1. Präsenzübung, **Statistische Physik**

zu bearbeiten am Donnerstag, 13.10.2011

Aufgabe P1 *Conditional probabilities*

Consider a system of N spins, which can each be "up" or "down". As you know nothing about the state of the system, all configurations are equally likely and have probability $p = 2^{-N}$.
Suppose you now measure the difference between the number of up and down spins (say by measuring the magnetic field), and find it has a definite value $2s$. With this new information in hand, what probability must you now assign to each configuration?

Aufgabe P2 *Individual spin*

Consider a very large number N of spins which can be each be "up" or "down". Let $2s$ be the difference between the number of up spins and the number of down spins. In the presence of an external magnetic field B, the total energy of the system is

$$U = -2smB$$

for some contant m.
Suppose that each accessible configuration is equally likely. Estimate the probability of one given spin being up, given that the total energy is U.

Aufgabe P3 *Shannon entropy*

To a probability distribution $P = \{p_1, \ldots, p_n\}$ (with $p_i > 0$ and $\sum_i p_i = 1$), one can assign a function called that *Shannon entropy* of that distribution, defined by

$$S(P) = \sum_i p_i \log \frac{1}{p_i} = -\sum_i p_i \log p_i$$

a. What is the entropy of a uniform distribution?

b. Find the minimum and maximum of S (for a fixed number of samples n) and for which distributions these are attained. Hint: the logarithm is a concave function.

c. Show that the entropy of a distribution made of two independent systems is the sum of the entropy of each system.

d. Note that the eigenvalues of a quantum state ρ (density matrix) form a probability distribution. Let $S(\rho)$ be the Shannon entropy of this distribution. $S(\rho)$ is called the *von Neumann entropy* of ρ. How can you express $S(\rho)$ concisely in terms of ρ?