Lecture 5: Ground state problems

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Outline of the course

1. Introduction to Python
2. SciPy/NumPy packages
3. Plotting and fitting
4. QuTiP: states and operators
5. **Ground state problems**
6. Non-equilibrium dynamics: quantum quenches
7. Quantum master equations
8. Generation of squeezed states
9. Quantum computing
10. Grover's algorithm and quantum machine learning
11. Student presentations

The Ising model in a transverse field

$$H = g \sum_{i=0}^{N-1} \sigma_z^{(i)} - \sum_{\langle ij \rangle} \sigma_x^{(i)} \sigma_x^{(j)}$$

- g: Strength of the transverse magnetic field in units of the ferromagnetic interaction
- Relevant for ferromagnets, ferroelectics, ultracold atoms
- Decomposition into single site operators

$$\sigma_z^{(i)} = \underbrace{1 \otimes 1 \otimes \cdots}_{i \text{ terms}} \otimes \sigma_z \otimes \underbrace{1 \otimes 1 \otimes \cdots}_{N-i \text{ terms}}$$

```
from qutip import tensor, qeye, sigmaz
sz3 = tensor(qeye(2), qeye(2), qeye(2), sigmaz(), qeye(2))
```

Ground state properties

- Ground state important for understanding low-temperature behavior
- First step towards the full solution
- Here: Ising symmetry ($\mathbb{Z}_2$): Consider the unitary transformation $U = \sigma_z^{(0)}\sigma_z^{(1)}\cdots\sigma_z^{(N-1)}$

$$[U, H] = 0$$

- $g \gg 1$: Paramagnet

$$|\psi_0\rangle = |{\leftarrow}{\leftarrow}\cdots{\leftarrow}\rangle$$

- $g \ll 1$: Ferromagnet (two possible ground states)

$$|\psi_0^A\rangle = |{\uparrow}{\uparrow}\cdots{\uparrow}\rangle$$

$$|\psi_0^B\rangle = |{\downarrow}{\downarrow}\cdots{\downarrow}\rangle$$

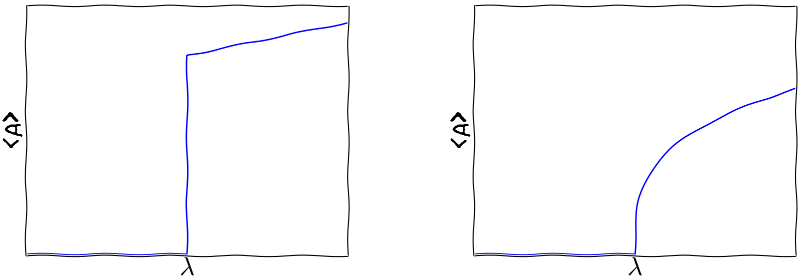
- Spontaneous breaking of the Ising symmetry!

Quantum phase transitions

- Order parameter: magnetization m

$$m = \frac{\langle\psi_0| \left| \sum_{i=0}^{N-1} \sigma_z^{(i)} \right| |\psi_0\rangle}{N}$$

- Paramagnet: $m = 0$, ferromagnet: $m \neq 0$
- Quantum phase transition: ground state energy becomes nonanalytic
- In general: order parameter can jump or be continuous at the phase transition



Critical exponents

- Scale invariance close to a continuous phase transition

$$f(\lambda s) = c(\lambda) f(s)$$

- Series expansion around $\lambda = 1$

$$f(\lambda s) = f(s) + \frac{df}{ds} s(\lambda - 1) + O\left([\lambda - 1]^2\right)$$

- Solutions in the form of power laws $f \sim s^\kappa$
- Microscopic properties become irrelevant
- Scaling of observables

$$O = O_0 |g - g_c|^\kappa$$

- κ: critical exponent

Finite size scaling

- Nonanalytic behavior only in the thermodynamic limit ($N \to \infty$)
- Finite size scaling allows to determine critical exponents
  J. Cardy, *Scaling and Renormalization in Statistical Physics*, (CUP, 1996)
- Correlation length exponent

$$\xi \sim |g - g_c|^{-\nu}$$

- Universal scaling function $\chi(N/\xi)$

$$m(g, N) \sim |g - g_c|^{\beta} \chi\left(\frac{N}{\xi}\right)$$

$$m(g, N) \sim \xi^{-\beta/\nu} \chi(N[g - g_c]^{\nu})$$

$$m(g, N) \sim N^{-\beta/\nu} \tilde{\chi}([g - g_c]N^{1/\nu})$$