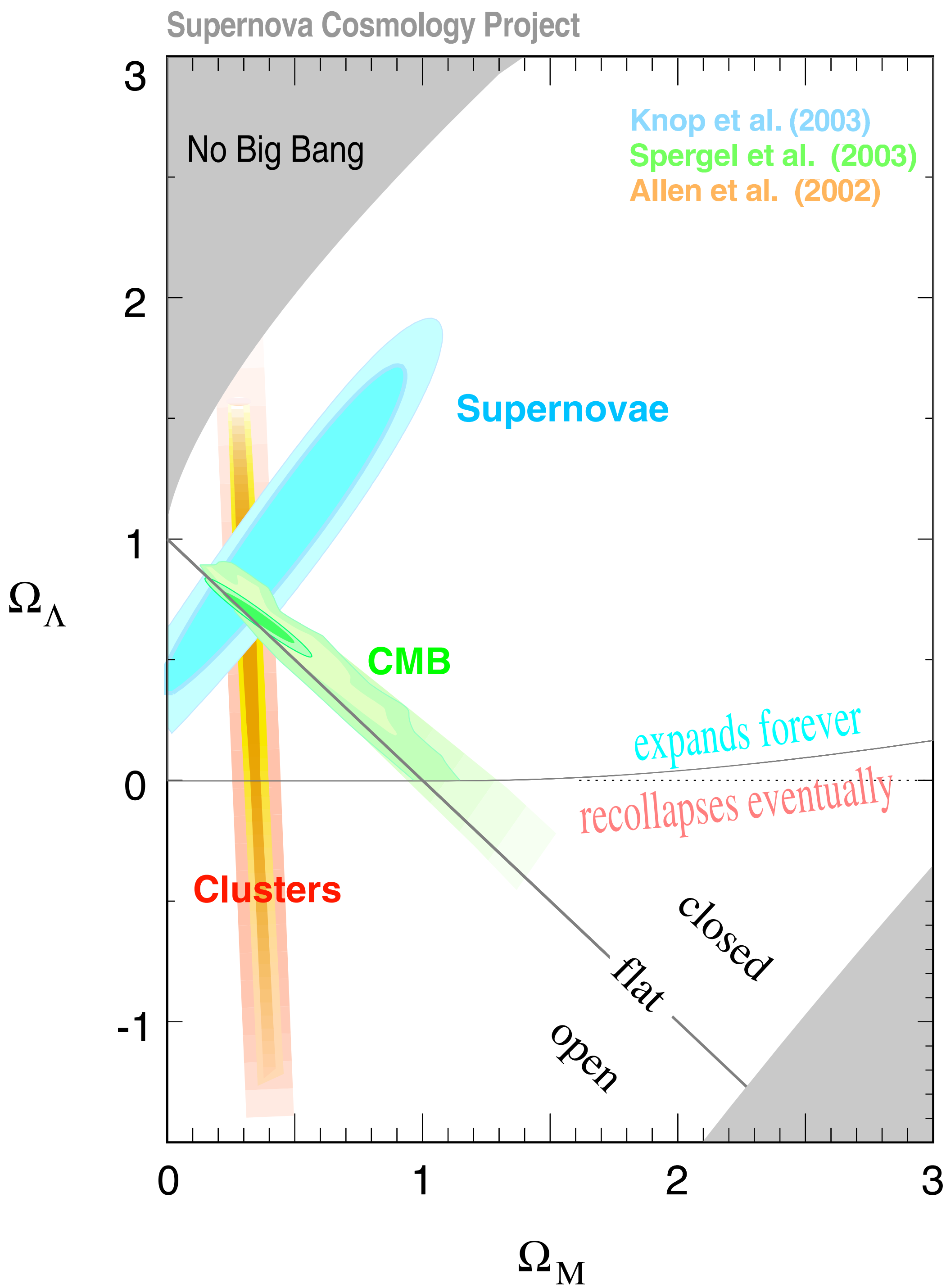


Data from Maxima, Boomerang, and WMAP

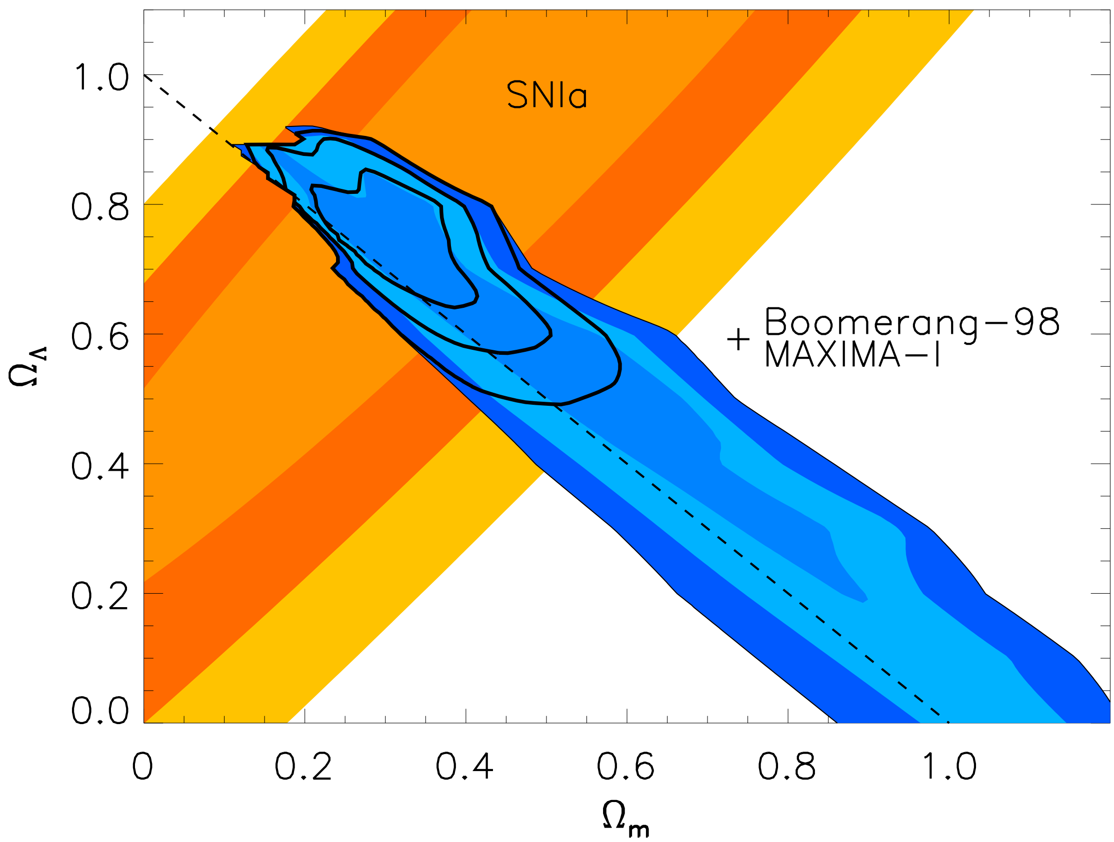




Combined SN 1a and CMB data



Combined SN 1a and CMB data
– magnification –



Vacuum fluctuations in QFT

QT:

$$H = p^2/2m + m\omega^2 q^2/2 \quad \Rightarrow \quad \langle H \rangle_{\rm vac} = \tfrac{1}{2}\omega\hbar$$

due to uncertainty relations $\Delta x \cdot \Delta p \geq \hbar/2$. Vacuum energy is entirely due to fluctuations:

$$\langle q^2 \rangle_{\rm vac} = \langle (\Delta q)^2 \rangle_{\rm vac} = \hbar/2m\omega$$

$$\langle p^2 \rangle_{\rm vac} = \langle (\Delta p)^2 \rangle_{\rm vac} = \hbar m\omega/2$$

QFT:

$$\vec{E}_f(\vec{x}) = \int d^3y \vec{E}_f(\vec{x}+\vec{y}) f(\vec{y}) \quad \text{(quantum field)}$$

Energy density

$$\langle \vec{E}_f(\vec{x}) \rangle_{\rm vac} = 2 \int \frac{d^3k}{(2\pi)^3} \frac{\hbar\omega}{2} |\tilde{f}(\vec{k})|^2$$

is ultraviolet-divergent. Choose e.g. cutoff function

$$\tilde{f}(\vec{k}) = \begin{cases} 1 & \text{for} \quad |\vec{k}| \leq K \\ 0 & \text{for} \quad |\vec{k}| \geq K \end{cases}$$

then

$$\langle \vec{E}_f(\vec{x}) \rangle_{\rm vac} = \hbar K^4/8\pi^2 \,.$$

Λ and vacuum fluctuations

- In Poincaré-invariant QFT one has $\langle T_{\mu\nu}\rangle_{\text{vac}} = C \cdot g_{\mu\nu}$, hence

  $$\Lambda_{\text{QFT}} = \hbar G K^4 = (K \cdot \ell_{\text{Planck}})^4 / \ell^2_{\text{Planck}}$$

  For $K = 1/\ell_{\text{Planck}}$, i.e. QFT=QG, have:

  'worst prediction ever'

  $$\boxed{\Lambda_{\text{QG}}/\Lambda_{\text{exp}} = \left(\frac{c/H_0}{\ell_{\text{Planck}}}\right)^2 \approx \left(\frac{15 \cdot 10^{25}\,\text{m}}{4 \cdot 10^{-35}\,\text{m}}\right)^2}$$

- However, in QFT on general background (M, g) with regularisation (point-splitting) of $T_{\mu\nu}$:

  $$\langle T_{\mu\nu}\rangle_{\text{loc}-\text{vac}} = C_1 \cdot G_{\mu\nu} + C_2 \cdot g_{\mu\nu}$$

  which generally leads to theoretically *unpredicted* redefinitions of G and Λ.

$\Rightarrow$ Current QFT does not predict a value for Λ.

Summary

- Distance measurements on type 1a supernova and determinations of the power spectrum of the temperature anisotropies on the microwave sky lead to

$$\Omega_M \approx 0.3 \qquad \Omega_\Lambda \approx 0.7 \qquad \Omega_K \approx 0$$

- We are presently in a phase of never ending accelerating expansion.
- Spatial geometry is almost flat ($\rightarrow$ Inflation). Spatial topology is restricted - to a degree depending on size of universe - but not determined.
- About 30% of gravitationally detectable energy is located in 'non relativistic' matter. If current big-bang models of baryogenesis are correct, only 1/6 of that, hence a mere 5% of all energy, are of baryonic nature ('made of atoms'). That raises the deep problem of clarifying the nature of the 'dark matter' ($\rightarrow$ incompleteness of standard model of elementary particle physics.)
- Presently there are only speculations on the physical nature of the energy represented by Ω_Λ: Quintessence (scalar field), higher-dimensional cosmology with 'branes',...
- Gravitation couples to absolute value of all energies. With current interpretations of QFT, the smallness of the vacuum energy density remains a big mystery.

Outlook on present and future work

- Exclusion of possible systematic errors in SN 1a distance measurements: Galactic extinction, evolutionary effects...

- Further, physically independent measurements of $\Omega_{\mathrm{M}}, \Omega_{\Lambda}$; e.g. Ω_{M} by 'counting' and gravitational-lensing statistics.

- Precision measurements of cosmological parameters by MAP (Microwave Anisotropy Probe; launched 30.6.2001, L2 1.10.2001) and PLANCK (ESA, scheduled $\approx$ 4/1/2007).

- Developing dynamical models for Λ: Quintessence, higher-dimensional (string, brane) cosmology...

THE END