12. Hausübung, **Statistische Physik**

abzugeben am Donnerstag, 19.1.2012

Aufgabe H21 *Transfer matrix* (12 Punkte)

We consider the one-dimensional Ising model with Hamiltonian

$$\mathcal{H} = -\sum_i J\sigma_i\sigma_{i+1}$$

with $J > 0$. We assume that there are N sites, with periodic boundary conditions: we identify σ_{N+1} with σ_1. The partition function Z for this system can be calculated exactly by observing that it can be written as

$$Z = \mathrm{tr}\,(T^N) \tag{1}$$

where T is a diagonalizable 2-by-2 matrix called the *transfer matrix*.

a. Show that eq. 1 is satisfied by the matrix

$$T = \begin{pmatrix} e^{J/\tau} & e^{-J/\tau} \\ e^{-J/\tau} & e^{J/\tau} \end{pmatrix}.$$

b. Use the transfer matrix T to compute Z.

   Hint: compute the eigenvalues of T.

c. Compute the free-energy f per spin in the thermodynamic limit $N \to \infty$. Is there a phase transition?

d. Use the transfer matrix to exactly compute the correlation function $\langle\sigma_i\sigma_j\rangle$ for any i, j.

   Hint: show that $\langle\sigma_i\sigma_j\rangle = \mathrm{tr}\,(P_z T^{j-i} P_z T^{N-(j-i)})$, where $P_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Also, observe that the eigenvectors of T are independent of τ or J.

e. Show that in the thermodynamic limit $N \to \infty$,

$$\langle\sigma_i\sigma_j\rangle = e^{-|j-i|/\xi(\tau)}$$

   where $\xi(\tau)$ is the correlation length. Compute $\xi(\tau)$.