4. Präsenzübung, **Statistische Physik**

zu bearbeiten am Donnerstag, 3.11.2011

Aufgabe P10 *Differential forms*

Consider the manifold $\Omega = \{(T,V) \in \mathbb{R}^2 \mid T, V > 0\}$ and the function $p : \Omega \to \mathbb{R}$ defined by

$$p(T,V) = \alpha T/V$$

with $\alpha > 0$ a constant.
We consider the two coordinate systems (T,V) and (T,p) for Ω. Let $U : \Omega \to \mathbb{R}$ be a differentiable function, and consider it in both coordinate systems, i.e., $U(T,V)$ and $\tilde{U}(T,p) := U(T, V(T,p))$ and drop the tilde.

a. Express the differential forms dT and dV in the coordinates (T,p) (i.e., in the form $F_1 dT + F_2 dp$ where the F_i are functions of T and p.)

b. Write the partial derivatives $\left(\frac{\partial U}{\partial T}\right)_V$ and $\left(\frac{\partial U}{\partial V}\right)_T$ in the coordinates (T,p) (i.e., in terms of $\left(\frac{\partial U}{\partial T}\right)_p$ and $\left(\frac{\partial U}{\partial p}\right)_T$ and some functions of T and p.)

c. Write the differential form $\delta Q := dU + p dV$ in both coordinate systems.

Aufgabe P11 *Particle creation*

Imagine that particles can come into existence or disappear freely. If we assign to each particle a mass M, a (non-relativistic) kinetic energy, and a rest energy Mc^2, where c is the speed of light, then assuming the system is in thermal equilibrium at temperature τ, compute the expected density of particles n.

Aufgabe P12 *Two-level fermions*

Consider a system of independent indistinguishable particles in diffuse and thermal equilibrium at temperature τ and chemical potential μ, where each particle can be in one of two states. One state has energy 0 and one state has energy ϵ. Also we forbid two particles from occupying the same state.

a. Show that the grand-canonical partition function is

$$\mathcal{Z} = (1+\lambda)(1+\lambda e^{-\epsilon/\tau}),$$

where $\lambda = e^{\mu/\tau}$.

b. Show, by directly summing Gibbs factors, that the expected number of particles is

$$\langle N \rangle = \frac{\lambda[1 + e^{-\epsilon/\tau} + 2\lambda e^{-\epsilon/\tau}]}{\mathcal{Z}}.$$

c. In the same manner, show that the expected energy is

$$U = \frac{\epsilon\lambda}{1 + \lambda e^{\epsilon/\tau}}.$$

d. Observe that the partition function $\mathcal{Z}$ has the form $\mathcal{Z} = \mathcal{Z}_1\mathcal{Z}_2$ where $\mathcal{Z}_1 = 1 + \lambda$ and $\mathcal{Z}_2 = 1 + \lambda e^{-\epsilon/\tau}$. This suggests that we can identify two independent systems corresponding to the partitions functions $\mathcal{Z}_i$. What are these systems?