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Fundamental Interactions

Problem 12: neutral kaon system Consider the two operations: parity P and charge conjugation C. Parity is the inversion of vectors at the origin. As such there are two types of vectors: P odd vectors, also called *polar* vectors, change their sign under parity, while P even vectors, known as *pseudo* or *axial* vectors, are invariant under parity. Charge conjugation, on the other hand, reverses the sign of all internal quantum numbers, while mass, energy, momentum and spin are unaffected. In this problem, we consider neutral kaons $K^0 = d\bar{s}$ and $\bar{K}^0 = \bar{d}s$.

a) Gell-Mann and Pais noted that the conversion $K^0 \rightleftharpoons \bar{K}^0$ is possible by an one-loop weak interaction. Draw the Feynman diagrams for this process. Is the strangeness quantum number conserved? (The strangeness of a state equals the number of s-quarks minus the number of $\bar{s}$-quarks.)

b) Consequently, K^0 and $\bar{K}^0$ are not the states normally observed. *Assuming* CP-invariance, the observed kaon states have to be CP-eigenstates. Using

$$P|K^0\rangle = -|K^0\rangle \ , \qquad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle \ , \tag{1a}$$

$$C|K^0\rangle = |\bar{K}^0\rangle \ , \qquad C|\bar{K}^0\rangle = |K^0\rangle \ , \tag{1b}$$

derive the CP-eigenstates K_1 and K_2 which are defined by

$$CP|K_1\rangle = |K_1\rangle \ , \qquad CP|K_2\rangle = -|K_2\rangle \ . \tag{2}$$

c) Kaons decay into neutral pions, for instance. The C and P action is as follows:

$$P|2\pi^0\rangle = |2\pi^0\rangle \ , \qquad P|3\pi^0\rangle = -|3\pi^0\rangle \ , \tag{3a}$$

$$C|2\pi^0\rangle = |2\pi^0\rangle \ , \qquad C|3\pi^0\rangle = |3\pi^0\rangle \ . \tag{3b}$$

What are the CP-allowed decay processes of K_1 and K_2?

The K_1 component decays much faster than the K_2 component, such that the K_1's are typically gone after a few centimetres, whereas the K_2's can travel several metres. Consequently, one speaks of a long- and short-lived state K_L and K_S. If CP-invariance holds, we can identify $K_S \equiv K_1$ and $K_L \equiv K_2$.

d) Suppose an experiment starts with a pure beam of K^0 states at time $t = 0$, i.e.

$$|\psi(t = 0)\rangle = |K^0\rangle \tag{4}$$

and let us introduce the time dependent wave functions

$$|K_\alpha(t)\rangle = |K_\alpha\rangle \cdot e^{-im_\alpha t - \frac{1}{2}\Gamma_\alpha t} \qquad \text{for} \quad \alpha \in \{L, S\} \ , \tag{5}$$

where m_α denotes the masses and $\Gamma_\alpha = \frac{1}{\tau_\alpha}$ the decay rate. Experimentally, one finds for the masses $m_L - m_S \approx (3{,}506 \pm 0{,}006) \cdot 10^{-15}$GeV, but $\tau_S = 0{,}9 \cdot 10^{-10}$s compared to $\tau_S = 0{,}5 \cdot 10^{-7}$s. The wave function ψ evolves as

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left(|K_S\rangle \cdot e^{-im_S t - \frac{1}{2}\Gamma_S t} + |K_L\rangle \cdot e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) \tag{6}$$

Compute the decay rate $\Gamma(K^0|_{t=0} \to 2\pi^0)$ (as a function of time) for the state ψ, which was produced as K^0 at $t = 0$, into two neutral pions.

e) Again, assuming CP-invariance we can employ the above set-up to study *strangeness oscillations*. Suppose you start with $|\psi(t)\rangle$ such that $|\psi(0)\rangle = |K^0\rangle$. Then compute $\Gamma(K^0|_{t=0} \to K^0)$ and $\Gamma(K^0|_{t=0} \to \bar{K}^0)$. Interpret your result.

f) The experimentially measured mass difference $\Delta m = m_L - m_S$ corresponds to an oscillation period of $T_{osc} = \frac{2\pi\hbar}{\Delta m} \approx 1,2 \cdot 10^{-9}$s. Comparing this to the lifetimes, what is the consequence for the oscillations?

In summary, note that the neutral kaons are typically produced by strong interactions, in the eigenstates of strangeness (K^0 and $\bar{K}^0$), but they decay by the weak interactions, as eigenstates of CP (K_1 and K_2).

Problem 13: CP violation In 1964, Cronin and Fitch analysed the neutral kaon decay products at the end of a 57 feet ($\approx 17,37$ metre) long beam. They counted 45 two-pion-events in a total of 22.700 events. Hence, the long-lived state cannot be a pure CP-eigenstate and the (now to be defined) long-lived component K_L needs to admit a small contribution of K_1 as follows:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} \left(|K_2\rangle + \epsilon |K_1\rangle\right) \ , \tag{7}$$

and similarly for the short-lived state

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} \left(|K_1\rangle + \epsilon |K_2\rangle\right) \ . \tag{8}$$

a) Compute the value $|\epsilon|$ for the Cronin and Fitch experiment.

b) If one observes a neutral kaon beam a long time after production it will consist solely of the K_L component. The original kaons K^0 and $\bar{K}^0$ can also decay leptonically via

$$K^0 \to \pi^+ \ e^- \ \bar{\nu}_e \ , \qquad \bar{K}^0 \to \pi^- \ e^+ \ \nu_e \ . \tag{9}$$

Compute the decay widths $\Gamma(K_L \to \pi^+ \ e^- \ \bar{\nu}_e)$ and $\Gamma(K_L \to \pi^- \ e^+ \ \nu_e)$ up to leading order in ϵ. Compare the two rates.