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Fundamental Interactions

In this tutorial class we explore three fundamental steps towards the so-called Higgs mechanism. Firstly, you will study *spontaneous symmetry breaking* for a discrete symmetry. Secondly, we consider a continuous (global) symmetry and stumble upon an example of the Goldstone theorem. Lastly, you delve into the simplest example of the abelian Higgs effect.

Problem 5: spontaneous symmetry breaking I — discrete symmetry Consider the Lagrangian

$$L = T - V = \frac{1}{2}\,\partial_\sigma\varphi\,\partial^\sigma\varphi - \left(\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4\right) , \tag{1}$$

which is invariant under the $\mathbb{Z}_2$ symmetry $\varphi \to -\varphi$.

a) Find the minimum ν of the total energy $T + V$ for a constant field $\varphi(x) \equiv \varphi$. Sketch the potential for $\mu > 0$ and $\mu < 0$.

b) Expand the theory around the minima (i) $\nu = 0$ for $\mu > 0$ and (ii) $\nu = \sqrt{-\mu^2/\lambda}$ for $\mu < 0$. By that we mean to perform the replacement $\varphi(x) \mapsto \nu + \eta(x)$ in L.

c) Is the original $\mathbb{Z}_2$ symmetry visible in the re-written L? What are the masses of the scalar field $\eta(x)$ in both cases?

To this end, the choice of one of the two equivalent vacua $\nu = \pm\sqrt{-\mu^2/\lambda}$ for $\mu < 0$ breaks the original $\mathbb{Z}_2$ symmetry. This means that the vacua do not have the symmetry of the original Lagrangian, which is called *spontaneous symmetry breaking*.

Problem 6: spontaneous symmetry breaking II — Goldstone theorem For the complex scalar field $\phi = (\varphi_1 + i\varphi_2)/\sqrt{2}$ the Lagrangian takes the form

$$L = (\partial_\sigma\phi)^*(\partial^\sigma\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 , \tag{2}$$

which has a continuous global $U(1) \cong SO(2)$ symmetry given by $\phi(x) \mapsto \exp(i\chi)\phi(x)$.

a) Find the minima for the total energy for constant scalar field and sketch the potential in the $\varphi_1 - \varphi_2$ plane.

b) Now, there is an entire circle of equivalent vacua, and we may select any point arbitrarily, say $\varphi_1 = \nu$, $\varphi_2 = 0$ with $\nu^2 = -\mu^2/\lambda$. Thus, we now expand the Lagrangian as

$$\phi(x) \mapsto \frac{(\nu + \eta(x) + i\rho(x))}{\sqrt{2}} \tag{3}$$

around $\eta = 0$ and $\rho = 0$. What are the mass terms for the two real scalar fields $\eta(x)$ and $\rho(x)$?

As you should have seen in this example, spontaneously breaking of the continuous global $U(1)$ symmetry leads to a massless scalar field, called the *Goldstone boson*. Intuitively, the arising massless spin-zero particle corresponds to the flat direction of the potential in the vicinity of a chosen vacuum. The massive scalar field describes excitations in the radial direction.

Problem 7: spontaneous symmetry breaking III — abelian Higgs effect Now, we promote the global $U(1)$ symmetry to a local symmetry or gauge symmetry. This is done in three steps: (i) introduce local gauge transformations $\phi(x) \mapsto \exp(i\chi(x))\phi(x)$, (ii) replace the partial derivative by the gauge covariant derivative $\partial_\sigma \to D_\sigma = \partial_\sigma - igA_\sigma$, and (iii) introduce the kinetic term $F_{\sigma\rho}F^{\sigma\rho}$ for the abelian gauge field A_σ. The resulting Lagrangian reads

$$L = (D_\sigma\phi)^*(D^\sigma\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\sigma\rho}F^{\sigma\rho} \ . \tag{4}$$

As the situation becomes more evolved, we employ the insights gained in Problem 6. As the minima of the scalar potential remain the same, the minima for $\mu < 0$ are given by $v^2 = -\mu^2/\lambda$, and the complex scalar is re-written as in (3). Employing a suitable gauge transformation, this can be expressed as

$$\phi(x) \mapsto \frac{(v + h(x))}{\sqrt{2}} \ . \tag{5}$$

a) Insert (5) into the Lagrangian and sort the terms corresponding to kinetic terms, mass terms and interaction terms.

b) What do you observe for the scalar field h and for the gauge field A? Do they have masses?

c) Since we broke a continuous symmetry, one may ask the following: Where is the Goldstone boson?

In conclusion, breaking a *local* symmetry evades the Goldstone theorem, i.e. there is no massless scalar field. In addition, the gauge field has acquired a mass term due to the symmetry breaking. Note that a mass term for gauge fields is not gauge invariant, but can be introduced by spontaneous symmetry breaking. The encountered phenomenon is the celebrated *Higgs effect*.