3. Präsenzübung, **Statistische Physik**

zu bearbeiten am Donnerstag, 27.10.2011

Aufgabe P7 *Two-state system*

We consider a system with two states: one at energy 0 and one at energy ϵ, in thermal contact with a large reservoir at temperature τ. Express its free energy F, expected energy U and entropy σ as functions of τ.

Aufgabe P8 *Energy fluctuations*

Consider a system in thermal contact with a large reservoir at temperature τ. Let ϵ denote the energy observable of the system, $U = \langle \epsilon \rangle$ be the thermal average of the energy. Show that

$$\langle (\epsilon - \langle \epsilon \rangle)^2 \rangle = \tau^2 C.$$

where

$$C = \frac{dU}{d\tau}$$

is the system's heat capacity.

Aufgabe P9 *Partition function for independent systems*

a. For $x, y \in \mathbb{R}$, prove that $e^{x+y} = e^x e^y$ using the expansion

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

Hint: Use the relation

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

where

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}.$$

b. Given two matrices A and B, show that

$$\mathrm{tr}\,[A \otimes B] = \mathrm{tr}\,A\,\mathrm{tr}\,B.$$

c. For a quantum system with Hamiltonian H and at temperature τ, the partition function is

$$Z = \mathrm{tr}\,e^{-H/\tau}.$$

Consider two independent quantum systems with Hamiltonians H_1 and H_2 respectively, i.e., the total Hamiltonian is $H = H_1 \otimes \mathbf{1} + \mathbf{1} \otimes H_2$. If Z_i is the partition function of system i, show that the partition function of both systems together is

$$Z = Z_1 Z_2.$$

Hint: do something similar to P9a.