Effekte der Relativität

bewegte Uhren gehen langsamer

bewegte Massstäbe sind verkürzt

Geschwindigkeitsaddition anders

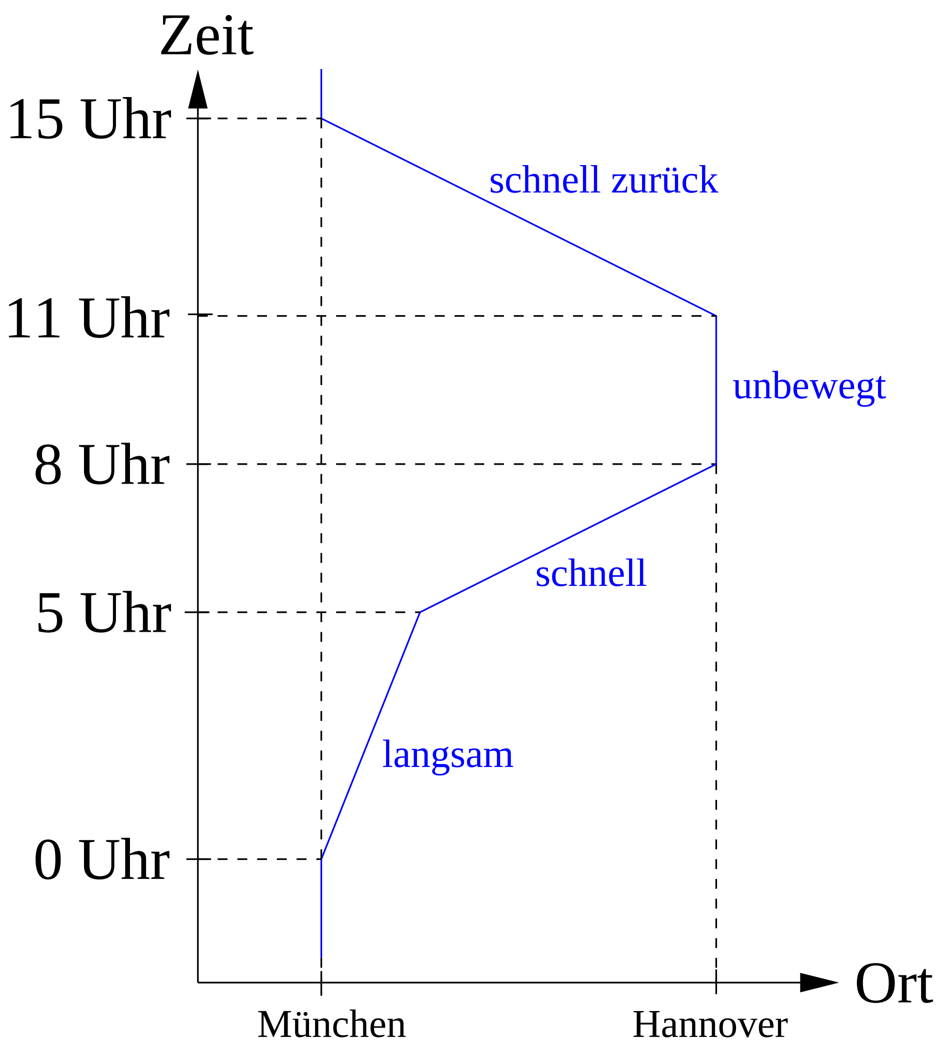
bewegte Beobachter sehen Lichtquellen

− aus anderer Richtung (Aberration)

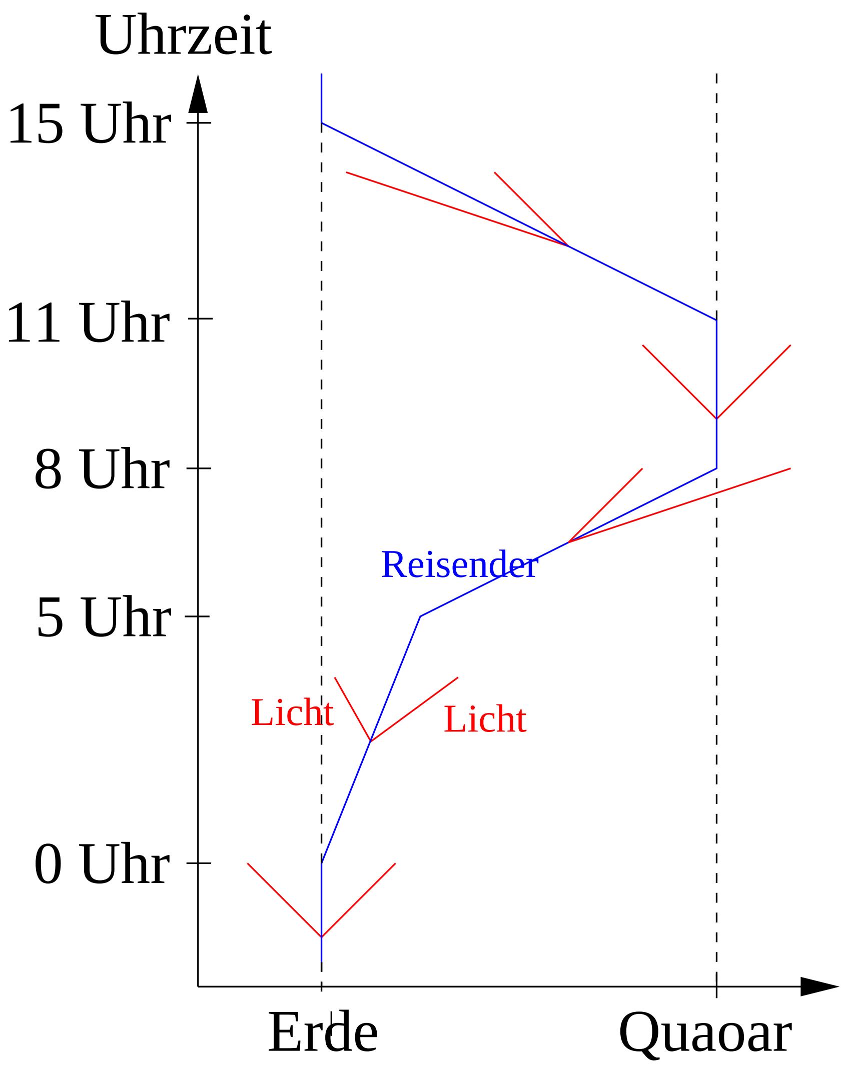
− in anderen Farben (Doppler−Effekt)

− in anderer Intensität

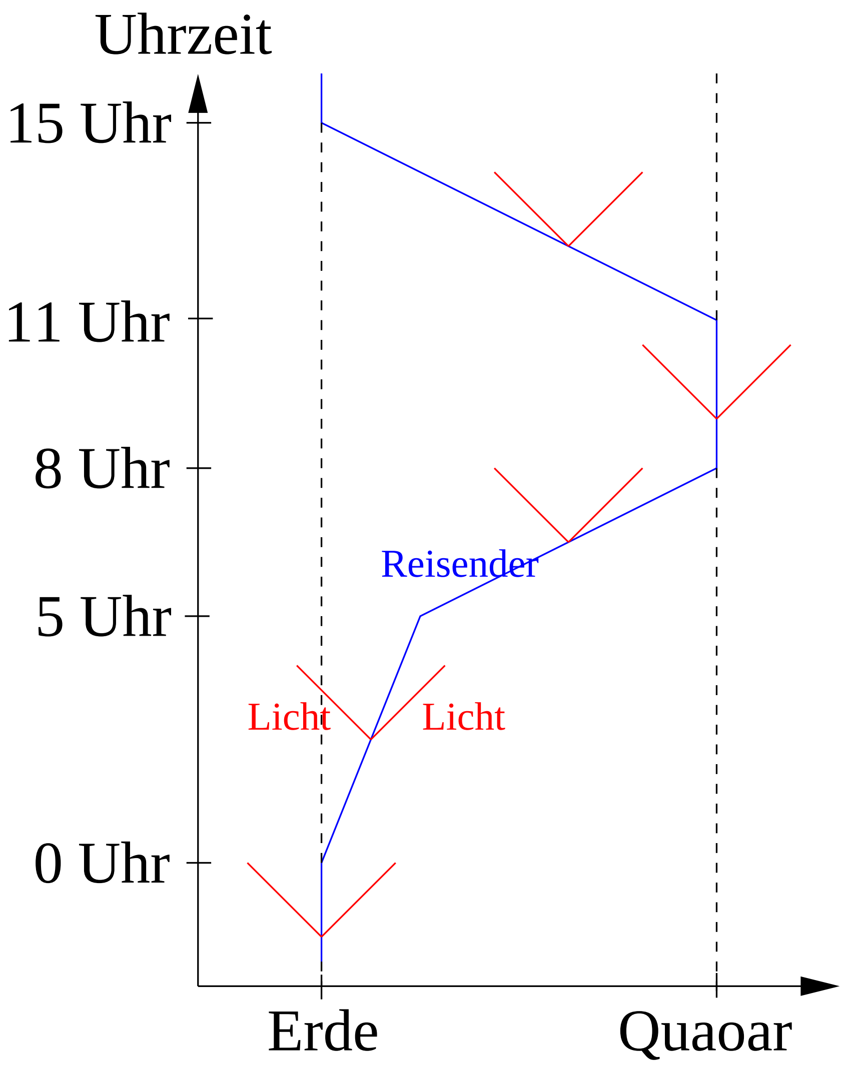
Raum−Zeit−Diagramm



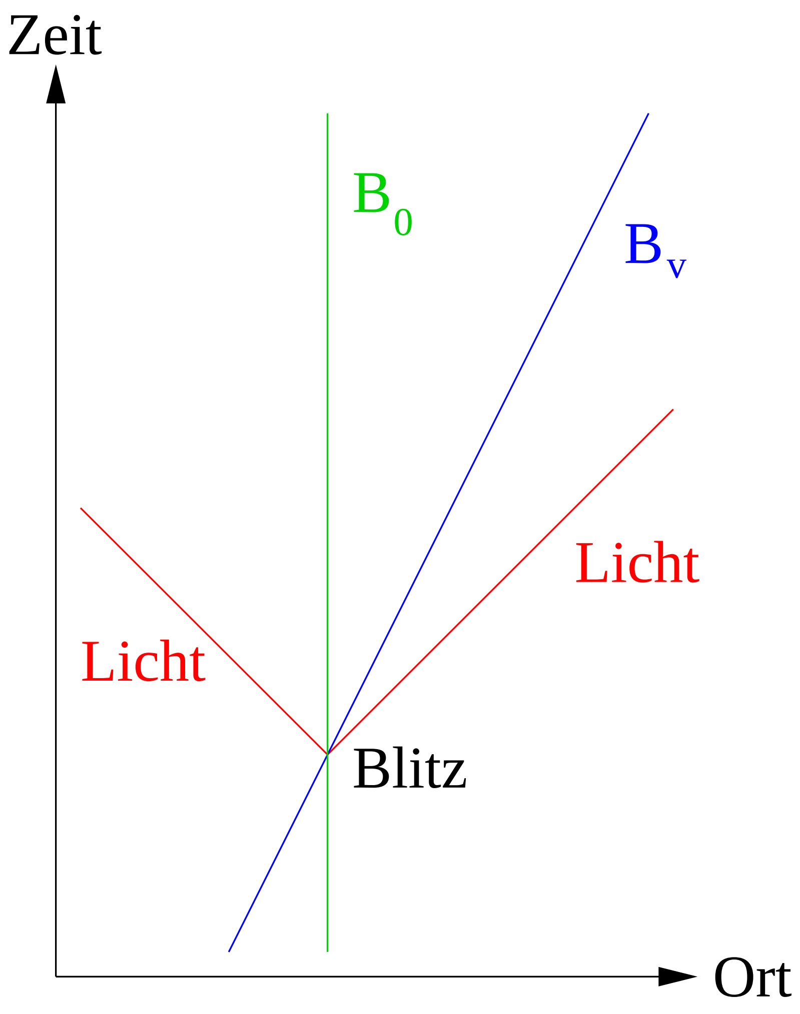
v o r   E i n s t e i n



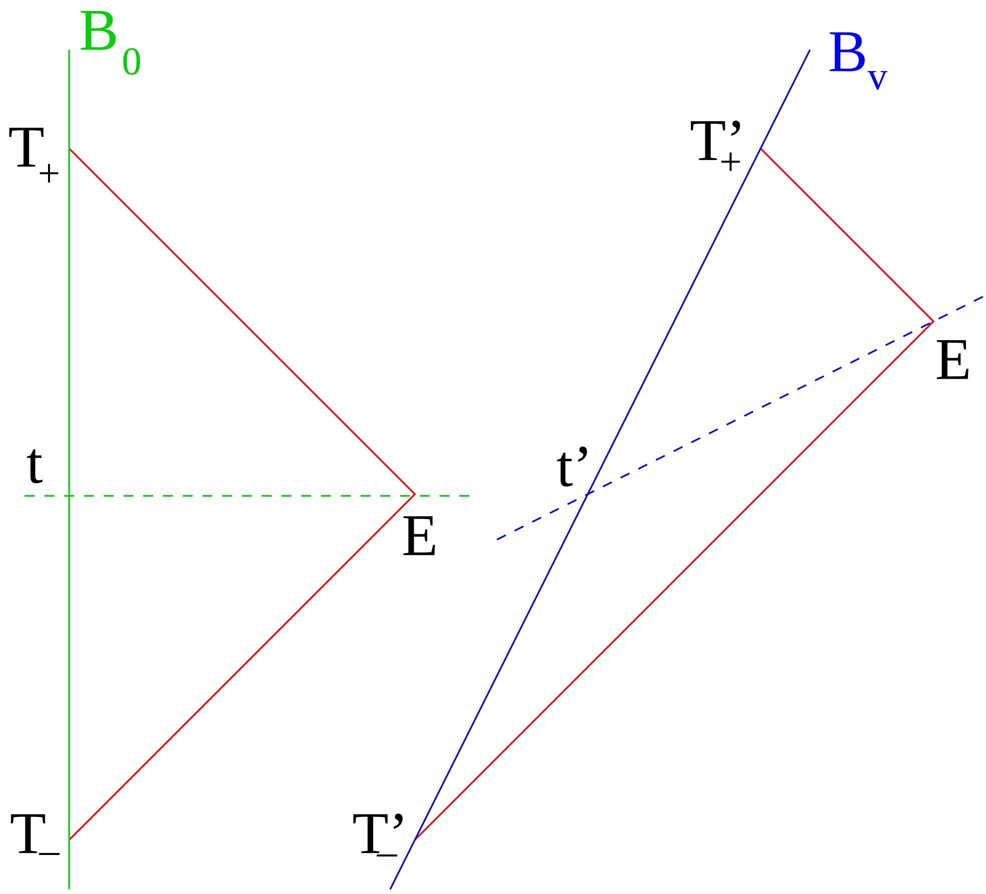
n a c h   E i n s t e i n



Beobachter und Licht



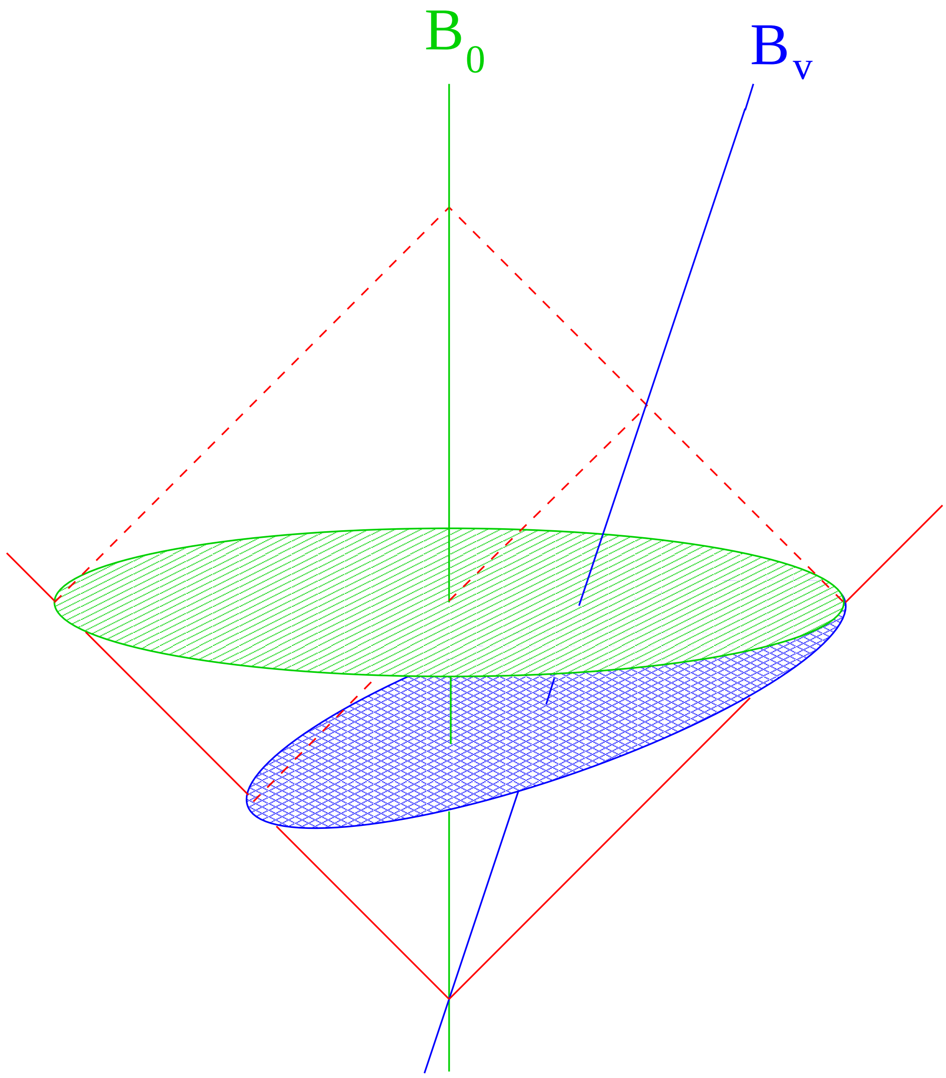
relativ gleichzeitig



$$r = (T_+ - T_-)/2 \qquad T_+ = t + r$$

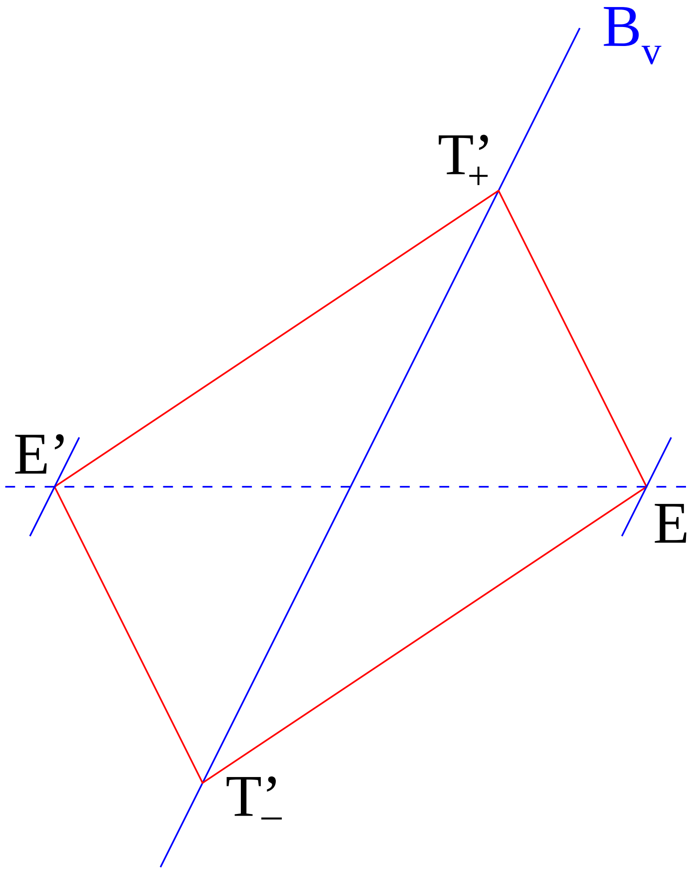
$$t = (T_+ + T_-)/2 \qquad T_- = t - r$$

Zwei Raum−Dimensionen



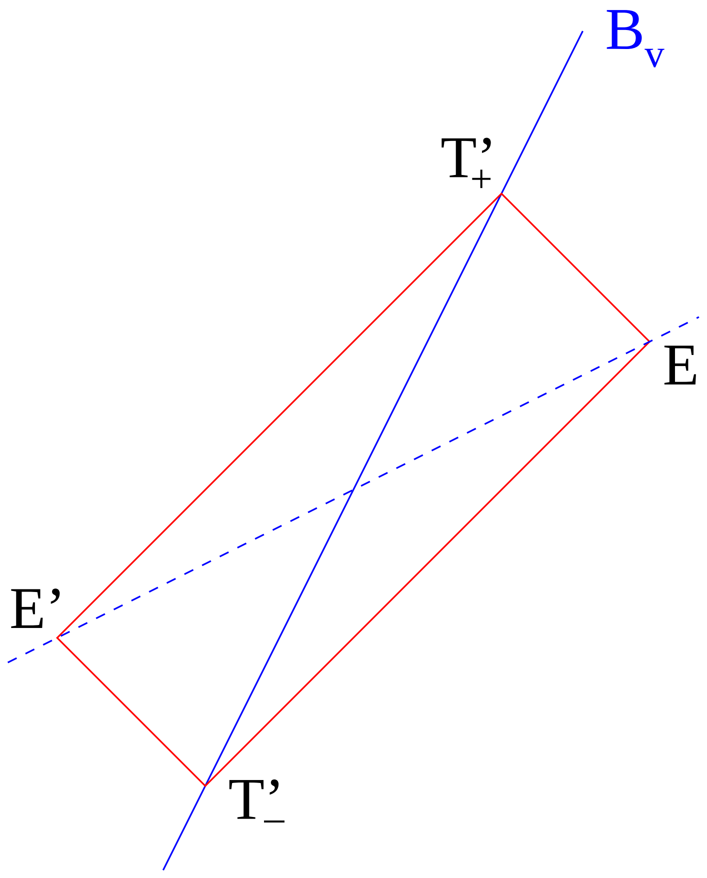
vor Einstein:

Licht−Parallelogramm

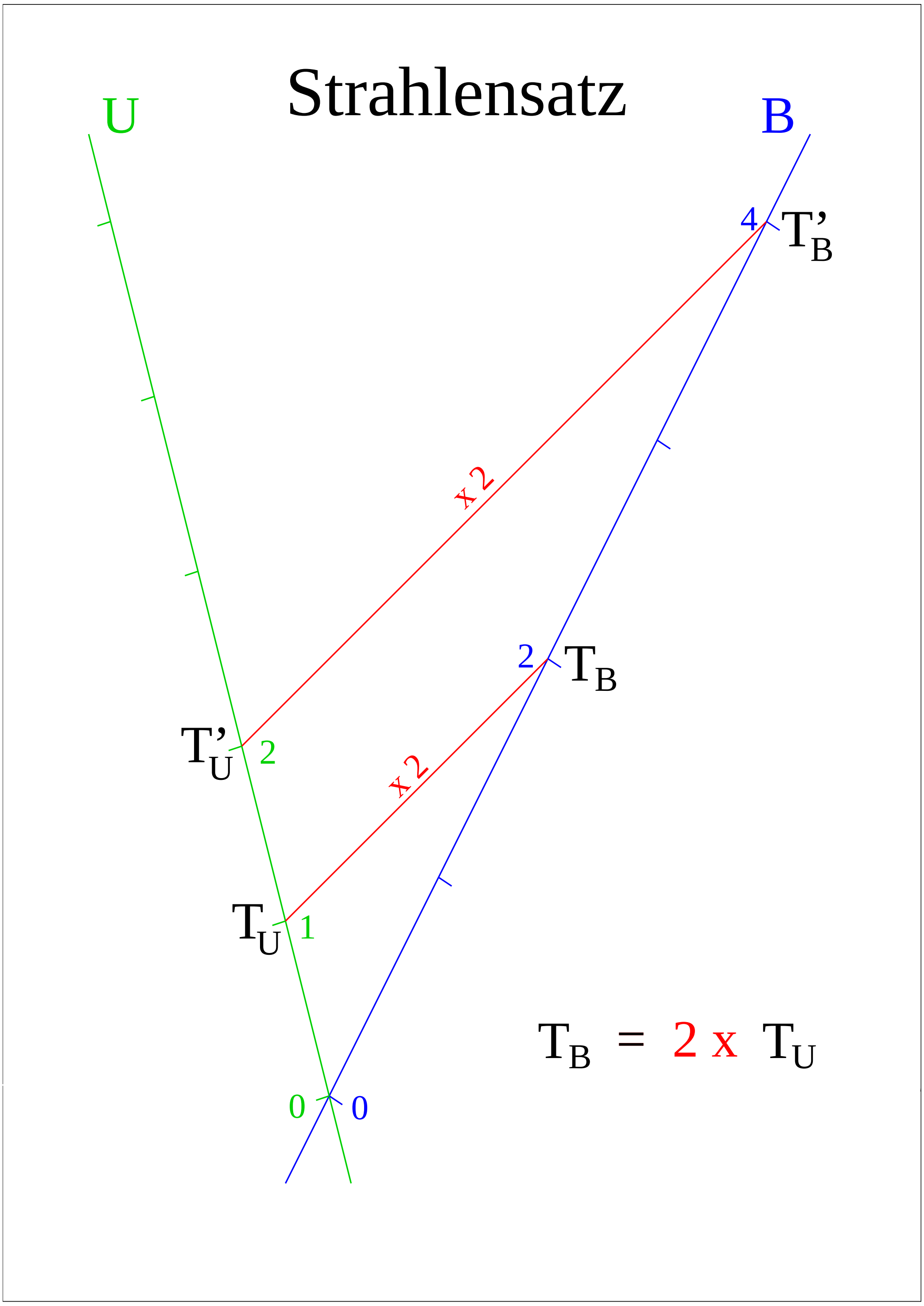


gleich−ortig & gleichzeitig

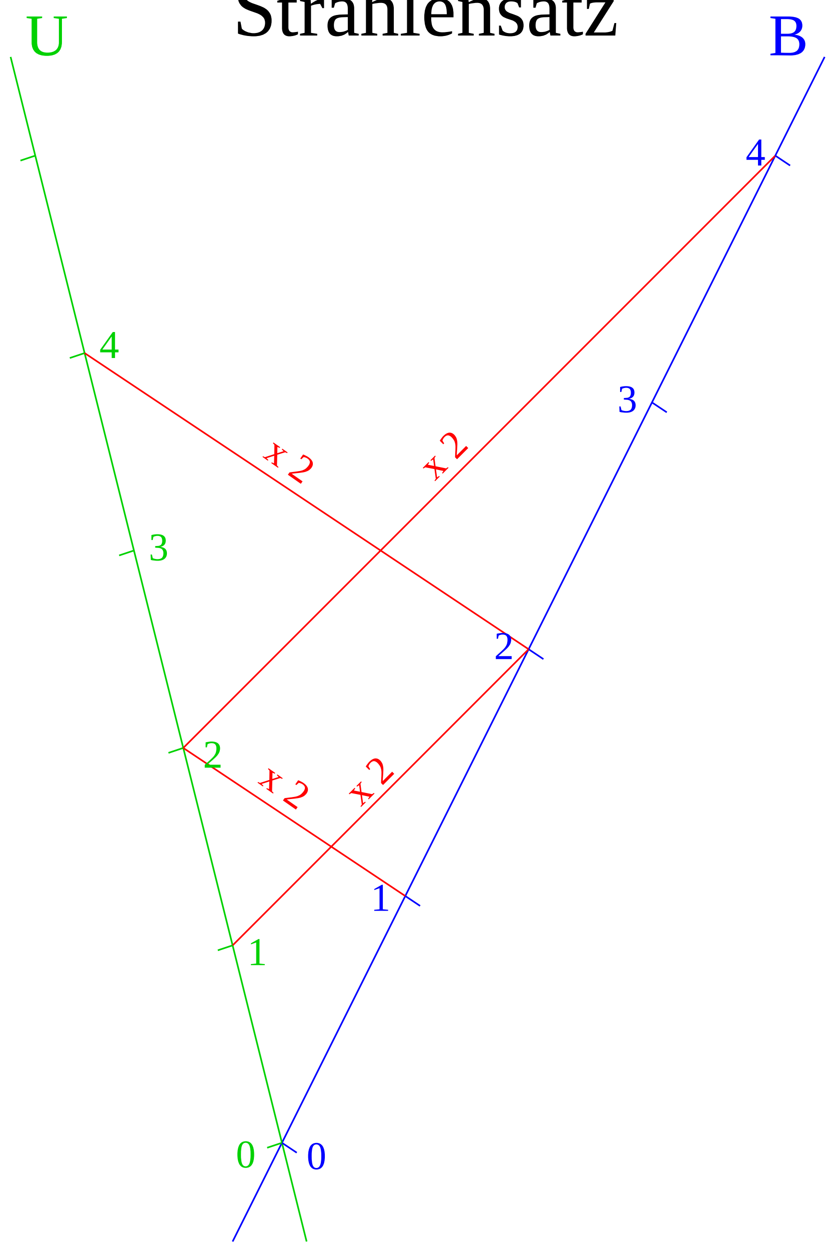
Licht−Eck



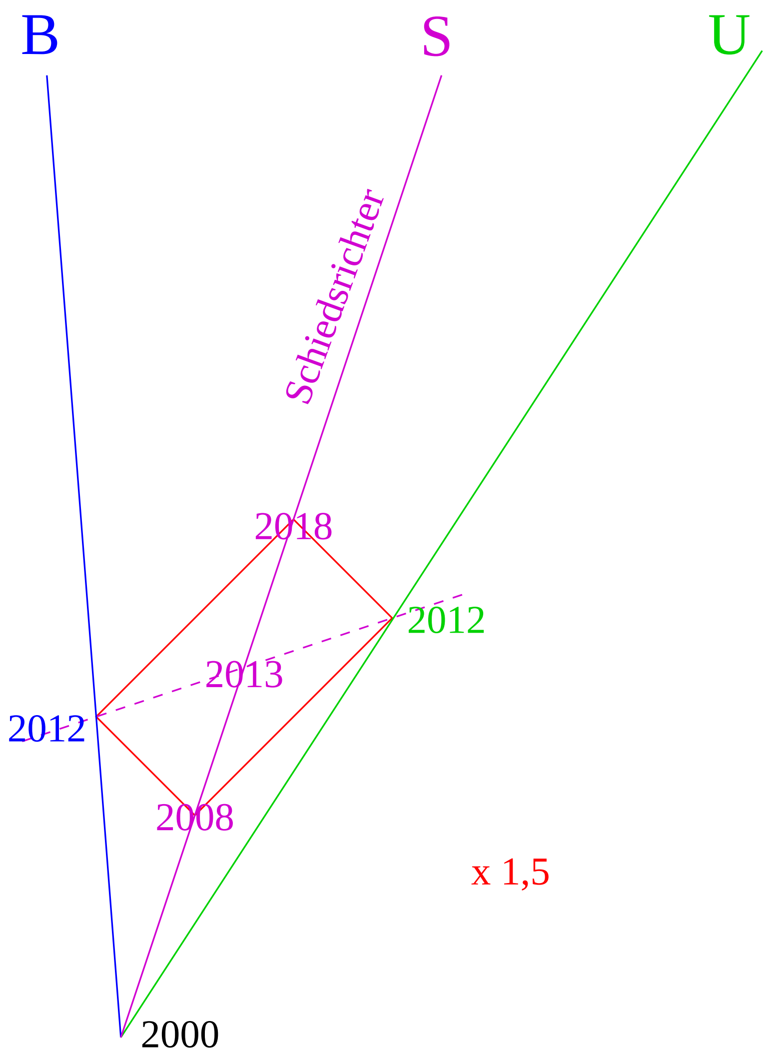
gleich−ortig & gleichzeitig


Strahlensatz
U
B
4
T'B
2
TB
x 2
x 2
T'U
2
TU
1
0
0
TB = 2 x TU

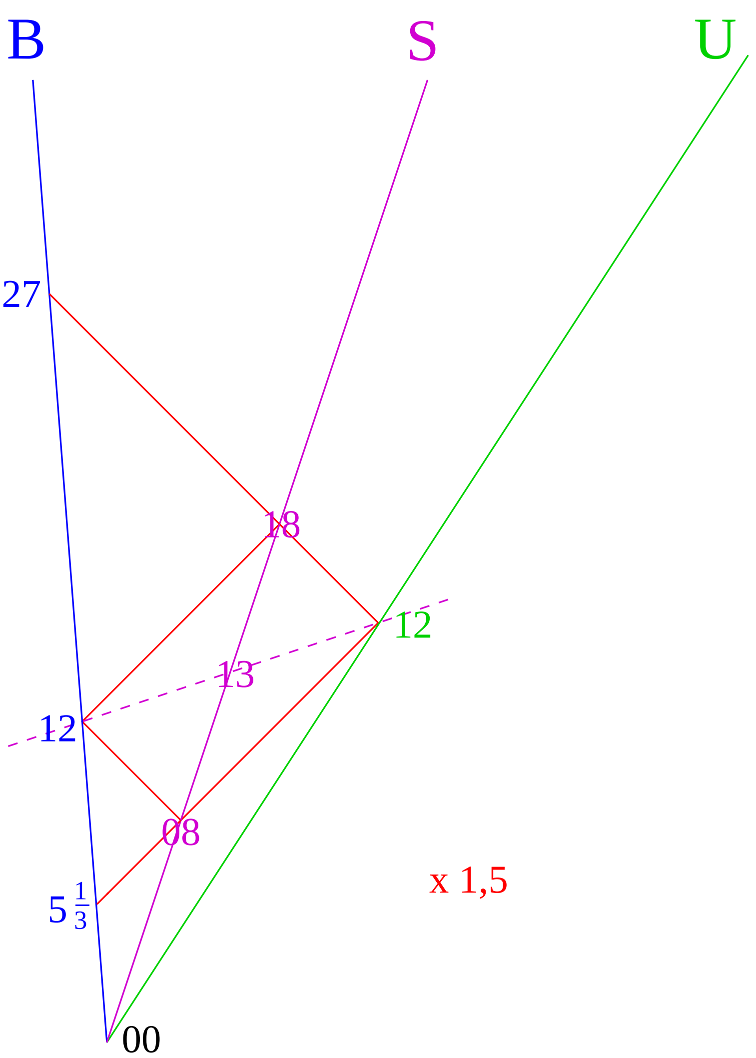
Strahlensatz



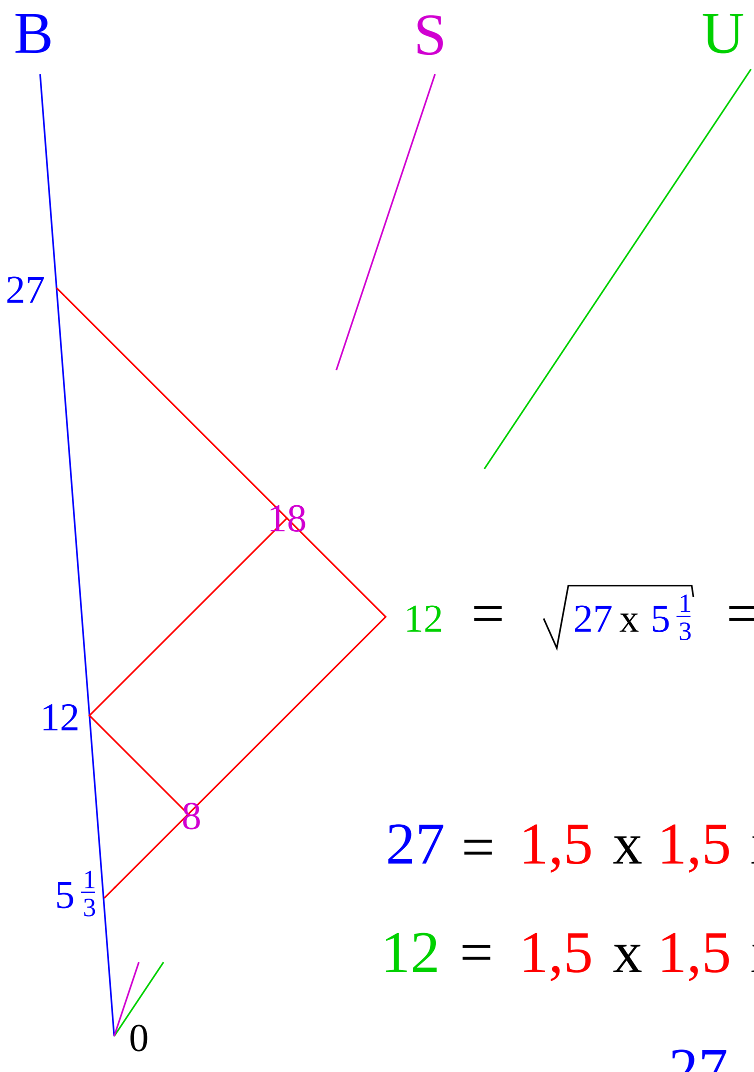
Uhrenvergleich



verlängertes Lichteck



geometrisches Mittel



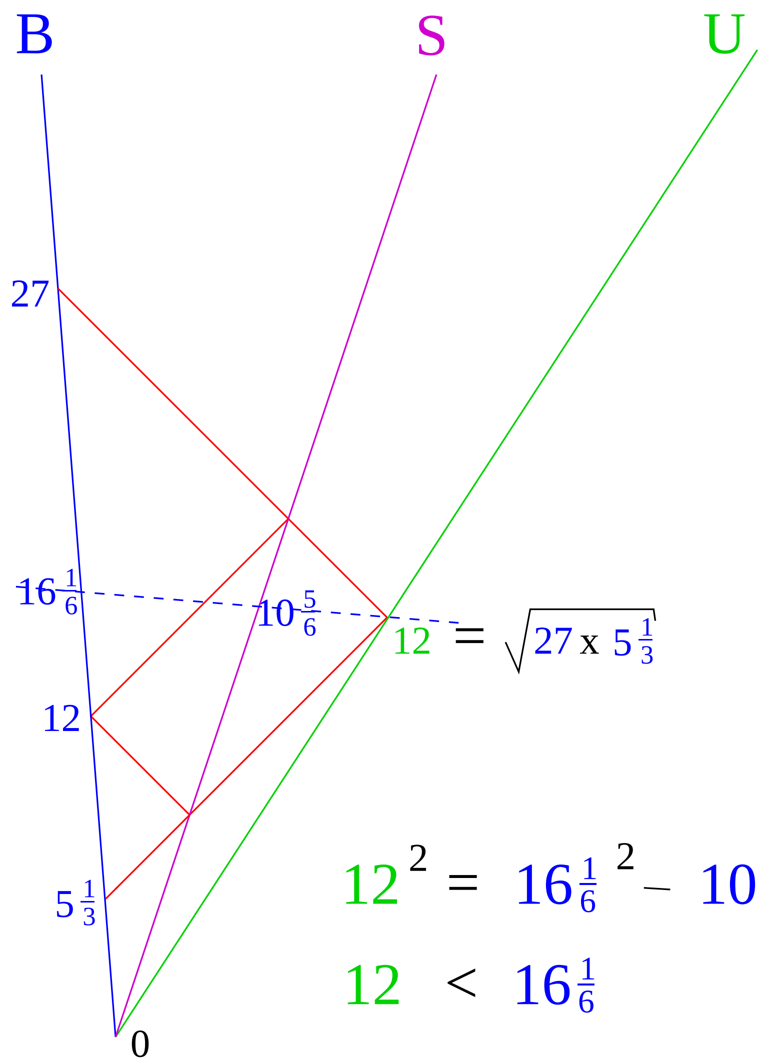
$$12 = \sqrt{27 \times 5\tfrac{1}{3}} = \sqrt{144}$$

$$27 = 1{,}5 \times 1{,}5 \times 12$$

$$12 = 1{,}5 \times 1{,}5 \times 5\tfrac{1}{3}$$

$$\Rightarrow \quad \frac{27}{12} = \frac{12}{5\tfrac{1}{3}}$$

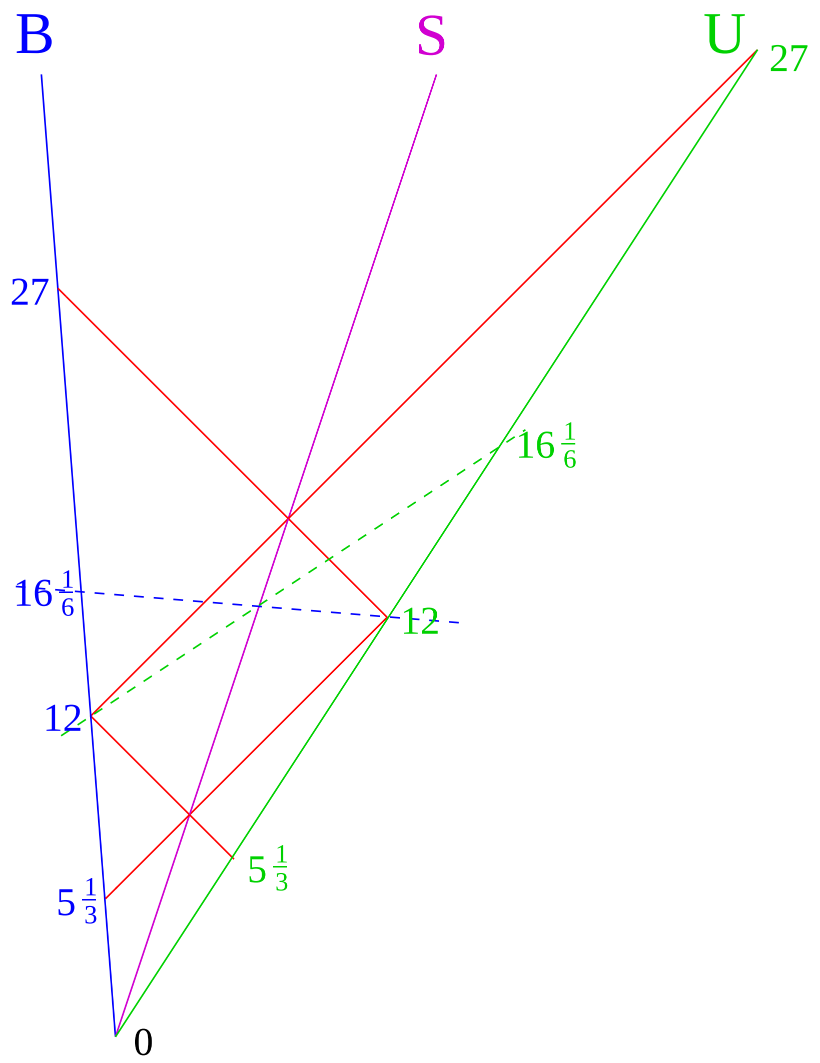
Zeit−Dehnung



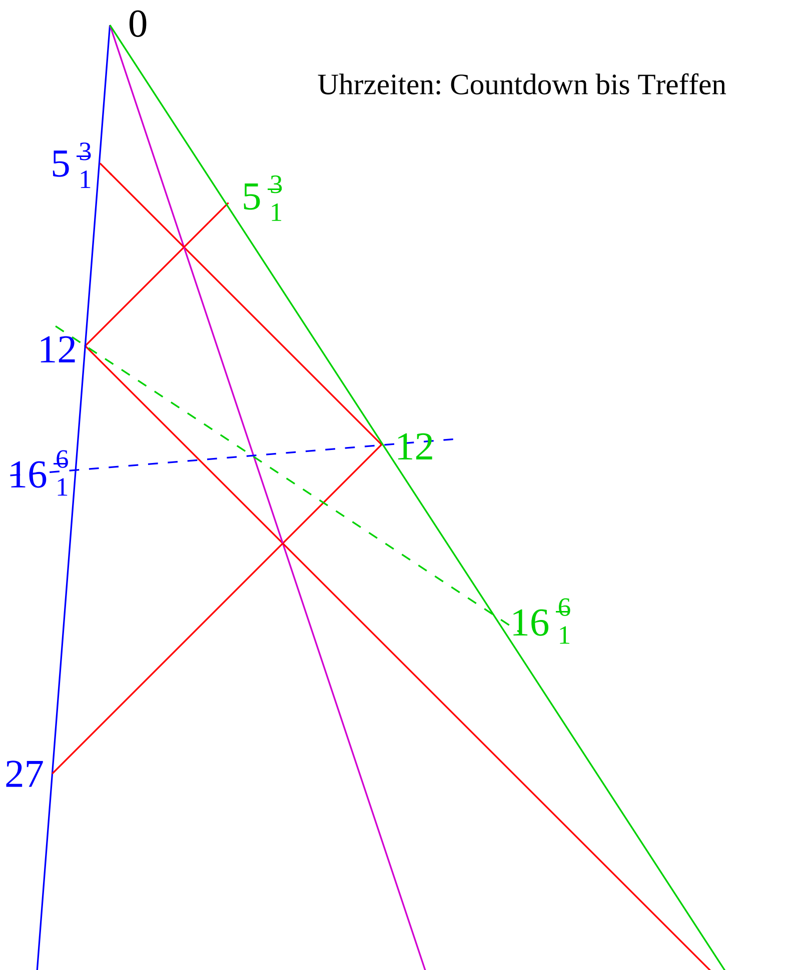
$$12^2 = 16\tfrac{1}{6}^{\,2} - 10\tfrac{5}{6}^{\,2}$$

$$12 < 16\tfrac{1}{6}$$

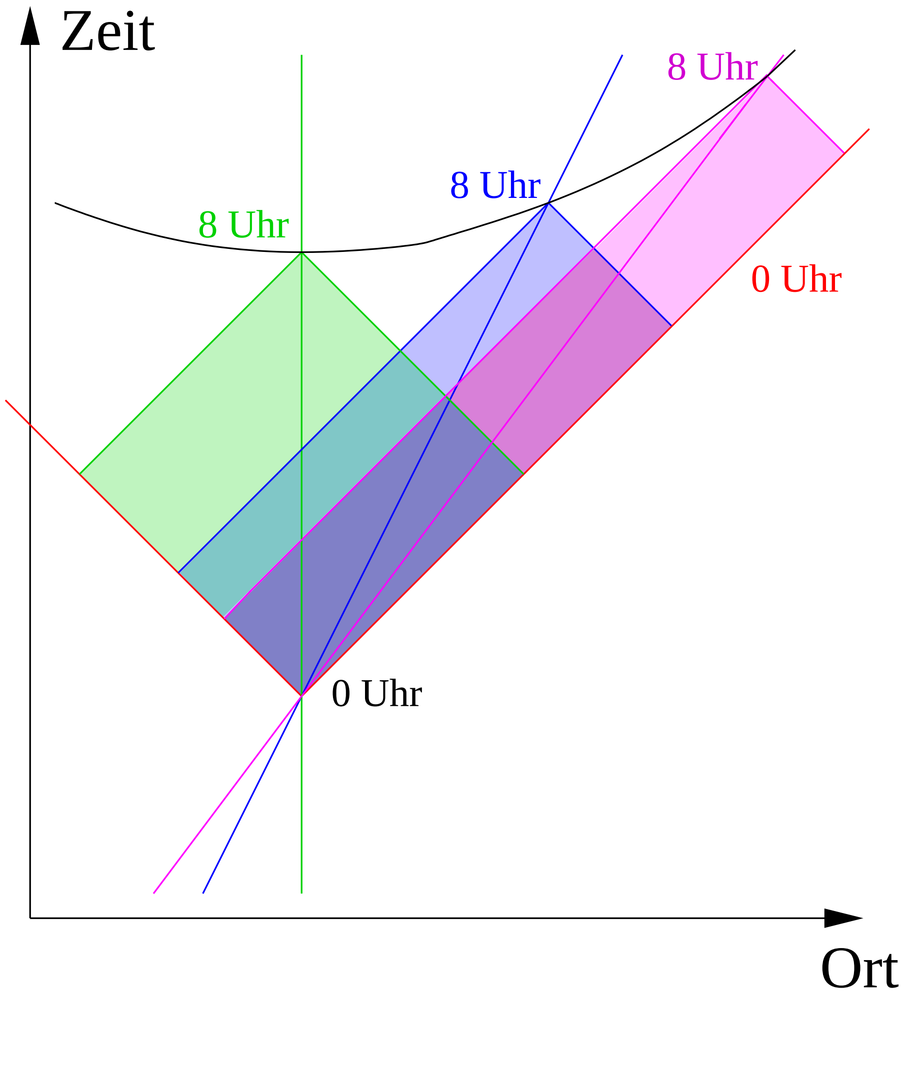
Zeit−Dehnung wechselseitig



umgekehrte Bewegung



Bewegte Uhren



Licht-Koordinaten $\quad r = \frac{1}{2}(T_+ - T_-)$ und $t = \frac{1}{2}(T_+ + T_-)$

$$T_\pm = t \pm r \quad \Longrightarrow \quad T_+ T_- = t^2 - r^2 = \tau^2 = (1-v^2)\, t^2 = t^2/\gamma^2$$

Streck-Faktor

$$T_+ = k\,\tau \quad \text{und} \quad \tau = k\, T_- \quad \Longrightarrow \quad k^2 = \frac{T_+}{T_-} = \frac{t+r}{t-r} = \frac{1+v}{1-v}$$

$$k(v) = \sqrt{\frac{1+v}{1-v}} = \frac{1+v}{\sqrt{1-v^2}} \quad \Longleftrightarrow \quad v = \frac{k^2-1}{k^2+1} = \frac{k-\frac{1}{k}}{k+\frac{1}{k}}$$

Rapidität

$$k =: \mathrm{e}^\theta \quad \Longrightarrow \quad v = \frac{\mathrm{e}^\theta - \mathrm{e}^{-\theta}}{\mathrm{e}^\theta + \mathrm{e}^{-\theta}} = \tanh\theta \quad \text{und} \quad \gamma = \frac{1}{\sqrt{1-v^2}} = \cosh\theta$$

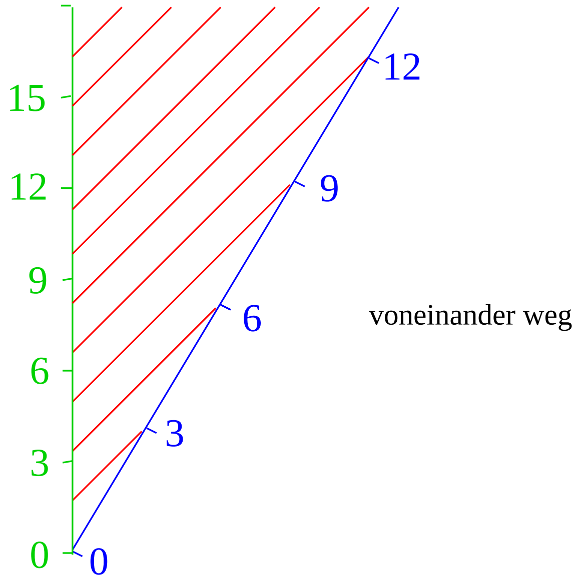
Lorentz-Transformation

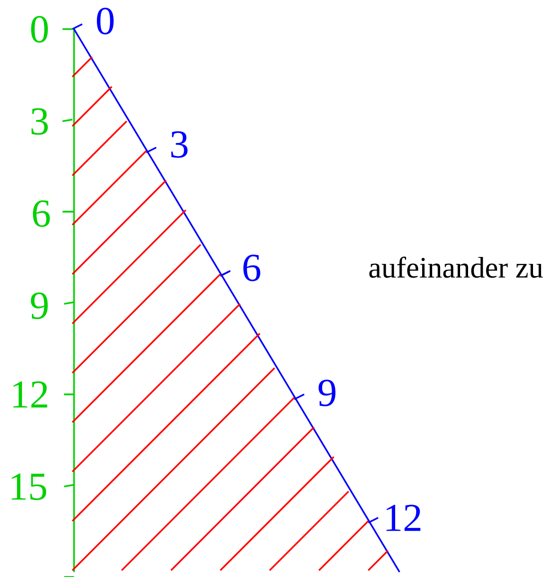
$$\begin{pmatrix} T'_+ \\ T'_- \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & \frac{1}{k} \end{pmatrix} \begin{pmatrix} T_+ \\ T_- \end{pmatrix} = \begin{pmatrix} \mathrm{e}^\theta & 0 \\ 0 & \mathrm{e}^{-\theta} \end{pmatrix} \begin{pmatrix} T_+ \\ T_- \end{pmatrix}$$

$$\begin{pmatrix} t' \\ r' \end{pmatrix} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} t \\ r \end{pmatrix} = \gamma \begin{pmatrix} 1 & v \\ v & 1 \end{pmatrix} \begin{pmatrix} t \\ r \end{pmatrix}$$

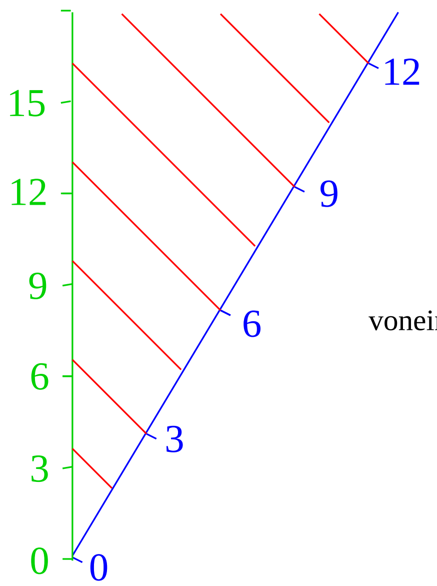
Aberration $\quad \tan\frac{\phi'}{2} = k \tan\frac{\phi}{2} = \sqrt{\frac{1+v}{1-v}} \tan\frac{\phi}{2}$

regelmäßige Lichtsignale

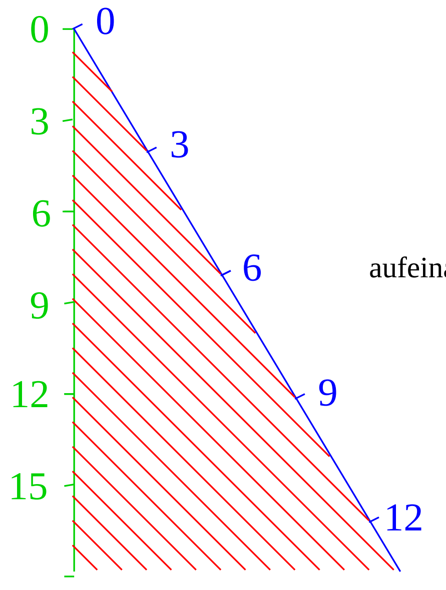




regelmäßige Lichtsignale

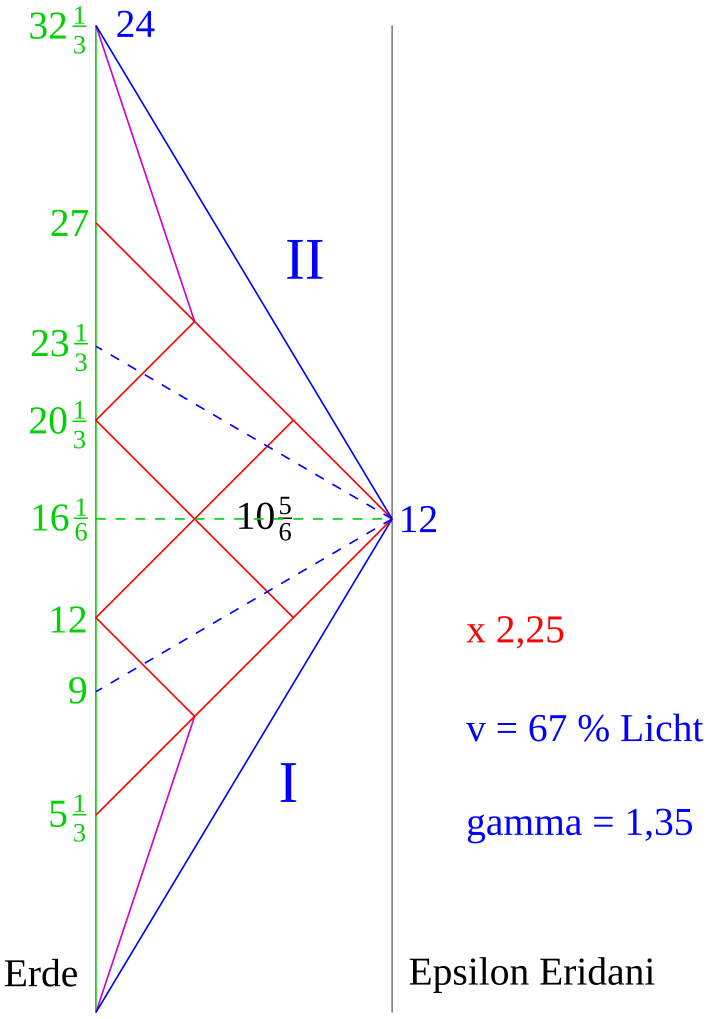


voneinander weg

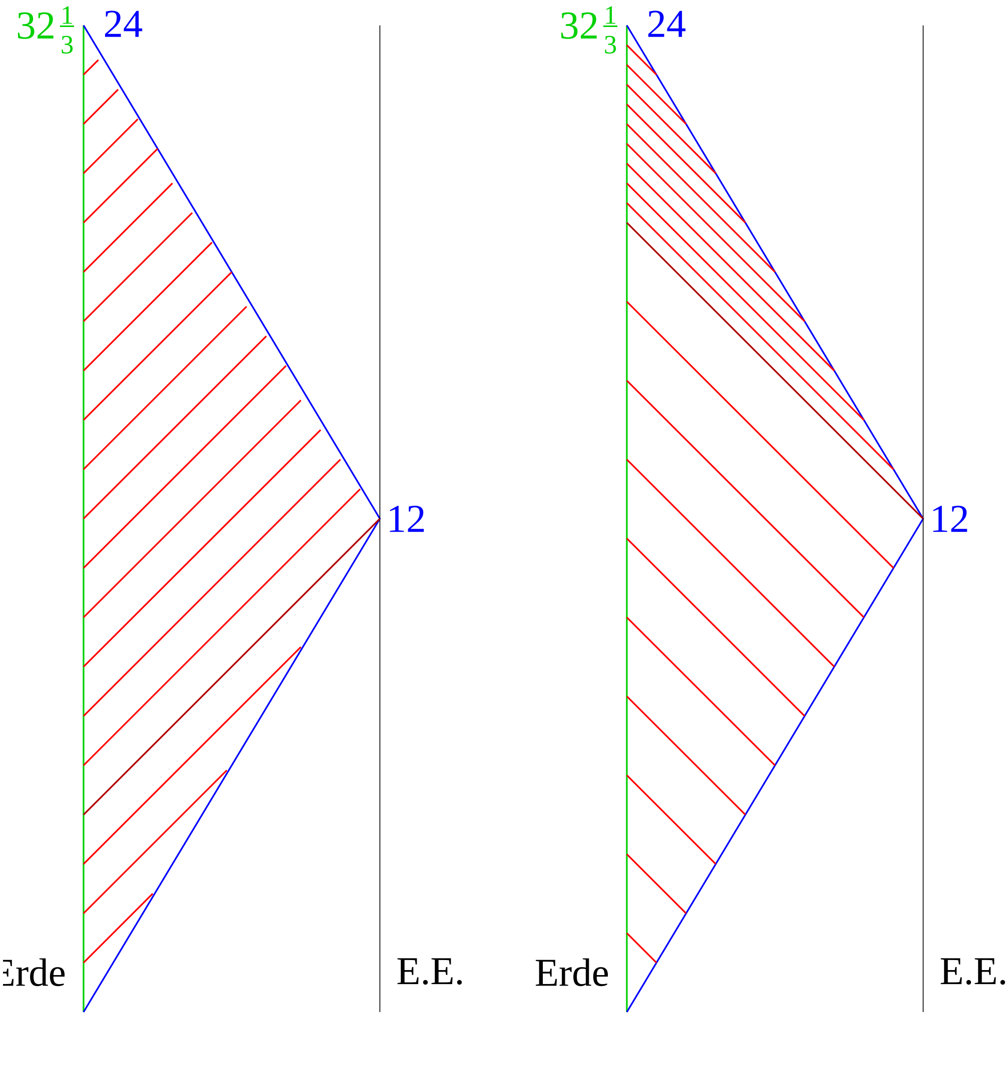


aufeinander zu

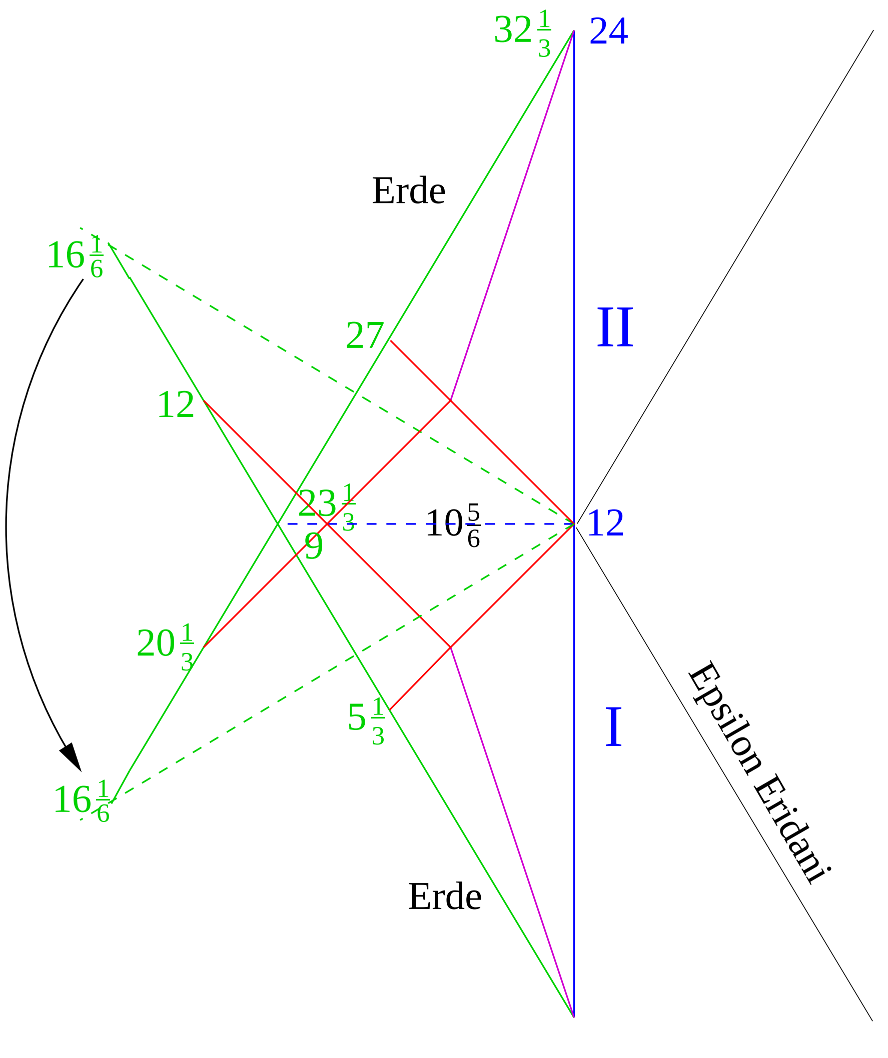
Zwillings−Effekt



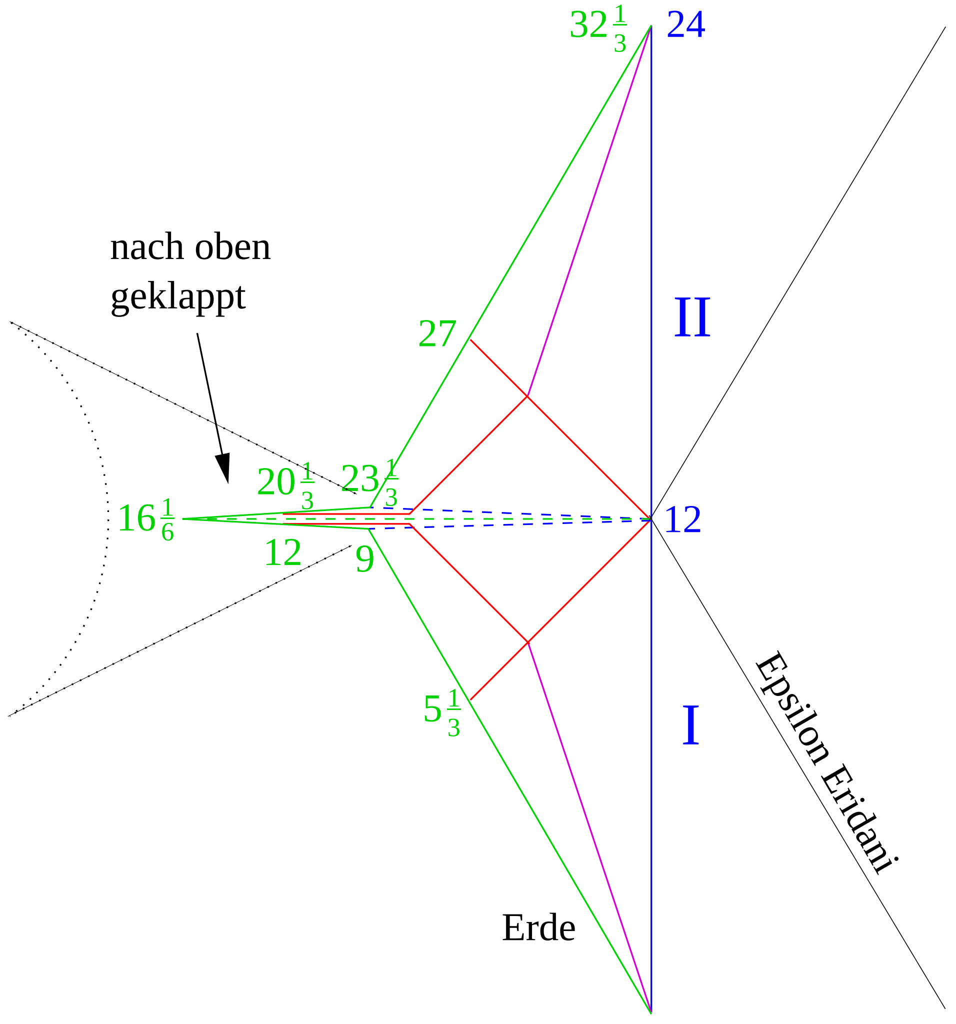
Austausch von Lichtsignalen



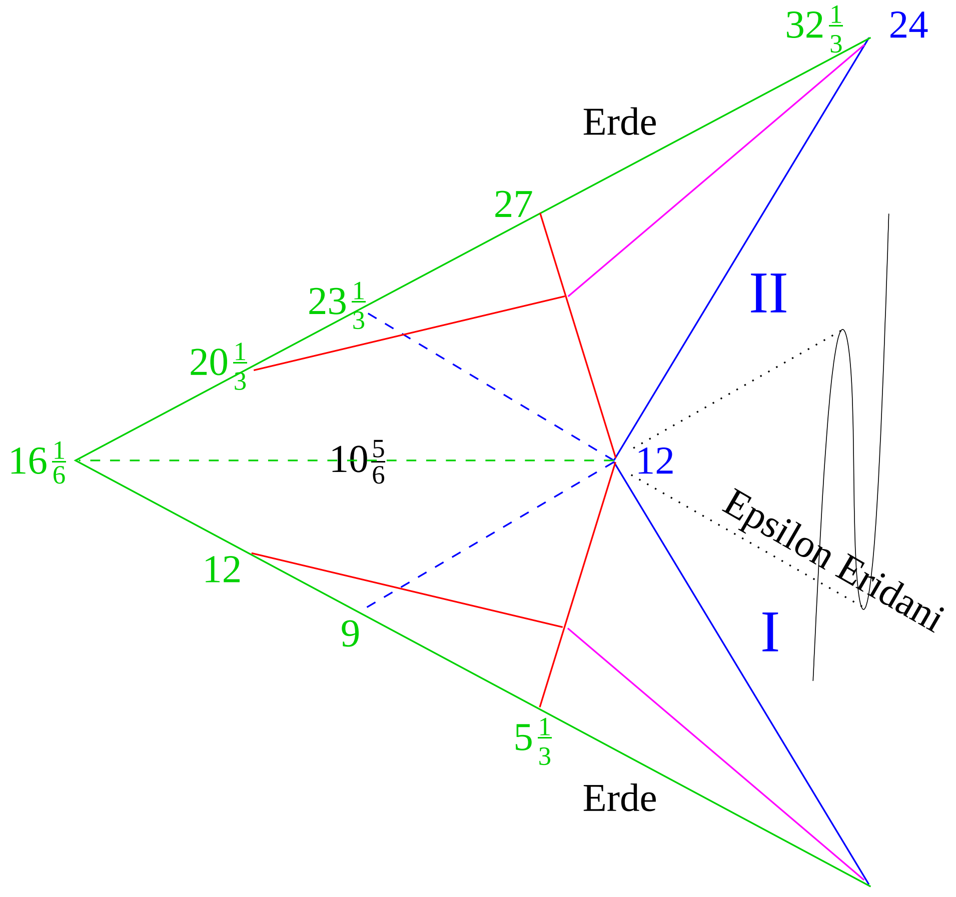
Zwillings−Effekt einseitig



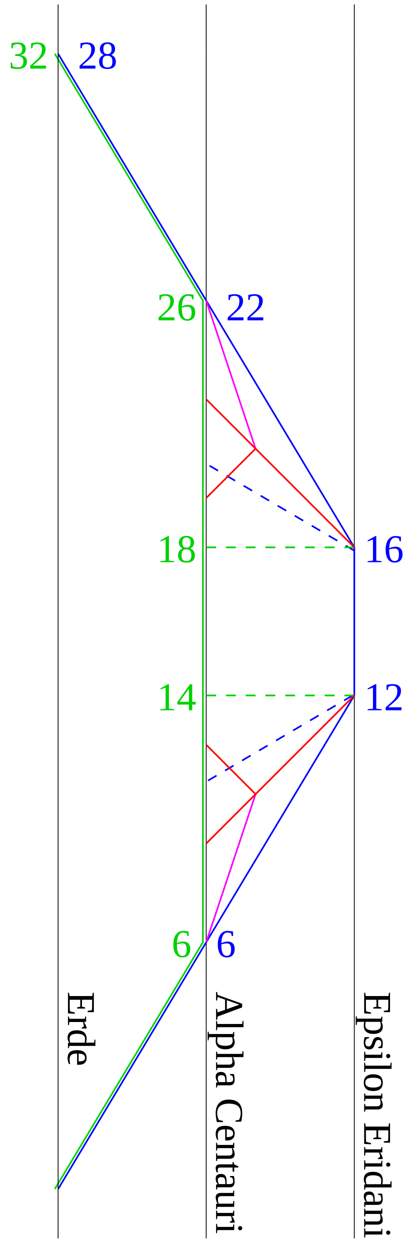
Space-Time Warp



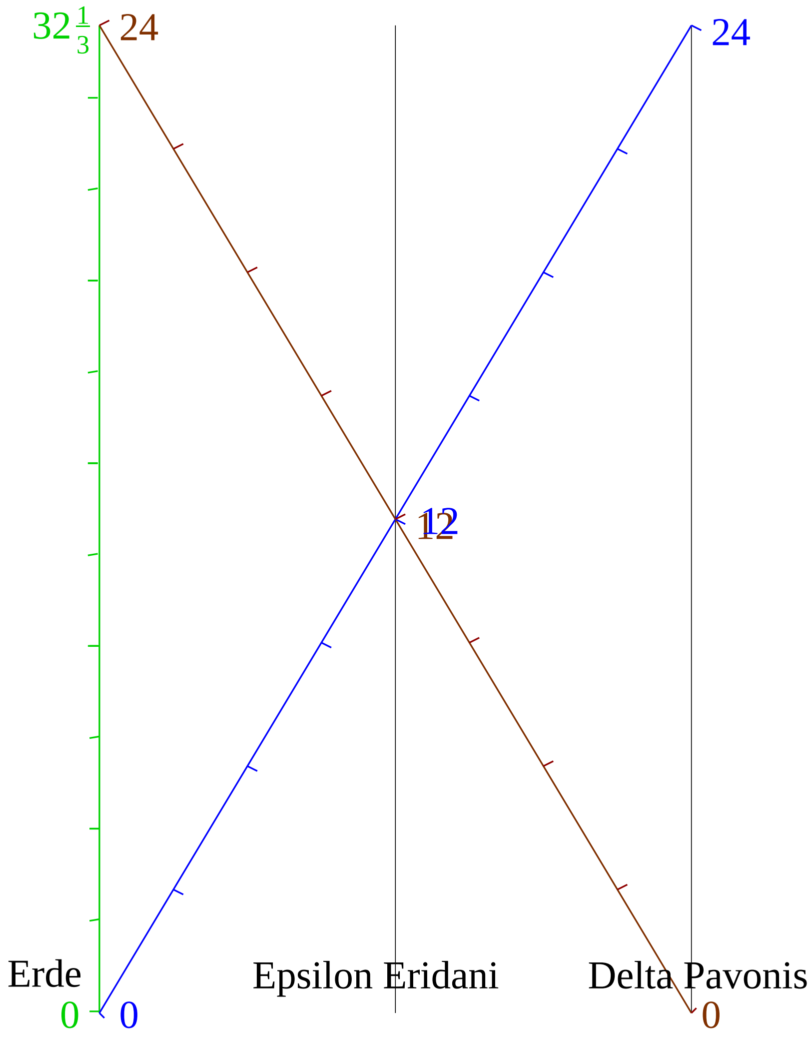
Warp geglättet?



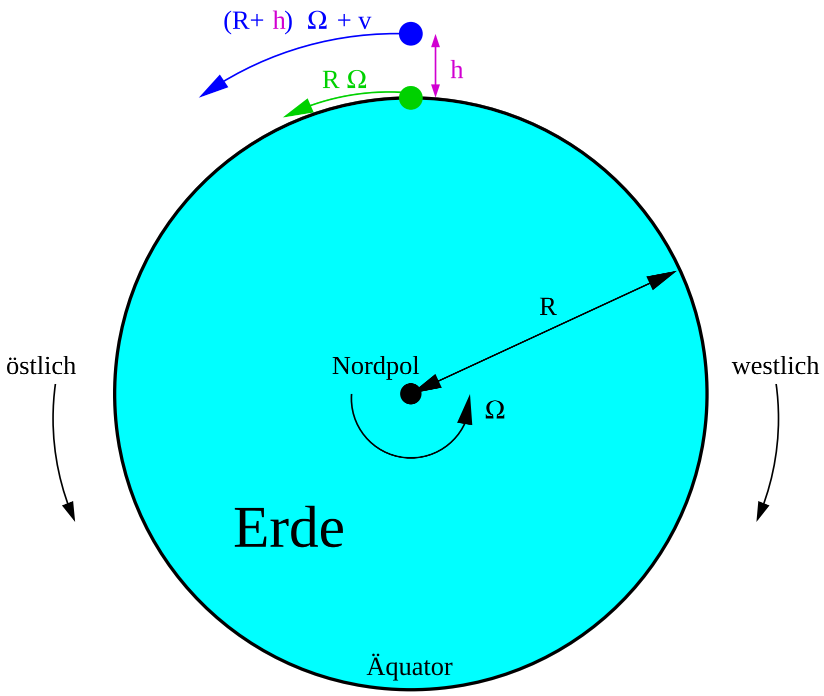
gleiche Beschleunigungen



ohne Beschleunigung



Hafele & Keating 1971



relative Differenz $\delta = \frac{T - T}{T} = \frac{g\,h}{c^2} - \frac{(2\,R\,\Omega + v)\;v}{2\,c^2}$

$c = 300000000$ m/s $\quad g = 9{,}8$ m/s ^{2} $\quad R\,\Omega = 470$ m/s

$h = 10000$ m $\quad v = \pm 300$ m/s

$\delta_+ = (1-2) \times 10^{-12}$

$\delta_- = (1+1) \times 10^{-12}$

Flug von 36 Stunden = 130000 Sekunden

Zeitdifferenz: − 130 ns bzw. + 260 ns

mit Caesium−Uhren gemessen