Quantum Field Theory: Exercise Session 7 29 January 2016

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Problem 1. The Renormalization Group

In the massless case, the Callan-Symanzik (or renormalization group, RG) equation for the renormalized n-point function $\tilde{\Gamma}^{(n)}(\{p_i\}, \lambda, \mu)$ (connected, truncated 1PI diagrams) reads

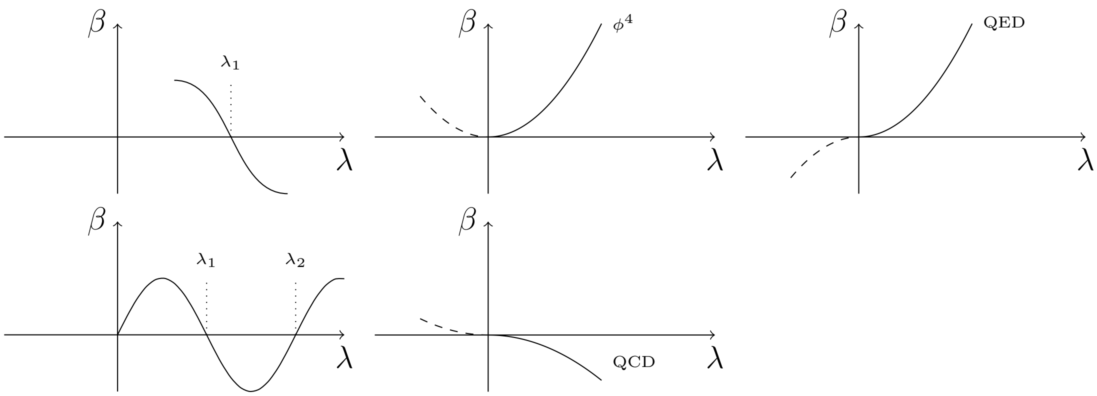
$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial\lambda} - \gamma\right)\tilde{\Gamma}^{(n)}(\{p_i\}, \lambda, \mu) = 0, \tag{1}$$

where μ is the renormalization scale.

(a) Using the RG equation, derive the scale dependence of $\tilde{\Gamma}^{(n)}(\{wp_i\}, \lambda, \mu)$ as $p_i \to wp_i$ in terms of a relation $\left(w\frac{\partial}{\partial w} + \dots\right)\tilde{\Gamma}^{(n)}(\{wp_i\}, \lambda, \mu) = 0$. Hint: In four spacetime dimensions, $\tilde{\Gamma}^{(n)}$ has mass dimension $4 - n$.

(b) What does the equation derived in (a) imply for the scale dependence of $\lambda = \lambda(w)$?

(c) What can you say qualitatively about the behavior of $\lambda(w)$ as $w \to 0, \infty$ for the following β-functions $\beta(\lambda(w))$?



Problem 2. Power Counting in Quantum Electrodynamics

(a) Derive a formula $D = D(d, L, F_i, P_i)$ for the superficial degree of divergence D of a QED diagram with L loops, F_i internal fermion lines and P_i internal photon lines in d dimensions.

(b) Express D in terms of d, the number of vertices n and the number of external lines F_e and P_e.

(c) Draw all 1PI divergent diagrams containing up to two loops in four dimensions.