Formelsammlung

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta\,,\;\; \sin(\alpha+\beta) = \cos\alpha\sin\beta + \sin\alpha\cos\beta\,,$

$\mathrm{e}^{\mathrm{i}\varphi} = \cos\varphi + \mathrm{i}\sin\varphi\;,\;\; (1+x)^\alpha = \sum_n \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!}x^n$

$\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}\,,\;\; \frac{\mathrm{d}}{\mathrm{d}x}\ln|x| = \frac{1}{x}\,,\;\; \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^x = \mathrm{e}^x\,,\;\; \frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x\,,\;\; \frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

$\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = \sinh(x)\,,\;\; \frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = \cosh(x)\,,\;\; \frac{\mathrm{d}}{\mathrm{d}x}\arctan(x) = \frac{1}{1+x^2}$

Taylor-Formeln: $f(x) = \sum_{n=0}^{N-1}\frac{1}{n!}(x-x_0)^n\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^n f_{|x_0} + \frac{1}{N!}(x-x_0)^N\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^N f_{|\bar{x}}$

$$f(x^1,x^2,\dots,x^n) = \sum_{l=0}^{N-1}\frac{1}{l!}(x^{i_1}-x_0^{i_1})(x^{i_2}-x_0^{i_2})\dots(x^{i_l}-x_0^{i_l})\partial_{i_1}\partial_{i_2}\dots\partial_{i_l}f_{|(x_0^1,x_0^2,\dots,x_0^n)}$$
$$+\frac{1}{N!}(x^{i_1}-x_0^{i_1})(x^{i_2}-x_0^{i_2})\dots(x^{i_N}-x_0^{i_N})\partial_{i_1}\partial_{i_2}\dots\partial_{i_N}f_{|(\bar{x}^1,\bar{x}^2,\dots,\bar{x}^n)}$$

Kettenregeln: $\frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) = \frac{\mathrm{d}g}{\mathrm{d}x}\frac{\mathrm{d}f}{\mathrm{d}g}_{|g(x)}\,,\;\; \frac{\partial z^i(y(x))}{\partial x^j} = \frac{\partial y^l(x)}{\partial x^j}_{|x}\frac{\partial z^i(y)}{\partial y^l}_{|y(x)}$

$$\int\limits_{x\text{-Bereich}}\mathrm{d}^n x\, f(x) = \int\limits_{z\text{-Bereich}}\mathrm{d}^n z\left|\det\frac{\partial x}{\partial z}\right| f(x(z))\,,\;\; x\text{-Bereich} = x(z\text{-Bereich})$$

Produktregeln: $\frac{\mathrm{d}}{\mathrm{d}x}(u\cdot v) = \frac{\mathrm{d}u}{\mathrm{d}x}\cdot v + u\cdot\frac{\mathrm{d}v}{\mathrm{d}x}\,,\;\; \int\mathrm{d}x\,u\cdot\frac{\mathrm{d}v}{\mathrm{d}x} = u\cdot v\Big| - \int\mathrm{d}x\,\frac{\mathrm{d}u}{\mathrm{d}x}\cdot v$

Kugelkoordinaten: $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}r\sin\theta\cos\varphi\\ r\sin\theta\sin\varphi\\ r\cos\theta\end{pmatrix}\,,$
$\dot{\vec{r}} = \dot{r}\vec{e}_r + \dot{\theta}\,r\,\vec{e}_\theta + \dot{\varphi}\,r\sin\theta\,\vec{e}_\varphi$
$\dot{\vec{r}}^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\varphi}^2\,,$
$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r^2\,\mathrm{d}r\,\mathrm{d}\cos\theta\,\mathrm{d}\varphi$
$0\le\theta\le\pi\;,\;\; -1\le\cos\theta\le 1$

Zylinderkoordinaten: $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}\rho\cos\varphi\\ \rho\sin\varphi\\ z\end{pmatrix}\,,$
$\dot{\vec{r}}^2 = \dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \dot{z}^2$
$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}z\,,\;\; 0\le\varphi\le 2\pi$

Summationskonvention: $i\in\{1,2\dots n\}\quad a_i\,b^i := a_1\,b^1 + a_2\,b^2 + \dots a_n\,b^n$

Vektoren: $\vec{a} = \vec{e}_i\,a^i$, *Skalarprodukt*: $\vec{e}_i\cdot\vec{e}_j = g(\vec{e}_i,\vec{e}_j) = g_{ij} \overset{\text{ONB}}{=} \delta_{ij}$

$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\alpha = g(\vec{e}_i,\vec{e}_j)\,a^i\,b^j \overset{\text{ONB}}{=} \sum_i a^i\,b^i =: a^i\,b^i$

$\vec{a} = \vec{a}_\parallel + \vec{a}_\perp = \frac{(\vec{a}\cdot\vec{b})}{(\vec{b})^2}\vec{b} + \left(\vec{a} - \frac{(\vec{a}\cdot\vec{b})}{(\vec{b})^2}\vec{b}\right)$

$\dim = 3$: $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})\,,\;\; (\vec{a}\times\vec{b})^i = \varepsilon^{ijk}a^j b^k\,,\;\; (\vec{a}\times\vec{b})^1 = a^2b^3 - a^3b^2$, und zyklisch

$(\vec{a}\times\vec{b}) \perp \vec{a}$ und $\perp \vec{b}\,,\ |\vec{a}\times\vec{b}| = |\vec{a}|\,|\vec{b}|\,\sin\alpha$

Vektorfelder: $(\operatorname{grad} f)^i = \frac{\partial}{\partial x^i} f\,,\ (\overrightarrow{\operatorname{rot} A})^i = \varepsilon^{ijk}\frac{\partial}{\partial x^j}A_k\,,\ \operatorname{div}\vec{B} = \partial_x B^x + \partial_y B^y + \partial_z B^z$

ε -Tensor: $\varepsilon_{i_1 i_2\dots i_n} = \operatorname{sign}(\pi)\,\varepsilon_{i_{\pi(1)} i_{\pi(2)}\dots i_{\pi(n)}}\,,\ i = 1, 2\dots n,$ antisymmetrisch, $\varepsilon_{1\,2\dots n} = 1$

$\dim = 3$:

$$\varepsilon_{ijk}\,\varepsilon_{lmn} = \delta_{il}\delta_{jm}\delta_{kn} + \delta_{im}\delta_{jn}\delta_{kl} + \delta_{in}\delta_{jl}\delta_{km} - \delta_{il}\delta_{jn}\delta_{km} - \delta_{in}\delta_{jm}\delta_{kl} - \delta_{im}\delta_{jl}\delta_{kn}$$

$$\varepsilon_{ijk}\,\varepsilon_{mnk} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}\,,\ \varepsilon_{ikl}\varepsilon_{jkl} = 2\delta_{ij}\,,\ \varepsilon_{ijk}\varepsilon_{ijk} = 6$$

Kronecker-δ: $a^i = \delta^i{}_j a^j\,,\ \delta^i{}_i = n =$ Laufbereich

Matrizen: $(\,M\cdot N\,)^i{}_j = M^i{}_k\,N^k{}_j\,,$ das heißt: Zeile $\times$ Spalte, $(M^{\mathrm{T}})_j{}^i = M^i{}_j$

$M^{i_1}{}_{j_1} M^{i_2}{}_{j_2}\dots M^{i_n}{}_{j_n}\varepsilon_{i_1 i_2\dots i_n} = \varepsilon_{j_1 j_2\dots j_n}\det M\,,$

$\det M = \det M^{\mathrm{T}}\,,\ \det(M\cdot N) = \det M\cdot\det N\,,\ \det M^{-1} = \dfrac{1}{\det M}\,,$

$\frac{\partial \det M}{\partial M^i{}_j} = \det M (M^{-1})^j{}_i\,,\ \operatorname{Sp} M = M^i{}_i\,,\ \operatorname{Sp}(A\cdot B) = \operatorname{Sp}(B\cdot A)\,,\ \operatorname{Sp}(A\cdot B\cdot A^{-1}) = \operatorname{Sp}(B)$

$\det(1+N) = 1 + \operatorname{Sp} N + O(N^2) \quad N^i{}_j = u^i\,w_j \Rightarrow \det(1+N) = 1 + u^i\,w_i$

Drehungen: $O^{\mathrm{T}} = O^{-1}\,,\ O^i{}_k O^j{}_l\,\delta^{kl} = \delta^{ij}\,,\ \det O = 1\,,\ \dim = 3:\ \operatorname{Sp} O = 2\cos\varphi + 1$

$$D_x = \begin{pmatrix}1 & & \\ & c & -s\\ & s & c\end{pmatrix}\,,\ D_y = \begin{pmatrix}c & & s\\ & 1 & \\ -s & & c\end{pmatrix}\,,\ D_z = \begin{pmatrix}c & -s & \\ s & c & \\ & & 1\end{pmatrix}$$

Eigenwerte: $M\vec{v} = \lambda\vec{v} \Leftrightarrow (\,M - \lambda\mathbf{1}\,)\,\vec{v} = 0\,,\ \vec{v} \neq 0 \Rightarrow \det(\,M - \lambda\mathbf{1}\,) = 0$

$$M = M^* = M^{\mathrm{T}} \Rightarrow M\vec{v}_i = \lambda_i\vec{v}_i\,,\ \vec{v}_i\cdot\vec{v}_j = \delta_{ij}\,,\ \lambda_i = \lambda_i^*$$

Eigenvektoren reelle Orthonormalbasis, Eigenwerte reell

$\int dx\,\mathrm{e}^{-x^2} = \sqrt{\pi}\,,\ \Gamma(s) = \int_0^\infty \mathrm{d}t\,t^{s-1}\,\mathrm{e}^{-t}\,,\ \Gamma(n+1) = n!\,,\ \Gamma(\frac{1}{2}) = \sqrt{\pi}\,,\ \Gamma(s+1) = s\,\Gamma(s)$

Volumen S^{n-1}: $2\,\pi^{\frac{n}{2}}/\Gamma(\frac{n}{2})$

Differentialformen: $\mathrm{d}x^i\wedge\mathrm{d}x^j = -\mathrm{d}x^j\wedge\mathrm{d}x^i\,,\ \omega = \frac{1}{p!}\,\omega_{i_1\dots i_p}(x)\,\mathrm{d}x^{i_1}\wedge\dots\wedge\mathrm{d}x^{i_p}\,,\ i = 1\dots D$

parametrisierte Untermannigfaltigkeit: $x^i(\lambda^1,\dots,\lambda^p)\,,\ \mathrm{d}x^i = \mathrm{d}\lambda^a\frac{\partial x^i}{\partial\lambda^a}\,,\ a = 1\dots p \leq D$

$\mathrm{d}x^{i_1}\wedge\dots\wedge\mathrm{d}x^{i_p} = \frac{\partial x^{i_1}}{\partial\lambda^{a_1}}\dots\frac{\partial x^{i_p}}{\partial\lambda^{a_p}}\,\mathrm{d}\lambda^{a_1}\wedge\dots\wedge\mathrm{d}\lambda^{a_p} = \frac{\partial x^{i_1}}{\partial\lambda^{a_1}}\dots\frac{\partial x^{i_p}}{\partial\lambda^{a_p}}\,\epsilon^{a_1\dots a_p}\,\mathrm{d}^p\lambda$

$\int_M\omega = \int_{\lambda\text{–Bereich}}\mathrm{d}^p\lambda\,\epsilon^{a_1\dots a_p}\,\frac{\partial x^{i_1}}{\partial\lambda^{a_1}}\dots\frac{\partial x^{i_p}}{\partial\lambda^{a_p}}\,\frac{1}{p!}\,\omega_{i_1\dots i_p}(x(\lambda))\,,\ M = \{x(\lambda), \lambda\in\lambda\text{–Bereich}\}$

Tangentialvektoren: $t_a{}^i = \frac{\partial x^i}{\partial\lambda^a}\,,\ a = 1\dots p\,,$

Metrik: $g_{ab} = \vec{t}_a\cdot\vec{t}_b$, Größe: $= \int_{\lambda\text{–Bereich}}\mathrm{d}^p\lambda\,\sqrt{\det g_{..}}$

$p = 1$: Weg Γ: $\int_\Gamma\omega = \int_{\lambda_a}^{\lambda_e}\mathrm{d}\lambda\,\frac{\mathrm{d}x^i}{\mathrm{d}\lambda}\,\omega_i(x(\lambda))$, Arbeit: $\int_\Gamma \mathrm{d}\vec{x}\cdot\vec{F}(\vec{x}) = \int\limits_{\lambda_1}^{\lambda_2}\mathrm{d}\lambda\,\frac{\mathrm{d}\vec{x}}{\mathrm{d}\lambda}\cdot\vec{F}(\vec{x}(\lambda))$

Tangentialvektor $t^i = \frac{\mathrm{d}x^i}{\mathrm{d}\lambda}$, Weglänge $l = \int_{\lambda_1}^{\lambda_2} \mathrm{d}\lambda \sqrt{\vec{t}^2}$

$p = 2$: Fläche F: $\int_F \omega = \int_{\lambda\text{–Bereich}} \mathrm{d}\lambda^1 \mathrm{d}\lambda^2 \, \frac{1}{2} \left(\frac{\partial x^i}{\partial \lambda^1} \frac{\partial x^j}{\partial \lambda^2} - \frac{\partial x^j}{\partial \lambda^1} \frac{\partial x^i}{\partial \lambda^2} \right) \omega_{i\,j}(x(\lambda))$

$D = 3 : \omega_{1\,2} = -\omega_{2\,1} = \Omega^3 \; \dots \, , \; \omega_{i\,j} = \epsilon_{ijk} \Omega^k$

$\int_F \omega = \int_{\lambda\text{–Bereich}} \mathrm{d}\lambda^1 \mathrm{d}\lambda^2 \, {t_1}^i {t_2}^j \Omega^k \epsilon_{ijk} = \int_{\lambda\text{–Bereich}} \mathrm{d}\lambda^1 \mathrm{d}\lambda^2 \, (\vec{t}_1 \times \vec{t}_2) \cdot \vec{\Omega} = \int_{\lambda\text{–Bereich}} \mathrm{d}^2 \vec{f} \cdot \vec{\Omega}$

äußere Ableitung: $\mathrm{d} = \mathrm{d}x^i \frac{\partial}{\partial x^i}$, : p–Form $\rightarrow$ $(p+1)$–Form

$\omega = \frac{1}{p!} \, \omega_{i_1 \dots i_p}(x) \, \mathrm{d}x^{i_1} \wedge \dots \wedge \mathrm{d}x^{i_p} \rightarrow \mathrm{d}\omega = \frac{1}{p!} \, (\partial_{i_0} \omega_{i_1 \dots i_p}(x)) \, \mathrm{d}x^{i_0} \wedge \mathrm{d}x^{i_1} \wedge \dots \wedge \mathrm{d}x^{i_p}$

$p = 0 :$ $\qquad \mathrm{d}f = \partial_i f \mathrm{d}x^i = \mathrm{grad}\, f \cdot \mathrm{d}\vec{x}$

$p = 1 \; D = 3 :$ $\mathrm{d}(\vec{A} \cdot \mathrm{d}\vec{x}) = \mathrm{rot}\, \vec{A} \cdot \overrightarrow{\mathrm{d}f} = (\mathrm{rot}\, \vec{A})_x \, \mathrm{d}y \, \mathrm{d}z + (\mathrm{rot}\, \vec{A})_y \, \mathrm{d}z \, \mathrm{d}x + (\mathrm{rot}\, \vec{A})_z \, \mathrm{d}x \, \mathrm{d}y$

$p = 2 \; D = 3 :$ $\mathrm{d}(\vec{B} \cdot \overrightarrow{\mathrm{d}f}) = \mathrm{d}(B_x \, \mathrm{d}y \, \mathrm{d}z + B_y \, \mathrm{d}z \, \mathrm{d}x + B_z \, \mathrm{d}x \, \mathrm{d}y) = \mathrm{div}\, \vec{B} \, \mathrm{d}^3 x$

$\mathrm{d}^2 = 0$, $\mathrm{rot}\,\mathrm{grad} = 0$, $\mathrm{div}\,\mathrm{rot} = 0$

sternförmig: $\mathrm{d}\,\eta = 0 \Rightarrow \eta = \mathrm{d}\,\omega$, $\mathrm{rot}\, \vec{A} = 0 \Rightarrow \vec{A} = \mathrm{grad}\, f$, $\mathrm{div}\, \vec{B} = 0 \Rightarrow \vec{B} = \mathrm{rot}\, \vec{C}$

Integralsätze: $\int_M \mathrm{d}\,\omega = \int_{\partial M} \omega$

$p = 1 :$ $\qquad \int_{\text{Weg}} \mathrm{grad}\, f \cdot \mathrm{d}\vec{x} = f|_{\text{Randpunkte}}$

$p = 2 \; D = 3 :$ $\int_{\text{Fläche}} \mathrm{rot}\, \vec{A} \cdot \overrightarrow{\mathrm{d}f} = \oint_{\text{Randkurve}} \mathrm{d}\vec{x} \cdot \vec{A}$, Rechtsschraube in $\mathrm{d}\vec{f}$–Richtung

$p = 3 \; D = 3 :$ $\int_{\text{Volumen}} \mathrm{div}\, \vec{B} \, \mathrm{d}^3 x = \oint_{\text{Randfläche}} \overrightarrow{\mathrm{d}f} \cdot \vec{B}$, $\mathrm{d}\vec{f}$ nach außen gerichtet

komplexe Funktionen: $z = x + \mathrm{i}\, y, f(z) = u(x,y) + \mathrm{i}\, v(x,y)$,

komplex differenzierbar: $\mathrm{d}f(z) = \mathrm{d}z \, \frac{\mathrm{d}f}{\mathrm{d}z} \Leftrightarrow (\partial_x u = \partial_y v \; , \; \partial_y u = -\partial_x v)$ Cauchy Riemann

$\int_\Gamma \mathrm{d}z \, f(z) = \int_\Gamma (\mathrm{d}x \, u - \mathrm{d}y \, v) + \mathrm{i}\, (\mathrm{d}y \, u + \mathrm{d}x \, v) = \int_{\lambda\text{-Bereich}} \mathrm{d}\lambda \, \big((\dot{x} \, u - \dot{y} \, v) + \mathrm{i}\, (\dot{y} \, u + \dot{x} \, v) \big)$

$\oint_\Gamma f(z) \mathrm{d}z = 0$ falls f im umlaufenen Gebiet komplex differenzierbar,

$\oint_\Gamma f(z) \mathrm{d}z = 2\pi \mathrm{i} \sum_{z_i} \operatorname*{Res}_{z = z_i} f(z)$, Γ im Gegenzeigersinn, $f(z) = \sum_{l=-\infty}^{\infty} c_l (z - z_i)^l$, $\operatorname*{Res}_{z_i} f = c_{-1}$

δ-Funktion: $\int \mathrm{d}x' \, \delta(x - x') \, t(x') = t(x)$, $\delta(ax) = \frac{1}{|a|} \delta(x)$, $\delta(f(x)) = \sum_{x_i :\, f(x_i) = 0} \frac{\delta(x - x_i)}{|\frac{\mathrm{d}f}{\mathrm{d}x}|}$

$\int \mathrm{d}x \, \frac{\mathrm{d}}{\mathrm{d}x} \delta(x) \, t(x) = -\frac{\mathrm{d}}{\mathrm{d}x} t(0)$, $f(x) \delta(x) = f(0) \delta(x)$, $f(x) \frac{\mathrm{d}}{\mathrm{d}x} \delta(x) = f(0) \frac{\mathrm{d}}{\mathrm{d}x} \delta(x) - \frac{\mathrm{d}f}{\mathrm{d}x}(0) \delta(x)$

$\theta(x) = \{0 \text{ falls } x < 0, 1 \text{ falls } x > 0\}$, $\frac{\mathrm{d}}{\mathrm{d}x} \theta(x) = \delta(x)$, $\int \mathrm{d}k \, \mathrm{e}^{\mathrm{i}kx} = 2\pi \, \delta(x)$

$\lim_{\epsilon \to 0+} \frac{1}{x + \mathrm{i}\epsilon} = \mathrm{P.V.} \frac{1}{x} - \mathrm{i}\pi \, \delta(x)$, $\int \mathrm{d}x \, \mathrm{P.V.} \frac{1}{x} \, t(x) = \frac{1}{2} \int \mathrm{d}x \, \frac{t(x) - t(-x)}{x}$

Entwicklung im Intervall: $I = \{x : -\frac{L}{2} \le x \le \frac{L}{2}\}$, Skalarprodukt $(f, g) = \int_I \mathrm{d}x \, f^*(x) g(x)$

f_l vollst. ONSystem: $(f_k, f_l) = \delta_{kl}$, $g(x) = \sum_l f_l(x) (f_l, g)$, $(g, g) = \sum_l |(f_l, g)|^2 \; \forall g$

Fourierreihe: $f_l = \frac{1}{\sqrt{L}} \mathrm{e}^{2\pi \mathrm{i} l \frac{x}{L}} \;\; l \in \mathbf{Z}$, $g(x) = \sum_{-\infty}^{\infty} \frac{1}{\sqrt{L}} \mathrm{e}^{2\pi \mathrm{i} l \frac{x}{L}} g_l$, $g_l = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \mathrm{d}x \, \frac{1}{\sqrt{L}} \mathrm{e}^{-2\pi \mathrm{i} l \frac{x}{L}} g(x)$

reelle Fourierreihe: $g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(2\pi n \frac{x}{L}) + \sum_{n=1}^{\infty} b_n \sin(2\pi n \frac{x}{L})$

$a_n = \frac{2}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} \mathrm{d}x \cos(2\pi n \frac{x}{L}) \cdot g(x)\,,\ b_n = \frac{2}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} \mathrm{d}x \sin(2\pi n \frac{x}{L}) \cdot g(x)$

Fourierdarstellung: $f(x) = \int \mathrm{d}k \, \frac{1}{\sqrt{2\pi}} \mathrm{e}^{\mathrm{i}kx} \tilde{f}(k)\,,\ \tilde{f}(k) = \int \mathrm{d}x \, \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\mathrm{i}kx} f(x)$

$\widetilde{\frac{\mathrm{d}f}{\mathrm{d}x}}(k) = \mathrm{i}k\tilde{f}(k)\,,\ (\widetilde{f \cdot g})(k) = \int \frac{\mathrm{d}p}{\sqrt{2\pi}} \, \tilde{g}(p) \, \tilde{f}(k-p)$

partielle Differentialoperatoren: (x^i kartesische Koord.) $\Delta = \sum_{i=1}^{3} \partial_{x^i}\partial_{x^i}\,,\ \Box = \partial_{x^0}\partial_{x^0} - \Delta$

$\Delta = \partial_r^2 + \frac{2}{r}\partial_r + \frac{L^2}{r^2}\,,\ L^2 = \frac{1}{\sin^2\theta}\partial_\varphi^2 + \partial_\theta^2 + \cot\theta\partial_\theta\,,\ \mathrm{grad} = \vec{e}_r\partial_r + \vec{e}_\varphi\frac{1}{r\sin\theta}\partial_\varphi + \vec{e}_\theta\frac{1}{r}\partial_\theta$

$\Delta\phi(\vec{x}) = -4\pi\rho(\vec{x})\,,\ \phi(\vec{x}) = \int \mathrm{d}^3y \, \frac{\rho(\vec{y})}{|\vec{x}-\vec{y}|}\,,\ \Delta\frac{1}{|\vec{x}|} = -4\pi\delta^3(\vec{x})$

$(\partial_t - D\,\Delta)\phi(t,\vec{x}) = 0\,,\ \phi(t,\vec{x}) = \frac{1}{\sqrt{4\pi D t}^3} \int \mathrm{d}^3y \, \mathrm{e}^{-\frac{(\vec{x}-\vec{y})^2}{4Dt}} \phi(0,\vec{y})\,,\ \vec{\jmath} = -D\ \mathrm{grad}\phi$

Mittelwert: $M_{t,\vec{x}}[v] = \frac{1}{4\pi} \int_{\cos\theta=-1}^{\cos\theta=+1} \mathrm{d}\cos\theta \int_0^{2\pi} \mathrm{d}\varphi \, v\big(\vec{x} + t\,\vec{n}(\cos\theta,\varphi)\big)$

$\Box\phi(t,\vec{x}) = 4\pi\rho(t,\vec{x})\ ,\ \phi(0,\vec{x}) = u(\vec{x})\ ,\ \dot{\phi}(0,\vec{x}) = v(\vec{x})\ ,\ c = 1$

$\phi(t,\vec{x}) = t\,M_{t,\vec{x}}[v] + \frac{\partial}{\partial t}\big(t\,M_{t,\vec{x}}[u]\big) + \int_{K_{|t|,\vec{x}}} \mathrm{d}^3y \, \frac{\rho(t - \mathrm{sign}(t)\,|\vec{x}-\vec{y}|,\vec{y})}{|\vec{x}-\vec{y}|}\ ,\ K_{r,\vec{x}} = \{\vec{y} : |\vec{x}-\vec{y}| \leq |r|\}$

$\phi_{\mathrm{ret}}(x^0,\vec{x}) = \int \mathrm{d}^3y \, \frac{\rho(x^0 - |\vec{x}-\vec{y}|,\vec{y})}{|\vec{x}-\vec{y}|} = 2\int \mathrm{d}^4y \, \theta(x^0-y^0)\,\delta((x-y)^2)\,\rho(y)$

$\Box\frac{1}{2\pi}\theta(x^0)\delta(x^2) = \delta^4(x^0,\vec{x}),\ x^2 = (x^0)^2 - \vec{x}^2$

Relativitätstheorie: $a = (a^0,a^1,a^2,a^3)\,,\ a\cdot b = a^0b^0 - a^1b^1 - a^2b^2 - a^3b^3 = a^m b^n \eta_{mn}$

$m \neq n : \eta_{mn} = 0,\ \eta_{00} = 1,\ \eta_{11} = \eta_{22} = \eta_{33} = -1,\ \eta_{mn} = \eta^{mn},\ a_m = \eta_{mn}a^n,\ a^n = \eta^{nr}a_r$

Lorentztransformation: $x \to x'\,,\ x'^m = \Lambda^m{}_n x^n\,,\ x'\cdot y' = x\cdot y\,,\ \Lambda^m{}_k\Lambda^n{}_l\,\eta_{mn} = \eta_{kl}$

kontragrediente Transformation: $\partial'_n = \frac{\partial}{\partial x'^n} = \Big(\Lambda^{-1\,\mathrm{T}}\Big)_n{}^m \frac{\partial}{\partial x^m} = \Lambda_n{}^m\partial_m\,,\ \Lambda^{-1\,\mathrm{T}} = \eta\Lambda\eta^{-1}$

$\Lambda_{2\times2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\begin{pmatrix}1 & \frac{v}{c}\\ \frac{v}{c} & 1\end{pmatrix}\ ,\ p = \begin{pmatrix}\frac{E}{c}\\ \vec{p}\end{pmatrix} = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}\begin{pmatrix}c\\ \vec{v}\end{pmatrix}\ ,\ p^2 = m^2c^2\,,\ \frac{\mathrm{d}x^m}{\mathrm{d}\tau} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\begin{pmatrix}c\\ \vec{v}\end{pmatrix}$

Vektorfeld: $j'^m(x'(x)) = \Lambda^m{}_n\,j^n(x)\,,\ A'^m(x'(x)) = \Lambda^m{}_n\,A^n(x)$

Tensorfeld: $F'^{mn}(x'(x)) = \Lambda^m{}_k\Lambda^n{}_l F^{kl}(x)$

Wirkung: $W[q] = \int \mathrm{d}t\,\mathscr{L}(q(t),\dot{q}(t),t)\,,$ Energie: $E = \sum E_n(q)\,\dot{q}^n \Rightarrow \mathscr{L} = \sum \frac{1}{n-1}E_n(q)\,\dot{q}^n$

Euler-Lagrange-Ableitung: $\frac{\hat{\partial}\mathscr{L}}{\hat{\partial}q^m} = \partial_{q^m}\mathscr{L} - \frac{\mathrm{d}}{\mathrm{d}t}\partial_{\dot{q}^m}\mathscr{L}$, Euler-Lagrange-Gleichung: $\frac{\hat{\partial}\mathscr{L}}{\hat{\partial}q^m} = 0$

Symmetrie und Erhaltungsgröße: $\delta q^m \frac{\hat{\partial}\mathscr{L}}{\hat{\partial}q^m} + \frac{\mathrm{d}}{\mathrm{d}t}Q = 0$

$\delta q^m\ \partial_{q^m}\mathscr{L} + (\frac{\mathrm{d}}{\mathrm{d}t}\delta q^m)\partial_{\dot{q}^m}\mathscr{L} + \frac{\mathrm{d}}{\mathrm{d}t}K(q,\dot{q},t) = 0$ Erhaltungsgröße: $Q = \delta q^m\,\partial_{\dot{q}^m}\mathscr{L} + K$

q^1 zyklisch: $\frac{\partial \mathscr{L}}{\partial q^1} = 0 \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathscr{L}}{\partial \dot{q}^1} = 0\,,\ E = \dot{q}^i \frac{\partial \mathscr{L}}{\partial \dot{q}^i} - \mathscr{L}\,,\ \partial_t \mathscr{L} = 0 \Rightarrow \frac{\mathrm{d}E}{\mathrm{d}t} = 0$

eindim. Bew.: $E = \frac{1}{2} m \dot{x}^2 + V(x)\,,\ t(x) - t(x_0) = \int\limits_{x_0}^{x} \frac{\mathrm{d}x'}{\sqrt{\frac{2}{m}(E - V(x'))}}\,,\ \omega^2_{\text{klein}} = \frac{V''(\bar{x})}{m}$

harmonischer Oszillator: $V(x) = \frac{1}{2}\kappa x^2\,,\ \ddot{x} + \omega^2 x = 0\,,\ \omega^2 = \frac{\kappa}{m}\,,\ x(t) = A\cos(\omega t + \varphi)$

gekoppelte Osz.: $\ddot{x}^i + (\Omega^2)^{ij} x^j = 0\,,\ (\Omega^2 - \omega_j^2) n_{(j)} = 0\,,\ x^i(t) = \sum_j a_j n^i_{(j)} \cos(\omega_j t + \varphi_j)$

Virialsatz: $V(\lambda \vec{r}) = \lambda^n V(\vec{r})\,,$ skalierte Bahn: $\vec{r}_\lambda(t) = \lambda \vec{r}(\lambda^{\frac{n}{2}-1} t)\,,\ 2\bar{E}_{\text{kin}} = n\bar{E}_{\text{pot}}$

Kepler: $V(\vec{r}) = -\frac{\alpha}{|\vec{r}|}\,,\ r(\varphi) = \frac{a(1-e^2)}{1+e\cos\varphi}\,,\ e = \sqrt{1 + \frac{2EL^2}{m\alpha^2}}\,,\ \frac{T^2}{(2\pi)^2} = \frac{a^3}{G_N(m_{\text{Sonne}} + m_{\text{Planet}})}$

Nichtinertialsystem: $\vec{x}_{\text{inertial}} = \vec{R}_{\text{Ursprung}} + O\,\vec{r}_{\text{Labor}}\ ,\ \dot{O} = O\,\Omega\ ,\ \Omega = -\Omega^T\ ,\ \Omega\,\vec{r} = \vec{\Omega} \times \vec{r}$

$\mathscr{L} = \frac{1}{2} m\,\dot{\vec{r}}^2 + m\,\vec{\Omega}\cdot(\vec{r} \times \dot{\vec{r}}) + \frac{1}{2} m\,(\vec{\Omega} \times \vec{r})^2 - O^{-1}\,\ddot{\vec{R}} \cdot \vec{r} - \hat{V}(r)$

$m\,\ddot{\vec{r}} = -\operatorname{grad}\hat{V} - m\,O^{-1}\,\ddot{\vec{R}} - 2\,m\,\Omega \times \dot{\vec{r}} - m\,\dot{\Omega} \times \vec{r} - m\,\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$

starrer Körper: $\vec{x}_\alpha = \vec{R} + O\,\vec{r}_\alpha\ ,\ \dot{\vec{x}}_\alpha = \dot{\vec{R}} + O(\vec{\Omega} \times \vec{r}_\alpha)\ ,\ \vec{\omega} = O\vec{\Omega}\ ,\ \vec{y}_\alpha = O\vec{r}_\alpha$

Masse: $M = \int \mathrm{d}^3x\,\rho(x)$, Schwerpunkt: $\vec{R} = \frac{1}{M}\int \mathrm{d}^3x\,\rho(x)\vec{x}\,,$ Impuls: $\vec{P} = M\dot{\vec{R}}$

Trägheitstensor: $\theta^{ij} = \int \mathrm{d}^3x\,\rho(x)(\delta^{ij}\vec{x}^2 - x^i x^j)\,,\ \theta^{ij} = O^i{}_k O^j{}_l\,\vartheta^{kl}\,,\ \theta^{11} + \theta^{22} \geq \theta^{33}$

$E_{\text{kin}} = \frac{1}{2} M\dot{\vec{R}}^2 + \frac{1}{2}\vec{\omega}\cdot\theta\vec{\omega} = \frac{1}{2} M\dot{\vec{R}}^2 + \frac{1}{2}\vec{\Omega}\cdot\vartheta\vec{\Omega}\,,\ \vec{L} = \vec{R} \times \vec{P} + \theta\vec{\omega} = \vec{R} \times \vec{P} + O(\vartheta\vec{\Omega})$

$\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = \sum_\alpha \vec{F}_\alpha\,,\ \frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = \sum_\alpha \vec{x}_\alpha \times \vec{F}_\alpha\ ,$ freier starrer Körper: $\theta_1\,\dot{\Omega}_1 = (\theta_2 - \theta_3)\,\Omega_2\,\Omega_3$

konjugierter Impuls: $p_i = \frac{\partial \mathscr{L}}{\partial \dot{q}^i}$, *Hamiltonfunktion*: $\mathscr{H}(q,p,t) = p_j \dot{q}^j(q,p) - \mathscr{L}(q, \dot{q}(q,p), t)$

Hamiltonsche G.: $\dot{q}^i = \frac{\partial \mathscr{H}}{\partial p_i}\,,\ \dot{p}_i = -\frac{\partial \mathscr{H}}{\partial q^i}\,,$ Poissonklammer: $\{A,B\} = \frac{\partial A}{\partial q^i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q^i}$

$\{aA + bB, C\} = a\{A,C\} + b\{B,C\}\,, \{A,B\} = -\{B,A\}\,, \{A,BC\} = \{A,B\}C + B\{A,C\}$

$\partial\,\{A,B\} = \{\partial\,A, B\} + \{A, \partial\,B\}\,,\ 0 = \{A,\{B,C\}\} + \{B,\{C,A\}\} + \{C,\{A,B\}\}$

Erhaltungsgrößen: $\frac{\mathrm{d}}{\mathrm{d}t}\phi = \{\phi, \mathscr{H}\} + \partial_t\phi = 0\,,\ \frac{\mathrm{d}\phi_1}{\mathrm{d}t} = 0 = \frac{\mathrm{d}\phi_2}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}\{\phi_1,\phi_2\}}{\mathrm{d}t} = 0$

kanonische, inf. Transformation: $\delta q^i = \{q^i, \phi\}\,,\ ,\ \delta p_i = \{p_i, \phi\}\,,$ Symmetrie $\Leftrightarrow \frac{\mathrm{d}}{\mathrm{d}t}\phi = 0$

flächentreue Entwicklung: $\omega(t) = \mathrm{d}q^i \wedge \mathrm{d}p_i\,,\ q^i(\lambda_1,\lambda_2,t)\,, p_i(\lambda_1,\lambda_2,t) \Rightarrow \partial_t\omega = 0\,,$

Liouville: $\partial_t \omega^n = 0$

Maxwell-Gleichungen: $\operatorname{div}\vec{B} = 0\ ,\ \operatorname{rot}\vec{E} + \frac{\partial}{c\,\partial t}\vec{B} = 0\ ,\ \operatorname{div}\vec{E} = 4\pi\rho\ ,\ \operatorname{rot}\vec{B} - \frac{\partial}{c\,\partial t}\vec{E} = \frac{4\pi}{c}\vec{j}$

$\vec{B} = \operatorname{rot}\vec{A}\ ,\ \vec{E} = -\operatorname{grad}\phi - \frac{\partial}{c\partial t}\vec{A}\ ,$ Eichtransformation: $\vec{A}' = \vec{A} - \operatorname{grad}\chi\ ,\ \phi' = \phi + \frac{\partial}{c\partial t}\chi$

Lorenz-Eichung: $\frac{\partial}{c\partial t}\phi + \operatorname{div}\vec{A} = 0 \ \Leftrightarrow\ \partial_m A^m = 0,\ x^m = (ct, \vec{x}),\ A^m = (\phi, \vec{A}),\ j^m = (c\rho, \vec{j})$

Kontinuitätsgleichung (lokale Ladungserhaltung): $\partial_t\rho + \operatorname{div}\vec{j} = 0 \ \Leftrightarrow\ \partial_m j^m = 0$

kovariante Beschreibung: $F_{mn} = -F_{nm}$, $F_{0i} = E^i$, $F_{ij} = -\epsilon_{ijk}B^k$, $F^{mn} = \eta^{mk}\eta^{nl}F_{kl}$

$\partial_m F^{mn} = \frac{4\pi}{c} j^n$, $\partial_k F_{lm} + \partial_l F_{mk} + \partial_m F_{kl} = 0$

$F_{mn} = \partial_m A_n - \partial_n A_m$ ist eichinvariant: $A'_m = A_m + \partial_m \chi \to F'_{mn} = F_{mn}$

$\Box A^m = \frac{4\pi}{c} j^m$, $A^m(x^0, \vec{x}) = \frac{1}{c}\int \mathrm{d}^3y \, \frac{j^m(x^0 - |\vec{x}-\vec{y}|, \vec{y})}{|\vec{x}-\vec{y}|} + \int \frac{\mathrm{d}^3k}{2k^0} \{a^m(\vec{k})\, \mathrm{e}^{-\mathrm{i}\,k\,x} + a^{m*}(\vec{k})\, \mathrm{e}^{\mathrm{i}\,k\,x}\}$

$k^0 = |\vec{k}|$, $k^0 a^0(\vec{k}) - \vec{k}\vec{a}(\vec{k}) = 0$, $k\,x = k^0x^0 - \vec{k}\cdot\vec{x}$

Multipolmomente: $Q = \int \mathrm{d}^3x\, \rho(\vec{x})$, $\vec{d} = \int \mathrm{d}^3x\, \vec{x}\rho(\vec{x})$, $Q^{ij} = \int \mathrm{d}^3x\, (3x^ix^j - \delta^{ij}\vec{x}^2)\rho(\vec{x})$

Energiedichte: $\frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2)$ *Poynting-Vektor*: $\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B}$

Energiesatz: $\frac{\partial}{\partial t}\left[\frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2)\right] + \mathrm{div}\left[\frac{c}{4\pi}\vec{E}\times\vec{B}\right] + \vec{j}\cdot\vec{E} = 0$

Impuls: $\frac{\partial}{\partial t}\left[\frac{1}{4\pi c}\vec{E}\times\vec{B}\right]^j + (\frac{-1}{4\pi})\partial_i\left[E^iE^j + B^iB^j - \frac{1}{2}\delta^{ij}(\vec{E}^2 + \vec{B}^2)\right] = -\left[\rho\vec{E} + \frac{1}{c}\vec{j}\times\vec{B}\right]^j$

Energie-Impuls-T.: $T^{mn}_{\text{Feld}}(x) = -\frac{1}{4\pi}(F^{ml}F^n{}_l - \frac{1}{4}\eta^{mn}F^{kl}F_{kl})$, *Lorentzk.*: $\vec{F} = q(\vec{E} + \frac{\vec{v}}{c}\times\vec{B})$

ebene elektromagnetische Welle: $\vec{E}(t,\vec{x}) = \Re\vec{E}_0\mathrm{e}^{\mathrm{i}(\vec{k}\vec{x} - \omega t)}$, $\vec{B} = \frac{\vec{k}}{|\vec{k}|}\times\vec{E}(t,\vec{x})$

$\frac{\omega}{c} = |\vec{k}|$, $\vec{k}\cdot\vec{E}_0 = 0$, $\vec{S} = \frac{c}{4\pi}\frac{\vec{k}}{|\vec{k}|}|\vec{E}(t,\vec{x})|^2$, $\vec{S}$, $\vec{E}$, $\vec{B}$ orthogonales Rechtssystem

$\vec{k} = \frac{\omega}{c}\begin{pmatrix}0\\0\\1\end{pmatrix}$: $\vec{E}_0 = \frac{|\vec{E}_0|}{\sqrt{1+a^2}}\begin{pmatrix}1\\ \mathrm{i}\,a\\0\end{pmatrix}$, $-1 \le a \le 1$, linear: $a = 0$, zirkular: $a = \pm 1$

skalare Kugelwelle: $\phi(t,\vec{x}) = \Re\,\frac{\phi_0}{|\vec{x}|}\,\mathrm{e}^{\mathrm{i}(k|\vec{x}| - \omega t)}$, $k = \frac{\omega}{c}$

Punktladung: Weltlinie $r^m(s) = (ct(s), \vec{r}(s))$, $j^m(x) = cq\int \mathrm{d}s\, \frac{\mathrm{d}r^m}{\mathrm{d}s}\delta^4(x - r(s))$

Liènard-Wiechert: $A^m_{\text{ret}}(x) = q\frac{\frac{\mathrm{d}r^m}{\mathrm{d}s}}{\frac{\mathrm{d}r^n}{\mathrm{d}s}\cdot(x_n - r_n)}\Big|_{s=\bar{s}(x)\,:\,(x - r(\bar{s}))^2 = 0,\ x^0 > r^0(\bar{s})}$

Energieimpulstensor: $T^{mn}(x) = \int \mathrm{d}s\, mc^2\frac{\dot{r}^m\dot{r}^n}{\sqrt{\dot{r}^2}}\delta^4(x - r(s))$, $P^m = \frac{1}{c}\int \mathrm{d}^3x\, T^{0m}$

Bewegungsgleichung ohne Strahlungsrückwirkung: $mc\frac{\mathrm{d}}{\mathrm{d}s}\frac{\dot{r}^m}{\sqrt{\dot{r}^2}} = \frac{q}{c}F^{mn}\dot{r}_n$

Hertzscher Dipol: Fernzone $\vec{B}(t,\vec{x}) = \frac{1}{|\vec{x}|}\frac{1}{c^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{d}(t - \frac{|\vec{x}|}{c})\times\frac{\vec{x}}{|\vec{x}|}$, $\vec{E} = \vec{B}\times\frac{\vec{x}}{|\vec{x}|}$

elektrische Dipolstrahlung: $\vec{S}(t,\vec{x}) = \frac{c}{4\pi}\frac{\vec{x}}{|\vec{x}|^3}\sin^2\theta\left|\frac{1}{c^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{d}(t - \frac{|\vec{x}|}{c})\right|^2$, $\theta = \theta(\vec{x}, \frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{d})$