8. Präsenzübung, **Statistische Physik**

zu bearbeiten am Donnerstag, 1.12.2011

Aufgabe P20 *Black hole thermodynamics*

If a system of energy $E = Mc^2$ is smaller than its Schwarzschild radius

$$R = \frac{2GM}{c^2}$$

where G is Newton's gravitational constant, then it forms a black hole. According to general relativity, any additional matter or information passing inside a sphere of radius R around the black hole (its event horizon) will never come out.
It is conjectured, based on quantum field theory calculations on a Schwarzschild background, that the event horizon of a black hole emits black body radiation at temperature

$$\tau = \frac{\hbar c^3}{8\pi GM},$$

which is called the Hawking-Unruh temperature.

a. Based on this result, express the entropy σ of a black hole as a function of the radius R of its event horizon. Observe that, surprisingly, σ grows like the area of the horizon rather than its volume.

b. How long would it take for a black hole of mass M to evaporate into photons?

Aufgabe P21 *Irreversible expansion of a Fermi gas*

When the sun runs out of material to fuse, it will collapse under its own gravitational pull until it is stopped by the degeneracy pressure of its electrons (i.e., the fact that electrons cannot occupy the same quantum state) and form a "white dwarf". It will then slowly radiate away its remnant heat until it becomes a "black dwarf". Let's model this state by a fermi gas of electrons at zero temperature, assuming there are $n = 10^{36}$ electrons per cubic meter.
Suppose we suddenly turn off gravity, what will happen to the electron gas (ignoring the nuclei as well as any interaction between the electrons)? What temperature will it tend to, assuming it doesn't have time to exchange heat or work with any other system?
The mass of an electron is $m_e \simeq 9 \cdot 10^{-31}$ kg, $\hbar \simeq 1 \cdot 10^{-34}$ m^2 kg/s and $k_B \simeq 1.38 \cdot 10^{-23}$ J/K.