Nonpolynomial gauge invariant interactions of 1-form and 2-form gauge potentials

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Abstract: A four dimensional gauge theory with nonpolynomial but local interactions of 1-form and 2-form gauge potentials is constructed. The model is a nontrivial deformation of a free gauge theory with nonpolynomial dependence on the gauge coupling constant.

Gauge invariant interactions of ordinary gauge fields (= 1-form gauge potentials) are very well known, the most famous one being undoubtedly the Yang–Mills interaction. Much less is known about the possible gauge invariant couplings between 1-form and 2-form gauge potentials, although an important coupling of that type is known for a long time: it is the celebrated coupling of a 2-form gauge potential to Chern–Simons forms which underlies among others the Green–Schwarz anomaly cancellation mechanism [1]. Here we shall construct a rather different interacting gauge theory for 1-form and 2-form gauge potentials in four dimensions. As the model has local but nonpolynomial interactions and gauge transformations, its structure is in some respect more reminiscent of gravitational interactions than of Yang–Mills theory or couplings of Chern–Simons forms to a 2-form gauge potential.

The same model was found by different means as a particular example in a more general class of theories (Eq. (17) of [2]) where it is formulated in a polynomial first-order form.

Although we will not study supersymmetric field theories here, our construction was partly motivated by the aim to gauge the central charge of the rigid N=2 supersymmetry algebra realized on the vector-tensor multiplet [3] which arises naturally in string compactifications [4]. This problem was investigated already in [5] and could be relevant among others in order to classify the still unknown couplings of the vector-tensor multiplet to N=2 supergravity.

The central charge occurs in the anticommutator of two supersymmetry transformations with the same chirality and is called central because it commutes with all generators of the supersymmetry algebra. This rigid bosonic symmetry acts nontrivially only on the 1- and 2-form gauge fields of the vector-tensor multiplet, as it is on-shell trivial on the remaining component fields of the vector-tensor multiplet. Therefore one can ask already in the nonsupersymmetric case whether it can be gauged in a reasonable way. This question is interesting in its own right and underlies our construction.

Our starting point is the standard free action for two abelian 1-form gauge potentials $A = dx^\mu A_\mu$ and $W = dx^\mu W_\mu$ and a 2-form gauge potential $B = (1/2)dx^\mu \wedge dx^\nu B_{\mu\nu}$ in flat four dimensional spacetime. The Lagrangian reads

$$\mathcal{L}_0 = -\frac{1}{4}\left(G_{\mu\nu}G^{\mu\nu} + F_{\mu\nu}F^{\mu\nu}\right) - H_\mu H^\mu \tag{1}$$

where

$$G_{\mu\nu} \quad = \quad \partial_\mu W_\nu - \partial_\nu W_\mu \,,$$

$$
\begin{aligned}
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \,, \\
H^\mu &= \frac{1}{2}\,\varepsilon^{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma} \,.
\end{aligned} \tag{2}
$$

The action with Lagrangian (1) has, among others, a rigid symmetry generated by

$$
\delta_z A_\mu = 2H_\mu,\ \delta_z B_{\mu\nu} = \frac{1}{2}\,\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma},\ \delta_z W_\mu = 0. \tag{3}
$$

This rigid symmetry coincides indeed on-shell with the central charge of the N=2 supersymmetry algebra for the vector-tensor multiplet, cf. [3].

Our aim will now be to gauge the rigid symmetry (3). With this end in view, we look for appropriate extensions $\Delta_z A_\mu$ and $\Delta_z B_{\mu\nu}$ of $\delta_z A_\mu$ and $\delta_z B_{\mu\nu}$ transforming covariantly under the sought gauge transformations generated by

$$
\begin{aligned}
\delta_\xi W_\mu &= \partial_\mu \xi \,, \\
\delta_\xi A_\mu &= g\,\xi\,\Delta_z A_\mu \,, \\
\delta_\xi B_{\mu\nu} &= g\,\xi\,\Delta_z B_{\mu\nu}
\end{aligned} \tag{4}
$$

where ξ is an arbitrary field and g is a gauge coupling constant. Following a standard recipe in gauge theories, we try to covariantize partial derivatives of A_μ and $B_{\mu\nu}$ by means of a covariant derivative

$$
\mathcal{D}_\mu = \partial_\mu - gW_\mu\Delta_z \tag{5}
$$

where Δ_z is the sought extension of δ_z. We now try to covariantize (3) by replacing there $\delta_z A_\mu$ and $\delta_z B_{\mu\nu}$ with $\Delta_z A_\mu$ and $\Delta_z B_{\mu\nu}$ respectively, and ∂_μ with $\mathcal{D}_\mu$. Explicitly this yields

$$
\begin{aligned}
\Delta_z A_\mu &= \varepsilon_{\mu\nu\rho\sigma}(\partial^\nu B^{\rho\sigma} - gW^\nu\Delta_z B^{\rho\sigma}), \\
\Delta_z B_{\mu\nu} &= \varepsilon_{\mu\nu\rho\sigma}(\partial^\rho A^\sigma - gW^\rho\Delta_z A^\sigma).
\end{aligned} \tag{6}
$$

(6) determines $\Delta_z A_\mu$ and $\Delta_z B_{\mu\nu}$. Indeed, inserting the second equation (6) in the first one, we get an equation for $\Delta_z A_\mu$,

$$
(E\delta_\mu^\nu + 2g^2 W_\mu W^\nu)\Delta_z A_\nu = 2Z_\mu \,, \tag{7}
$$

where

$$
E = 1 - 2g^2 W_\mu W^\mu,\ Z_\mu = H_\mu + gF_{\mu\nu}W^\nu. \tag{8}
$$

To solve (7) for $\Delta_z A_\nu$, we only need to invert the matrix $E\delta_\mu^\nu + 2g^2 W_\mu W^\nu$. The inverse is

$$
V_\mu{}^\nu = E^{-1}(\delta_\mu^\nu - 2g^2 W_\mu W^\nu). \tag{9}
$$

(7) and (6) yield now

$$
\Delta_z A_\mu = 2\mathcal{H}_\mu \ ,\ \Delta_z B_{\mu\nu} = \frac{1}{2}\,\varepsilon_{\mu\nu\rho\sigma}\mathcal{F}^{\rho\sigma} \tag{10}
$$

where

$$
\begin{aligned}
\mathcal{H}_\mu &= E^{-1}(Z_\mu - 2g^2 W_\mu W_\nu H^\nu), \\
\mathcal{F}_{\mu\nu} &= F_{\mu\nu} - 4gE^{-1}W_{[\mu}Z_{\nu]} \,.
\end{aligned} \tag{11}
$$

Recall that our goal was to find gauge transformations (4) under which $\Delta_z A_\mu$ and $\Delta_z B_{\mu\nu}$ transform covariantly. We can now examine whether we have reached this goal. This amounts to check whether $\mathcal{H}_\mu$ and $\mathcal{F}_{\mu\nu}$ transform covariantly, i.e. without derivatives of ξ, under the gauge transformations

$$
\begin{aligned}
\delta_\xi A_\mu &= 2g\,\xi\,\mathcal{H}_\mu \ , \\
\delta_\xi B_{\mu\nu} &= \frac{1}{2}\,g\,\xi\,\varepsilon_{\mu\nu\rho\sigma}\mathcal{F}^{\rho\sigma}, \\
\delta_\xi W_\mu &= \partial_\mu \xi \,.
\end{aligned} \tag{12}
$$

The answer is affirmative: neither $\delta_\xi \mathcal{H}_\mu$ nor $\delta_\xi \mathcal{F}_{\mu\nu}$ contain derivatives of ξ. Indeed, an elementary, though somewhat lengthy calculation yields

$$\delta_\xi \mathcal{H}_\mu = g\,\xi\,\Delta_z \mathcal{H}_\mu\,,\ \ \delta_\xi \mathcal{F}_{\mu\nu} = g\,\xi\,\Delta_z \mathcal{F}_{\mu\nu} \tag{13}$$

with

$$\begin{aligned}\Delta_z \mathcal{H}_\mu &= V_\mu{}^\nu(\partial^\rho \mathcal{F}_{\rho\nu} - 4gW^\rho \partial_{[\rho}\mathcal{H}_{\nu]}),\\ \Delta_z \mathcal{F}_{\mu\nu} &= 4(\partial_{[\mu}\mathcal{H}_{\nu]} - gW_{[\mu}\Delta_z \mathcal{H}_{\nu]}).\end{aligned} \tag{14}$$

To construct an action which is invariant under the gauge transformations (12), it is helpful to realize that the transformations (14) are nothing but

$$\Delta_z \mathcal{H}_\mu = \mathcal{D}^\nu \mathcal{F}_{\nu\mu},\ \ \Delta_z \mathcal{F}_{\mu\nu} = 4\mathcal{D}_{[\mu}\mathcal{H}_{\nu]} \tag{15}$$

where the second identity is obvious from (14), whereas the verification of the first one is slightly more involved. Combining (13) and (15) it is now easy to verify that

$$\begin{aligned}&\delta_\xi\left(-\frac{1}{4}\,\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \mathcal{H}_\mu\mathcal{H}^\mu\right)\\ &= 2g\,\xi\,\mathcal{D}_\nu(\mathcal{H}_\mu\mathcal{F}^{\mu\nu})\\ &= 2g\,\xi\,\partial_\nu(\mathcal{H}_\mu\mathcal{F}^{\mu\nu}) - 2g^2\xi\,W_\nu\Delta_z(\mathcal{H}_\mu\mathcal{F}^{\mu\nu})\\ &= \partial_\nu(2g\,\xi\,\mathcal{H}_\mu\mathcal{F}^{\mu\nu}) - \delta_\xi(2gW_\nu\mathcal{H}_\mu\mathcal{F}^{\mu\nu}).\end{aligned}$$

This implies immediately that the Lagrangian

$$\mathcal{L} \ = \ -\frac{1}{4}\,(G_{\mu\nu}G^{\mu\nu} + \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}) - \mathcal{H}_\mu\mathcal{H}^\mu - 2gW_\mu\mathcal{H}_\nu\mathcal{F}^{\mu\nu} \tag{16}$$

transforms under δ_ξ into a total derivative,

$$\delta_\xi \mathcal{L} = \partial_\nu(2g\,\xi\,\mathcal{H}_\mu\mathcal{F}^{\mu\nu}). \tag{17}$$

Hence, the action with Lagrangian (16) is gauge invariant under δ_ξ. Evidently it is also invariant under the following standard gauge transformations acting only on A_μ and $B_{\mu\nu}$ respectively:

$$\delta_\Lambda A_\mu = \partial_\mu\Lambda,\ \ \delta_\lambda B_{\mu\nu} = \partial_{[\mu}\lambda_{\nu]}\,. \tag{18}$$

Inserting finally the explicit expressions (11) in (16), the Lagrangian reads

$$\mathcal{L} \ = \ -\frac{1}{4}\,(G_{\mu\nu}G^{\mu\nu} + F_{\mu\nu}F^{\mu\nu}) - E^{-1}Z_\mu Z^\mu + 2E^{-1}(gW_\mu H^\mu)^2 \tag{19}$$

with E and Z_μ as in (8).

It is now easy to compute the Euler–Lagrange derivatives of $\mathcal{L}$ with respect to the fields. The result is

$$\frac{\hat\partial\mathcal{L}}{\hat\partial A_\mu} \ = \ \partial_\nu\mathcal{F}^{\nu\mu}, \tag{20}$$

$$\frac{\hat\partial\mathcal{L}}{\hat\partial B_{\mu\nu}} \ = \ -\varepsilon^{\mu\nu\rho\sigma}\partial_\rho\mathcal{H}_\sigma\,, \tag{21}$$

$$\frac{\hat\partial\mathcal{L}}{\hat\partial W_\mu} \ = \ \partial_\nu G^{\nu\mu} + 2g\mathcal{F}^{\mu\nu}\mathcal{H}_\nu\,. \tag{22}$$

Note that the equations of motion for A_μ and $B_{\mu\nu}$ obtained from (20) and (21) are not covariant under δ_ξ, in contrast to the equation of motion for W_μ following from (22). However, from (14) and (15) it is obvious that (20) and (21) can be combined to covariant expressions too, which holds generally for the equations of motion in gauge theories [6]. The covariant form of the equations of motion reads

$$\mathcal{D}_\nu \mathcal{F}^{\mu\nu} = 0,$$
$$\mathcal{D}_{[\mu} \mathcal{H}_{\nu]} = 0,$$
$$\partial_\nu G^{\nu\mu} + 2g\mathcal{F}^{\mu\nu}\mathcal{H}_\nu = 0.$$

To summarize, we have constructed an interacting four dimensional gauge theory with Lagrangian (16) or (19) for two ordinary gauge fields A_μ and W_μ and an antisymmetric gauge field $B_{\mu\nu}$. The key feature of this gauge theory is its gauge invariance under the transformations (12) which gauge the rigid symmetry (3) of the free action with Lagrangian (1). Both the Lagrangian and the gauge transformations (12) are nonpolynomial in the gauge coupling constant g and the gauge field W_μ. Nevertheless they are local, for the Lagrangian and the gauge transformations are still quadratic and linear in differentiated fields respectively. Note that the Lagrangian and the gauge transformations constitute a consistent deformation of the free Lagrangian (1) and its gauge symmetries in the sense of [7]. In particular one recovers the free theory and its gauge symmetries for $g = 0$. The generalization of all the above formulas to curved spacetime is obvious.

Let us finally compare our results to those of [5] where the central charge of the vector-tensor multiplet was gauged. First we have presented the action and the gauge transformations in an explicit and manifestly local form. In contrast, in [5] both the action and the gauge transformations are only implicitly defined (the formulas given in [5] result in local expressions too [8]). Furthermore our results appear to differ from those of [5], even when the latter are restricted to the particular nonsupersymmetric case studied here. In particular, neither the Lagrangian (19) nor the gauge transformations (12) contain Chern–Simons terms of the type occurring in [5]. This might signal that such terms are actually not needed in order to gauge the central charge of the vector-tensor multiplet. Of course, in contrast to [5], we did not study the supersymmetric case, and therefore we cannot clarify this issue here.

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Literatur

[1] M.B. Green and J.H. Schwarz, Phys. Lett. B 149 (1984) 117.

[2] M. Henneaux and B. Knaepen, All consistent interactions for exterior form gauge fields, to appear in Phys. Rev. D (hep-th/9706119).

[3] M. Sohnius, K.S. Stelle and P.C. West, Phys. Lett. B 92 (1980) 123.

[4] B. de Wit, V. Kaplunovsky, J. Louis and D. Lüst, Nucl. Phys. B 451 (1995) 53 (hep-th/9504006).

[5] P. Claus, B. de Wit, M. Faux, B. Kleijn, R. Siebelink and P. Termonia, Phys. Lett. B 373 (1996) 81 (hep-th/9512143); P. Claus, B. de Wit, M. Faux and P. Termonia, Nucl. Phys. B 491 (1997) 201 (hep-th/9612203).

[6] F. Brandt, Local BRST cohomology and covariance, to appear in Commun. Math. Phys. (hep-th/9604025).

[7] G. Barnich and M. Henneaux, Phys. Lett. B 311 (1993) 123 (hep-th/9304057).

[8] P. Claus, M. Faux and P. Termonia, private communication.