‘Down-to-Earth’ Issues in Atom Interferometry

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Regensburg, October 1st 2010

Outline

- In this talk I wish to discuss the recent (Feb. 18. 2010) Nature paper by Holger Müller, Achim Peters, and Steven Chu: *A precision measurement of the gravitational redshift by the interference of matter waves.*
- In this paper it is claimed that a re-interpretation of some 10-years old experiments using vertical beams of laser cooled atoms give rise to a dramatic improvement in measurement of gravitational redshift and hence of Einstein's equivalence principle and the geometric nature of gravity.
- This paper has started a still ongoing controversy with a criticism in Nature (September 2.) by Peter Wolf, Luc Blanchet, Christian Bordé, Serge Reynaud, Christophe Salomon, and Claude Cohen-Tannoudji and a reply to that by the authors: *"We stand by our result"*.
- Controversial is the answer to the question: What has been measured?

Reminder: Principles of Equivalence

Three formulations of the equivalence principle should be clearly distinguished

1. The **weak equivalence principle (WEP)** states the universality of free fall **(UFF)** for test particles.
2. The **strong equivalence principle (SEP)** states the universality of free fall also for bodies whose gravitational self-energy is not negligible.
3. The **Einstein equivalence principle (EEP)** states that for all non-gravitational interactions, which do not couple to tidal gravitational fields, the usual laws (special relativistic) hold in a local inertial (freely falling and non rotating) reference frame.

$\Rightarrow$ Geometrisation of gravitational interaction and universal coupling-scheme for interaction between gravity and matter.

$$\eta_{\mu\nu} \mapsto g_{\mu\nu} \qquad \partial_\mu \mapsto \nabla_\mu := \partial_\mu + D_*(\Gamma_\mu) \tag{1}$$

We see EEP as the foundation of the statements that space-time is curved and that gravity and inertia are merely attributes of space-time's geometry.

The EEP canonised

The Einstein Equivalence Principle is usually canonised in the following form (cf. C. Will: Living Reviews 2006):
EEP is equivalent to

- WEP is valid.
- The outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.
  $\Rightarrow$ **Local Lorentz invariance (LLI)**.
- The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed.
  $\Rightarrow$ **Local position invariance (LPI)**.

Equivalence principle(s) and QM

- According to EEP, a homogeneous gravitational field cannot be distinguished from uniform acceleration wrt. an inertial system. The single-particle Schrödinger equation in a homogeneous gravitational field $\vec{g} = -g\vec{e}_z$ is given by

$$i\hbar\partial_t\Psi = \Big(-\frac{\hbar^2}{2m_i}\Delta + m_g gz\Big)\Psi \tag{2}$$

- Let K be an inertial reference frame without gravitational field. Let K' be constantly accelerated by $\vec{a} = g\vec{e}_z$ relative to K. Then

$$\vec{x}\,' = \vec{x} - \tfrac{1}{2}gt^2\,, \quad t' = t \tag{3}$$

In terms of $(\vec{x}\,', t')$ the free one-particle Schrödinger equation is equivalent to

$$i\hbar\partial_{t'}\Psi' = \left(-\frac{\hbar^2}{2m_i}\Delta' + m_i gz'\right)\Psi' \tag{4}$$

where

$$\Psi'(\vec{x}\,', t') = \Psi(\vec{x}, t)\ \exp\left(-i\,\frac{m_i g}{\hbar}\big(z't' - \tfrac{1}{6}gt'^3\big)\right) \tag{5}$$

$\Rightarrow$ If $m_i = m_g$, evolution of *rays* is identical to (2).

LPI and redshift -1

- Let there be a static gravitational field $\vec{g} = -g\vec{e}_z$. Assume validity of $EEP - LPI = WEP + LLI$. Then WEP guarantees local existence of freely-falling frame F^3 with coordinates $\{x_f^\mu\}$ whose acceleration is the same as that of test particles:

$$
\begin{aligned}
ct_f &= (z_s + c^2/g)\,\sinh(gt_s/c)\,,\\
x_f &= x_s\,,\\
y_f &= y_s\,,\\
z_f &= (z_s + c^2/g)\,\coth(gt_s/c)\,.
\end{aligned}
$$

- LLI guarantees that, *locally*, time measured by, e.g., an atomic clock is proportional to Minkowskian proper length in F^3. If we consider violations of LPI, the constant of proportionality might depend on the space-time point (e.g. via dependence on gravitational potential ϕ) as well as the type of clock:

$$
\begin{aligned}
c^2\,d\tau^2 &= F^2(\phi)\left[c^2dt_f^2 - dx_f^2 - dy_f^2 - dz_f^2\right] &\quad (6)\\
&= F^2(\phi)\left[\left(1 + \frac{gz_s}{c^2}\right)^2 c^2dt_s^2 - dx_s^2 - dy_s^2 - dz_s^2\right]. &\quad (7)
\end{aligned}
$$

LPI and redshift -2

- The *same* time interval $dt_s = dt_s(z_s^{(1)}) = dt_s(z_s^{(2)})$ on the two static clocks at rest wrt. $\{x_s^\mu\}$, placed at different heights $z_s^{(1)}$ and $z_s^{(2)}$, correspond to *different* intervals $d\tau^{(1)}, d\tau^{(2)}$ of the inertial clock, giving rise to the redshift (all coordinates are $\{x_S^\mu\}$ now, so we drop the subscript s):

$$\zeta := \frac{d\tau^{(2)} - d\tau^{(1)}}{d\tau^{(1)}} = \frac{F(z^{(2)})(1 + gz^{(2)}/c^2)}{F(z^{(1)})(1 + gz^{(1)}/c^2)} - 1 \tag{8}$$

- For small $\Delta z = z^{(2)} - z^{(1)}$ this gives to first order in Δz

$$\boxed{\Delta\zeta = (1 + \beta) g \Delta z / c^2} \tag{9}$$

where

$$\beta = \frac{c^2}{g}\left(\vec{e}_z \cdot \vec{\nabla}\ln(F)\right) \tag{10}$$

parametrises the deviation from GR result. β may depend on position, gravitational potential, and the type of clock one is using.

Redshift, WEP, and energy conservation

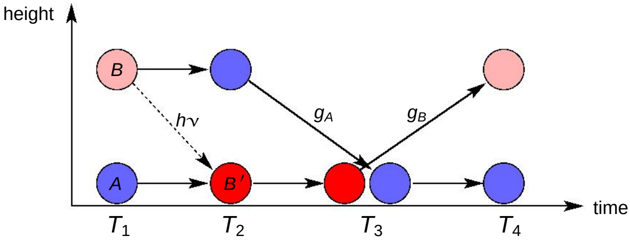

Figure 1: Gedankenexperiment by Nordtvedt to show that energy conservation connects anomalous redshift and violation of WEP. Considered are two copies of a system that is capable of 3 energy states A, B, and B' (blue, pink, and red), with $E_A < E_B < E_{B'}$. Initially system 2 is in state B and placed a height h above system 1 which is in state A. At time T_1 system 2 makes a transition $B \rightarrow A$ and sends out a photon of energy $h\nu = E_B - E_A$. At time T_2 system 1 absorbs this photon, which is now blue-shifted, and makes a transition $A \rightarrow B'$. At T_3 system 2 has been dropped from height h with acceleration g_A, has hit system 1 inelastically, leaving one system in state A and at rest, and the other system in state B with an upward motion with kinetic energy $E_{\rm kin} = M_A g_A h + (E_{B'} - E_B)$. The latter motion is decelerated by g_B, which may differ from g_A. At T_4 the system in state B has climbed to the **same** height h by energy conservation. Hence have $E_{\rm kin} = M_B g_B h$ and therefore $M_A g_A h + M_{B'} c^2 = M_B c^2 + M_B g_B h$, from which we get

$$\frac{\delta\nu}{\nu} = \frac{(M_{B'} - M_a) - (M_B - M_A)}{M_B - M_A} = \frac{g_B h}{c^2}\left[1 + \frac{M_A}{M_B - M_A}\,\frac{g_B - g_A}{g_B}\right] \tag{11a}$$

$$\Rightarrow \beta = \frac{M_A}{M_B - M_A}\,\frac{g_B - g_A}{g_B} =: \frac{\delta g/g}{\delta M/M} \tag{11b}$$

Dependencies

- Equations (11) answer the question of how accurate a clock must be in order to test the metric nature of gravity to the same level of accuracy than Eötvös-type experiments.
- Given that for the latter we have $\delta g/g < 10^{-13}$, this depends on the specific situation (interaction) through $\delta M/M$. For magnetic interaction have typically $\delta M/M \approx 10^{-4}$ and hence $\beta < 10^{-9}$.
- We also note

$$(SRT) \quad \text{and} \quad (UFF) \Rightarrow (\text{redshift})$$

$$(EEP) \Rightarrow (UFF) \quad \text{and} \quad (\text{redshift})$$

$$(EEP) \Leftarrow (UFF) \quad \text{and} \quad (\text{redshift}) \quad [\text{Schiff's conjecture}]$$

The Argument of Müller, Peters, and Chu

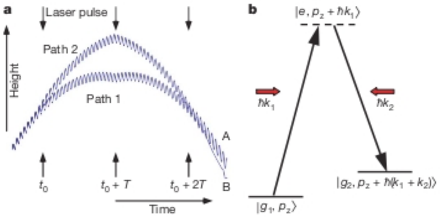

Figure 2: Atom interferometer and 2-photon Raman beam-splitter (Fig. 1 of Müller *et al.*). If $k_1 := \|\vec{k}_1\| > k_2 := \|\vec{k}_2\|$, then the transition $g_1 \to g_2$ is accompanied by a four-momentum change of $\Delta p = \hbar(\vec{k}_1 - \vec{k}_2\ ,\ \omega_1 - \omega_2)$, the transition $g_2 \to g_1$ by $-\Delta p$.

$$\Delta\phi = \underbrace{\Delta\phi_{\text{redshift}} + \Delta\phi_{\text{time}}}_{\Delta\phi_{\text{free}} \Leftrightarrow \text{geometry}} + \Delta\phi_{\text{light}} \qquad (12)$$

The Argument of Müller, Peters, and Chu (cont'd)

- *"As the purpose of our analysis is to study violations of local position invariance, it is useful to re-derive the phase from first principles"*

$$\Delta\phi_{\text{free}} = \frac{1}{\hbar}\int L\,dt = \underbrace{\frac{mc^2}{\hbar}}_{\omega_C}\int d\tau \qquad (13)$$

  where

$$d\tau = \frac{1}{c}\sqrt{|g_{\mu\nu}(x)dx^\mu dx^\nu|} \qquad (14)$$

- *"This shows that the phase is the integral of the Comptonunit frequency $\omega_C := mc^2/\hbar$ over the proper time $d\tau$ as it varies over the trajectory:"*
- *"An atom interferometer thus provides a textbook test case of general relativity: a neutral atom is almost ideal as a light test particle and contains a built-in quantum clock."*

The Argument of Müller, Peters, and Chu (cont'd)

- *"If the gravitational redshift is conventional [as predicted by GR], it turns out that they [contributions in (12)] have the same magnitude but opposite sign"*

$$\Delta\phi = \Delta\phi_{\text{redshift}} = -\Delta\phi_{\text{time}} = \Delta\phi_{\text{light}} \tag{15}$$

- This allows to regard the phase shift as entirely due to either redshift (Müller *et al.*). However, one might just as well regard it as due to the interaction with light:

$$\Delta\phi = \Delta\phi_{\text{redshift}} + \underbrace{\Delta\phi_{\text{time}} + \Delta\phi_{\text{light}}}_{=0} = \Delta\phi_{\text{redshift}} \tag{16a}$$

$$\Delta\phi = \underbrace{\Delta\phi_{\text{redshift}} + \Delta\phi_{\text{time}}}_{=0} + \Delta\phi_{\text{light}} = \Delta\phi_{\text{light}} \tag{16b}$$

- Müller *et al.* state that the former cancellation generalises to cases of anomalous redshift. **This is their essential point.**

The Argument of Müller, Peters, and Chu (cont'd)

Table 1 | Interferometer phase shifts larger than 0.3 p.p.b.

Effect	Equation	Phase shift (rad)
Leading order	$(1+\beta)kgT^2$	**3,698,530.529**
Redshift, $\Delta\varphi_{\text{redshift}}$	$(1+\beta)kgT^2$	
Time dilation, $\Delta\varphi_{\text{time}}$	$-kg'T^2$	
Atom–light interaction, $\Delta\varphi_{\text{light}}$	$+kg'T^2$	
Systematic effects[10]		**−0.018 ± 0.013**
Fundamental effects[17]		
Gravity gradient	$-k\frac{dg}{dz}T^3\left[v_L T+\frac{7}{12}gT\right]$	−0.108
Finite speed of light	$3kg^2T^2$	−0.058
Doppler shift	$-3kgv_LT^2$	0.059
First gradient recoil	$-\frac{\hbar k^2}{2m}\frac{dg}{dz}T^3$	−0.001
Sum of other effects*		<**0.0001**

Figure 3: Table 1 in Müller *et al.*, p. 927. The overall signs of the quantities $\Delta\varphi$ are conventional.

The Argument of Müller, Peters, and Chu (cont'd)

$$\boxed{\Delta\phi = \Delta\phi_{\text{redshift}} = (1+\beta)\kappa T^2 g} \tag{17}$$

- Hence the redshift per unit length is

$$z := (1+\beta)\frac{g}{c^2} = \frac{\Delta\phi}{\kappa T^2 c^2} \tag{18}$$

- The measured versus the predicted (taking systematic corrections into account) values are

$$z_{\text{meas}} = (1.090\,322\,683 \pm 0.000\,000\,003) \times 10^{-16}\,\mathrm{m}^{-1} \tag{19a}$$

$$z_{\text{pred}} = (1.090\,322\,675 \pm 0.000\,000\,006) \times 10^{-16}\,\mathrm{m}^{-1} \tag{19b}$$

which translates to

$$\beta = \frac{z_{\text{meas}}}{z_{\text{pred}}} - 1 = (7 \pm 7) \times 10^{-9}. \tag{20}$$

This should be compared to previous tests (Gravity-Probe-A, 1976) using hydrogen masers in rockets at altitude 10 000 Km (7×10^{-5}) and planned ones (launch 2013) on the ISS (ACES, 2×10^{-6}).

The Argument of Müller, Peters, and Chu (cont'd)

> "In summary, we improved the precision of measurements of the gravitational redshift by a factor of 10 000.
> This compares favourably to the European Space Agency's ACES mission, where it is anticipated that the gravitational redshift can be tested to a precision of 2 p.p.m."
>
> Müller et al. 2010

How to calculate phase shifts: Path-integral representation of Schrödinger evolution

$$\psi(z_b, t_b) = \int_{\text{space}} dz_a \, K(z_b, t_b\,;\, z_a, t_a)\, \psi(z_a, t_a) \tag{21}$$

where

$$K(z_b, t_b\,;\, z_a, t_a) := \langle z_b | \exp\big(-iH(t_b - t_a)/\hbar\big) | z_a \rangle\,. \tag{22}$$

The path-integral representation of the propagator K is

$$K(z_b, t_b\,;\, z_a, t_a) = \int_{\Gamma(a,b)} \mathcal{D}z(t)\ \exp\big(iS[z(t)]/\hbar\big) \tag{23}$$

where

$$\Gamma(a,b) := \big\{ z : [t_a, t_b] \to M \mid z(t_{a,b}) = z_{a,b} \big\} \tag{24}$$

and $S : \Gamma(a,b) \to \mathbb{R}$ is the action.

P_2-Lagrangians

$$L(z,\dot{z}) = a(t)\dot{z}^2 + b(t)\dot{z}z + c(t)z^2 + d(t)\dot{z} + e(t)z + f(t) \tag{25}$$

Examples are: 1) The free particle, 2) particle in a homogeneous gravitational field, 3) particle in a rotating frame of reference.
Let $z_* \in \Gamma(a,b)$ be the solution to the classical equations of motion:

$$\frac{\delta S}{\delta z(t)}\bigg|_{z(t)=z_*(t)} = 0 \tag{26}$$

Writing $z(t) = z_*(t) + \xi(t)$, so that

$$K(z_b,t_b\,;\,z_a,t_a)\int_{\Gamma(0,0)} \mathcal{D}\xi(t)\ \exp\bigl(iS[z_*(t)+\xi(t)]/\hbar\bigr) \tag{27}$$

Taylor-expansion around $z_*(t)$ gives

$$\begin{aligned} K(z_b,t_b\,;\,z_a,t_a) &= \exp\left\{\frac{i}{\hbar}S_*(z_b,t_b\,;\,z_a,t_a)\right\} \\ &\times\int_{\Gamma(0,0)} \mathcal{D}\xi(t)\exp\left\{\frac{i}{\hbar}\int_{\Gamma(0,0)} dt\ \left[a(t)\dot{\xi}^2 + b(t)\dot{\xi}\xi + c(t)\xi^2\right]\right\} \end{aligned} \tag{28}$$

Form of propagator for P_2-Lagrangians

- For polynomial Lagrangians of at most quadratic order the propagator has the exact representation

$$K(z_b, t_b\,;\, z_a, t_a) = F(t_b, t_a)\ \exp\left\{\frac{i}{\hbar} S_*(z_b, t_b\,;\, z_a, t_a)\right\} \tag{29}$$

  where $F(t_b, t_a)$ does not depend on the initial and final position and S_* is the action for the extremising path (classical solution).
- Using this expression, the phase-change in a Kasevich-Chu situation can be calculated **exactly** for Newtonian Lagrangians of at most quadratic order.
- In the following we shall briefly forget about the derivation of (29) and use this formula to calculate the phase change along *any* path, even if it is not a stationary point of the action functional. **This seems to be the rationale behind the argument of Müller *et al.***

Spacetime paths in Kasevich-Chu situation

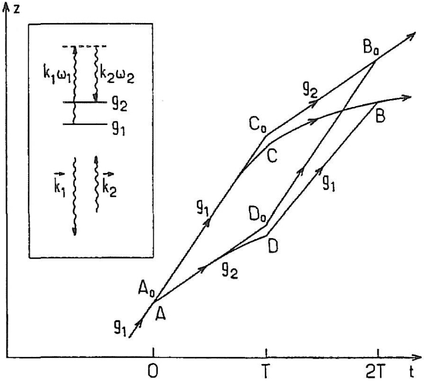

Figure 4: Spacetime paths followed by the atoms in the experiment of Kasevich and Chu. Raman pulses occur at times 0, T, and $2T$ with four-momenta $p_1 = \hbar(-k_1\vec{e}_z, \omega_1)$ and $p_2 = \hbar(k_2\vec{e}_z, \omega_2)$. The insert shows the atomic level scheme and the directions of the laser beams. Transitions $g_1 \rightarrow g_2$ and $g_2 \rightarrow g_1$ are accompanied by four-momentum transfers $\Delta_{12}p = (-\kappa, \omega)$ and $\Delta_{21}p = -\Delta_{12}p$ respectively, where $\kappa = k_1 + k_2 > 0$ and $\omega = \omega_1 - \omega_2 > 0$.

Exact NR calculation of $\Delta\phi_{\rm free}$

- The non-relativistic Lagrangian for a particle of mass m in a homogeneous (vertical) gravitational field $\vec{g} = -g\vec{e}_z$ is given by (taking only into account the z-degree of freedom):

$$L(z,\dot{z}) = \tfrac{1}{2}m_i\dot{z}^2 - m_g g z \tag{30}$$

  Here and in the following we shall separate the kinetic (“time”) from the potential (“redshift”) contribution by writing the latter in redred.

- From this the action along a parabolic path with acceleration $\vec{g} = -g'\vec{e}_z$, **where** g' **is not necessarily equal to** $(m_g/m_i)g$, and connecting the initial event (z_a, t_a) with the final event (z_b, t_B) can be obtained by straightforward computation:

$$\begin{aligned} S_{g'}(z_b,t_b;z_a,t_a) = {} & \frac{m_i}{2}\frac{(z_b - z_a)^2}{t_b - t_a} \\ & - \frac{m_g g}{2}(z_b + z_a)(t_b - t_a) \\ & + \frac{g'}{24}(t_b - t_a)^3(m_i g' - 2m_g g) \end{aligned} \tag{31}$$

  Terms in red ($\propto m_g$) originate from the potential part, those $\propto m_i$ from the kinetic part.

Exact NR calculation of $\Delta\phi_{\text{free}}$ (cont'd)

- Applied to $A = (z_A, t_A = 0)$, $B = (z_B, t_B = 2T)$, $C = (z_C, t_C = T)$, and $D = (z_D, t_D = T)$ (see Figure 4) and noting that the $(t_b - t_a)^3$ - term is independent of the z_Xs and hence does not contribute to differences for equal time lapses, we find

$$\begin{aligned}\Delta\phi_{\text{free}} &= \hbar^{-1}\Big[S_{g'}(A;C) + S_{g'}(C;B) - \big(S_{g'}(A;D) + S_{g'}(D;B)\big)\Big]\\ &= \frac{m_i}{\hbar T}\Big(z_C - z_D\Big)\Big[(z_C + z_D - z_A - z_B) - (m_g/m_i)gT^2\Big]\end{aligned} \tag{32}$$

Exact calculation of $\Delta\phi_{\rm prop}$ (contd.)

- Let $A_0 = (z_A^0, t_A = 0)$, $B_0 = (z_B^0, t_B = 2T)$, $C_0 = (z_C^0, t_C = T)$, and $D_0 = (z_D^0, t_D = T)$ be the corresponding events for $g' = 0$ (vanishing gravitational field), then obviously

$$z_A = z_A^0 \quad z_C = z_C^0 - \tfrac{1}{2}g'T^2 \quad z_D = z_D^0 - \tfrac{1}{2}g'T^2 \quad z_B = z_B^0 - 2g'T^2 \tag{33}$$

where, since $A_0C_0B_0D_0A_0$ is a parallelogram,

$$z_A^0 + z_B^0 = z_C^0 + z_D^0 \,. \tag{34}$$

Hence

$$z_C + z_D - z_A - z_B = gT^2 \,. \tag{35}$$

- Using also that

$$z_C - z_D = z_C^0 - z_D^0 = \Delta v_z\, T = \frac{\hbar\kappa}{m}\, T \tag{36}$$

we finally get

$$\begin{aligned} \Delta\phi_{\rm free} &= \Delta\phi_{\rm time} + \Delta\phi_{\rm redshift} \\ &= \kappa T^2 \Big(g' - (m_g/m_i) g \Big) \end{aligned} \tag{37}$$

Intermediate result

- In metric theories, the classical action along a solution path is

$$S = \int dt\,\left(\tfrac{1}{2}m\dot{z}^2(t) - mgz(t)\right) \tag{38a}$$

$$= \hbar\omega_C \int dt\,\left(\tfrac{1}{2}(\dot{z}^2(t)/c^2) - (g/c^2)z(t)\right) \tag{38b}$$

$$\cong \hbar\omega_C \int dt\,\left[1 - \frac{1}{c}\sqrt{g_{\mu\nu}(z(t))\,\dot{z}^\mu(t)\dot{z}^\nu(t)}\,\right] \tag{38c}$$

- **Differences** for paths with same initial and final t-values can therefore be written as differences of proper-time integrals:

$$\Delta(S/\hbar) \cong -\Delta\left\{\omega_C \int d\tau\right\} \tag{39}$$

- If the path is a stationary point of the action and the redshift is non anomalous, the foregoing result implies $g' = (m_g/m_i)g = g$ and $\Delta\phi_{\text{free}} = 0$. It therefore states that the number of proper Compton periods along the upper path is the same as that on the lower path. The phase difference could then be argued to be entirely due to the laser interaction. But that is **not** the viewpoint of Müller *et al.*

Phases from laser interactions

- Along the "upper" path ACB the phase due to laser interaction is (here and in the following $\kappa := \|\vec{k}_1 - \vec{k}_2\|$ and : $\omega = \omega_1 - \omega_2$):

$$U^{(3)}_{g_2 g_2} U^{(2)}_{g_2 g_1} \underbrace{\exp\left\{ i\left[-\kappa\left(z^0_C - \tfrac{1}{2} g' T^2\right) - \omega T - \phi_{II}\right]\right\}}_{\text{at } C} U^{(1)}_{g_1 g_1} \qquad (40)$$

- Along the "lower" path ADB the phases are

$$\begin{aligned} & U^{(3)}_{g_2 g_1} \exp\left\{ i\left[-\kappa\left(z^0_B - 2g' T^2\right) - 2\omega T - \phi_{III}\right]\right\} && \text{(at B)} \\ \times\ & U^{(2)}_{g_1 g_2} \exp\left\{ -i\left[-\kappa\left(z^0_D - \tfrac{1}{2} g' T^2\right) - \omega T - \phi_{II}\right]\right\} && \text{(at D)} \\ \times\ & U^{(1)}_{g_2 g_1} \exp\left\{ i\left(-\kappa z^0_A - \omega \cdot 0 - \phi_I\right)\right\} && \text{(at A)} \qquad (41) \end{aligned}$$

- Hence the upper minus the lower phase is, up to U's and ϕ's:

$$\begin{aligned} \Delta\phi_{\text{interaction}} &= -\kappa\left[\left(z^0_c + z^0_D - z^0_A - z^0_B\right) + g' T^2\right] \\ &= -\kappa g' T^2 \qquad (42) \\ &= -(\vec{k}_1 - \vec{k}_2) \cdot \vec{g}'\, T^2 \end{aligned}$$

Total phase shift

- Since we assumed the trajectory to be parabolic wrt. g', the parameter g only enters in calculating the redshift part of $\Delta\phi$. Allowing for this also to be anomalous, we make the replacement $g \to (1+\beta)g$. Then we get

$$\Delta\phi = \underbrace{\kappa T^2 g'}_{\Delta\phi_{\text{time}}} - \underbrace{\kappa T^2 (m_g/m_i)(1+\beta)g}_{\Delta\phi_{\text{redshift}}} - \underbrace{\kappa T^2 g'}_{\Delta\phi_{\text{light}}} \tag{43}$$

- This equation contains an unknown g. It is eliminated through a nearby reference measurement of the acceleration $\bar{g} = (M_g/M_i)g$ of a corner cube of inertial mass M_i and gravitational mass M_g.
- Using the Nordtvedt parameter for the atom-cube pair,

$$\eta := \eta(\text{atom}, \text{cube}) := 2\frac{(m_g/m_i) - (M_g/M_i)}{(m_g/m_i) + (M_g/M_i)}\,, \tag{44}$$

we get for the total phase shift (43):

$$\Delta\phi = -\kappa T^2 \bar{g} \quad (1+\beta)\frac{2+\eta}{2-\eta} \approx -\kappa T^2 \bar{g} \quad (1+\beta)(1+\eta)\,. \tag{45}$$

- We see that violations of URS and WEP enter in precisely the same fashion. Possible variations of m_g/m_i between the hyperfine-split states are not taken into account here.

Conclusion

- A simple replacement $g \to (1+\beta)g$ in the action and then proceeding in standard fashion renders measurable quantities insensitive to β. Sensitivity to η remains however.
- If energy and momentum are conserved within the system we are considering, then violations of UFF and URS are linked ($\to$ Nordtvedt's Gedankenexperiment) and current limits on the former imply better limits on the latter than the atom interferometric experiment discussed here.
- Hence the whole consideration of Müller *et al.* is only relevant for those types of violations of URS where energy-momentum conservation does *not* hold within the system ($\to$ additional forces, e.g. scalar fields etc.). If fundamental energy - momentum conservation is rescued by introducing additional fields (forces), they must couple non minimally and violate UFF.
- Independent of all that, the argument of Müller *et al.* seems theoretically incomplete, for the following reason: When using the path integral to calculate the propagator, one may only replace the integral over actions along paths by a single action, i.e. use the representation (29), if the later extremises the action, which in our case means $g' = g$ and $\Delta\phi_{free} = 0$. Otherwise we cut the logical connection to the propagator and hence the phase shift.
- To me, and at this moment, the only logically consistent interpretation of what has been done is a measurement of the Eötvös factor.