6. Präsenzübung, **Statistische Physik**

zu bearbeiten am Donnerstag, 17.11.2011

Aufgabe P15 *Density of states in one and two dimensions*

a. Express the density of states $\mathcal{D}_1(\epsilon)$ for a free electron (with spin $\frac{1}{2}$) in a one-dimensional box of length L.

b. Express the density of states $\mathcal{D}_2(\epsilon)$ for a free electron in a two-dimensional square box of area A.

Aufgabe P16 *2D bosons*

Consider an ideal gas of spinless bosons in a two-dimensional box of area A. Compute the fugacity λ as a function of the temperature τ and particle density $n = N/A$. Observe that, at $\tau = 0$, $\lambda(\tau) = 1$ and all its derivatives with respect to τ are zero.

Aufgabe P17 *Relativistic Fermi gas*

When the energy of an electron is large compared to its rest energy mc^2, it is related to the particle's momentum p by

$$\epsilon \simeq pc.$$

If such relativistic electrons are in a cube of volume $V = L^3$, their momenta are quantized in exactly the same way as for non-relativistic electrons, i.e., the eigenvalues of the components p_1, p_2, p_3 of the momentum are

$$p_i = \frac{\pi\hbar}{L} n_i$$

where $n_i = 1, 2, \ldots$.
Compute the Fermi energy ϵ_f of a gas of N electrons in such a cube, assuming that their individual energies are all in this extreme relativistic limit.
Then show that the energy U_0 of the gas at zero temperature is

$$U_0 = \frac{3}{4} N \epsilon_f.$$