5. Hausübung, **Statistische Physik**

abzugeben am Donnerstag, 17.11.2011

Aufgabe H10 *Double occupancy statistics* (6 Punkte)

Imagine a "gas" of non-interacting indistinguishable particles with non-degenerate discrete energy levels ϵ_j, $j = 1, 2, \ldots, J$, where *two particles but no more* are allowed to occupy the same energy level.

a. Suppose for now that only the energy level j is accessible, i.e., assume that each particle can only be in the state with energy ϵ_j so that the only degree of freedom available is the total number of particles in existence. Compute the corresponding Gibbs sum $\mathcal{Z}_j$ at temperature τ and with chemical potential μ.

b. Now we allow all energy levels $j = 1, 2, \ldots, J$. Let $\mathcal{Z}$ be the corresponding Gibbs sum. Using the Gibbs sum, show that

$$\mathcal{Z} = \mathcal{Z}_1 \mathcal{Z}_2 \cdots \mathcal{Z}_J,$$

which implies that each energy level can be treated as an independent system, whose state is characterized by its occupancy number.

c. Compute the expected occupation number $f(\epsilon_j)$ of the jth energy level.

d. Let $f_c(\epsilon) = e^{-(\epsilon-\mu)/\tau}$ be the "classical" average occupancy for energy level ϵ. Show that $f(\epsilon) \sim f_c(\epsilon)$ when $(\epsilon - \mu)/\tau \to \infty$, i.e., that in this limit, $f(\epsilon)/f_c(\epsilon) \to 1$.

e. Is this system the same as one of fermions, but where each energy level has degeneracy 2?

Aufgabe H11 *Poisson distribution* (6 Punkte)

Consider a classical ideal gas. Recall that the Boltzmann sum (partition function) at temperature τ, for N particles and in a volume V, is

$$Z_N = \frac{1}{N!}(n_Q V)^N.$$

a. Suppose that the gas is at thermal and diffuse equilibrium and with expected (i.e. average) density n of particles. Express the probability of finding exactly N particles in a volume V. Show that it is

$$P(N) = \frac{(nV)^N e^{-nV}}{N!},$$

which is known as a Poisson distribution.

b. Check directly that indeed,

$$\sum_{N=0}^{\infty} P(N) = 1 \quad \text{and} \quad \sum_{N=0}^{\infty} N P(N) = nV.$$