Aspects of Classical Physics - Tutorial 5

Olaf Lechtenfeld, Gabriel Picanço

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Spontaneous Symmetry Breaking and Higgs Mechanism

Example 1: SU(2) with scalar in the adjoint representation

- Take an SU(2) Yang–Mills theory with matter field $\phi = \phi^a\, t_a$ in the adjoint representation, with potential $U = \frac{1}{4}\lambda(\phi^a\phi^a - v^2)^2$. What is the vacuum manifold of this theory? What is the remaining symmetry H after the SSB (spontaneous symmetry breaking)? How many massive and massless bosons will the theory have then?
- Rewrite the lagrangian after the SSB. Compute the mass matrix.
- Compute the masses of the final (Higgs) bosons.

Example 2: SU(3) with matter in the fundamental representation

- Take an SU(3) Yang–Mills theory with a scalar field $\phi = (\phi_1, \phi_2, \phi_3) \in \mathbb{C}^3$ in the fundamental representation, with potential $U = \frac{1}{4}\lambda(\bar{\phi}_i\phi_i - v^2)^2$. What is the vacuum manifold of this theory? What is the remaining symmetry H after the SSB? How many massive and massless bosons will the theory have then?
- Rewrite the lagrangian after the SSB. Compute the mass matrix.
- Compute the masses of the final (Higgs) bosons.

Example 3: Georgi-Glashow GUT model

The gauge group of the Standard Model of particle physics is $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, where C refers to "color", L means "left", and Y refers to "hypercharge". Here, $\mathrm{SU}(3)_C$ is the gauge group of QCD, and $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is the gauge group of the electroweak theory. A theory that tries to unify strong and electroweak interactions as a single interaction at high energies is called Grand Unified Theory (GUT for short). It embeds the Standard Model gauge group into one larger simple group. The easiest of those models is the SU(5) Georgi-Glashow model, with a scalar field in the adjoint (24-dimensional) representation, $\phi = \phi^a\, t_a$. Its effective (energy-dependent) potential is chosen to develop a nontrivial minimum below a certain energy. As a representative of the vacuum manifold one may choose $\phi_{\text{vac}} = \frac{1}{6}v\,\mathrm{diag}(-2,-2,-2,+3,+3)$, whose stabilizer subgroup is $\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2)) \subset \mathrm{SU}(5)$.

- Explain how $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is described as $\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2))$.
- With the information given above, compute the number of massive and massless gauge bosons in this model, after the SSB.
- How many more massive gauge bosons than the Standard Model does this GUT model predict? What could be a possible explanation for not having detected them yet?