$\boxed{y_i - y_j := y_{ij}}$

2) $$A_4^{tree}(k_1 k_2 k_3 k_4) = \int_{y_4}^{y_2} dy_3 \, y_{12} y_{14} y_{24} \langle 0 | V(\tau_1) V(\tau_2) V(\tau_3) V(\tau_4) | 0 \rangle$$

$$= \int_{y_4}^{y_2} dy_3 \; y_{12} y_{14} y_{24} \; \delta(k_1+k_2+k_3+k_4) \prod_{i<j} \underbrace{(y_i - y_j)^{2\alpha' k_i \cdot k_j}}_{\text{für } i,j = 1,\dots,4:\ 6\ \text{Terme}}$$

$$= \delta(\Sigma k_i) \int_{y_4}^{y_2} dy_3 \Big(y_{12}^{1+2\alpha' k_1 k_2} \; y_{14}^{1+2\alpha' k_1 k_4} \; y_{24}^{1+2\alpha' k_2 k_4} \; y_{13}^{2\alpha' k_1 k_3} \; y_{23}^{2\alpha' k_2 k_3} \; y_{34}^{2\alpha' k_3 k_4} \Big)$$

$$\overset{\substack{y_1 \to \infty \\ y_2 = 1 \\ y_4 = 0}}{=} \delta(\Sigma k_i) \lim_{y_1 \to \infty} \int_0^1 dy_3 \Big[(y_1 - 1)^{1+2\alpha' k_1 k_2} \; y_1^{1+2\alpha' k_1 k_4} \; (y_1 - y_3)^{2\alpha' k_1 k_3} \; (y_1 - y_3)^{2\alpha' k_1 k_3} \; (1 - y_3)^{2\alpha' k_2 k_3} \; y_3^{2\alpha' k_3 k_4} \Big]$$

$$= \delta(\Sigma k_i) \lim_{y_1 \to \infty} \int_0^1 dy_3 \; y_1^{2 + 2\alpha' \sum_{j=2}^{4} k_1 k_j} \; (1 - y_3)^{2\alpha' k_2 k_3} \; y_3^{2\alpha' k_3 k_4}$$

$$\boxed{\left.\begin{array}{l} \Sigma k_i = 0 \\ \alpha' k_i^2 = 1 \end{array}\right\} \Rightarrow 2\alpha' \sum_{j=2}^{4} k_1 k_j = 2\alpha' k_1^2 = 2}$$

$$= \delta(\Sigma k_i) \int_0^1 dy_3 \, (1 - y_3)^{2\alpha' k_2 k_3} \; y_3^{2\alpha' k_3 k_4}$$

Mandelstam-variablen: gl. (3.35) im Skript

$$2\alpha' k_2 k_3 = \alpha'(k_2 + k_3)^2 - 2 = -\alpha' t - 2$$

$$2\alpha' k_3 k_4 = \alpha'(k_3 + k_4)^2 - 2 = -\alpha' s - 2$$

$$= \underline{\delta(\Sigma k_i) \int_0^1 dy_3 \, (1 - y_3)^{-\alpha' t - 2} \; y_3^{-\alpha' s - 2}}$$

Dieses Integral hat folgende Lösung:
Wir definieren $a := -(\alpha' s + 1)$, $b := -(\alpha' t + 1)$.
Durch Einsetzen in den Integralausdruck sieht man, dass die Amplitude die Form der Eulerschen Beta-Funktion $B(a,b)$ annimmt:

$$A_4^{tree} = \delta(\Sigma k_i) \int_0^1 dy_3 \; y_3^{a-1} (1 - y_3)^{b-1}$$

$$= \delta(\Sigma k_i) \; B(a,b)$$

mit folgender Lösung (Polchinsky I, Kap. 6.4):

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \qquad \Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt$$