Tutorial 6 - Fundamental Interactions

Olaf Lechtenfeld, Gabriel Picanço

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1 Feynman diagrams

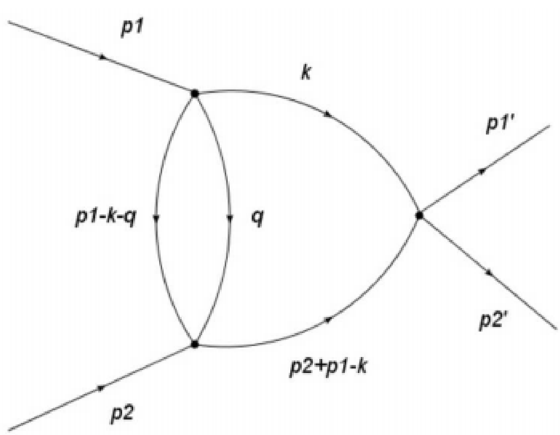
a) Elaborate momentum-space Feynman rules for the system described by the following lagrangian density:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda_1}{3!}\phi^3 - \frac{\lambda_2}{4!}\phi^4. \tag{1}$$

b) Elaborate momentum-space Feynman rules for the system described by the following lagrangian density:

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2. \tag{2}$$

c) Write the expression corresponding to the following Feynman diagram (no need to solve the integrals) for the real scalar $\lambda\phi^4$ model:



2 Non-abelian: infinitesimal gauge transformation

In this exercise let us derive the infinitesimal gauge transformations for fermion and gauge fields in a Yang-Mills theory. Analogous to what we've seen in class, consider the N color-components q_j of a quark field, $j = 1, ..., N$, that transforms under an $\mathrm{SU}(N)$ gauge group.

Let $\Omega(x) = e^{g\omega(x)} \in \mathrm{SU}(N)$, where $\omega(x) = \omega^a(x)\, t_a$ for $t_a \in \mathfrak{su}(N)$. We know that the quark fields transform via gauge transformations as

$$q(x) \to \Omega(x) q(x) \quad \text{and} \quad \bar{q}(x) \to \bar{q}(x)\Omega^\dagger(x), \quad \text{for} \quad q = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \end{pmatrix}. \tag{3}$$

a) Expanding in ω, find the infinitesimal (up to first order) gauge transformation for $q(x)$. Now find the infinitesimal transformation for its components $q_j(x)$ as well.

As in the abelian case, let us define a covariant derivative $\hat{D}_\mu$ (now a matrix) to make the term coming from the Dirac equation,

$$i\bar{q}\gamma^\mu \hat{D}_\mu q, \tag{4}$$

gauge invariant. Again, the extra term in the covariant derivative shall compensate the term coming from the fact that gauge transformations are local. Then let us define $\hat{D}_\mu = \mathbb{1}\partial_\mu + g\hat{A}_\mu$, with $\hat{A}_\mu(x) = A^a_\mu(x) t_a$. Let f_{abc} be the structure constants of the $\mathfrak{su}(N)$ Lie algebra.

b) How should $\hat{D}_\mu$ and $\hat{A}_\mu$ transform under a gauge transformation $\Omega(x)$ such that (4) is gauge invariant? Use this to find the infinitesimal gauge transformation for $\hat{A}_\mu$. Now find the infinitesimal transformation for its components A^a_μ as well.

c) Take the field strength $F^a_{\mu\nu} t_a = \hat{F}_{\mu\nu} = \frac{1}{g}[\hat{D}_\mu, \hat{D}_\nu]$. Find how $\hat{F}_{\mu\nu}$ transforms under gauge transformations. Use this to find how it transforms under infinitesimal gauge transformations. Now find the infinitesimal transformation for its components $F^a_{\mu\nu}$ as well.