Relativistic Quantum Physics

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Ψ_{in} , Ψ_{out} , Ψ: Interaction is a Property of Paths not of States

$\vec{P}' = \vec{P}$, $P'^0 \neq P^0$, $M'_{ij} = M_{ij}$, $M'_{0i} \neq M_{0i}$?? Haag's Theorem

Motion of Center unaffected: $P^m = U^m\, M$
$H = U^0\, M$, $H' = U^0\, M'$, $[U^m, M'] = 0$

Quantum Fields fixed up to Sign: No Dynamic Variation

Lie groups K: Subspace of Smooth States (Finite Energy), Differential Geometry

Charge Basis: $C\Phi(x)C^{-1} = \kappa\Phi^*(x)$, C is linear, Conjugation in K

Massive Fields $s \geq 1/2$ have no Massless Limit

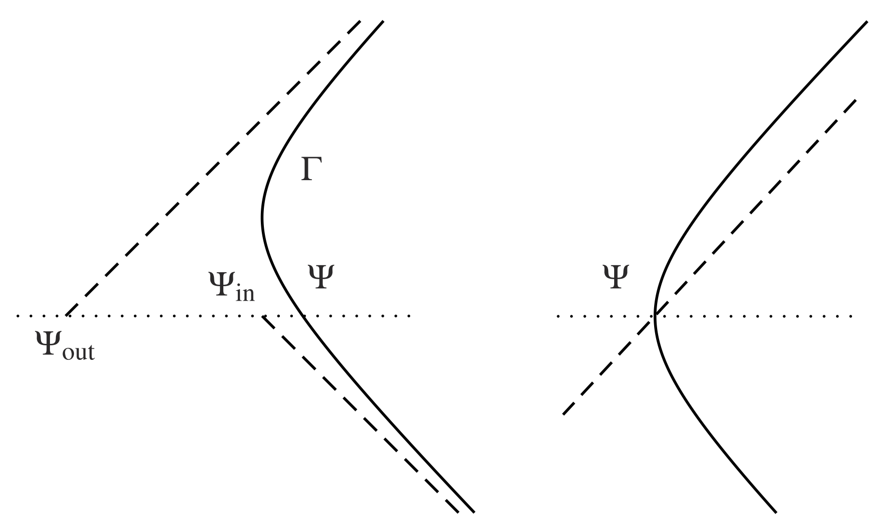
Vector Field implies BRST-Symmetry and Gauge Interactions

Scattering

$\mathrm{H} = \mathrm{P}^0$ separately conserved momenta: free motion

$$(\mathbf{P}^m \boldsymbol{\Psi})^{ij}(\mathbf{p}_1, \mathbf{p}_2) = (\mathbf{p}_1^m + \mathbf{p}_2^m)\, \boldsymbol{\Psi}^{ij}(\mathbf{p}_1, \mathbf{p}_2)$$

Interaction: $\mathrm{H}' \neq \mathrm{P}^0$, Unitary Motion $\mathrm{U}_t \mathrm{U}'_t = \mathrm{U}_{t+t'}$ has no limit



Interacting path Γ with asymptotes in $\mathcal{H} \times \mathbb{R}$, free and interacting path through Ψ

$$\boldsymbol{\Omega}(\mathbf{t}) = \mathrm{e}^{\mathrm{i}\mathbf{H}\,\mathbf{t}}\,\mathrm{e}^{-\mathrm{i}\mathbf{H}'\,\mathbf{t}}$$

$$\lim_{\mathbf{t}\to-\infty} \boldsymbol{\Omega}(\mathbf{t})\,\boldsymbol{\Psi} = \boldsymbol{\Omega}_+\boldsymbol{\Psi} = \boldsymbol{\Psi}_{\mathrm{in}} \ , \ \lim_{\mathbf{t}\to\infty} \boldsymbol{\Omega}(\mathbf{t})\,\boldsymbol{\Psi} = \boldsymbol{\Omega}_-\boldsymbol{\Psi} = \boldsymbol{\Psi}_{\mathrm{out}}$$

Møller Operators

strong limit weaker than norm limit

$$\mathbf{\Omega}_{\pm} = \underset{t\to\mp\infty}{\text{s-lim}}\ e^{i\mathbf{H}\,(t+a)}e^{-i\mathbf{H}'\,(t+a)} = e^{i\mathbf{H}a}\mathbf{\Omega}_{\pm}e^{-i\mathbf{H}'a}\ , \quad e^{i\mathbf{H}\,a}\mathbf{\Omega}_{\pm} = \mathbf{\Omega}_{\pm}e^{i\mathbf{H}'a}$$

$H = \Omega_{\pm} H' \Omega_{\pm}^{-1}$ unitarily equivalent (on scattering states), but $[H, H'] \neq 0$

$$\mathbf{S} : \mathbf{\Psi}_{\text{in}} \mapsto \mathbf{\Psi}_{\text{out}}\ ,\ \mathbf{S} = \mathbf{\Omega}_{-}\mathbf{\Omega}_{+}^{-1} = \underset{t,t'\to\infty}{\text{s-lim}}\ e^{i\mathbf{H}t}e^{-i\mathbf{H}'(t+t')}e^{i\mathbf{H}t'}$$

commutes with H and with P^m if relativistic i.e. if $[S, U_\Lambda] = 0$

$\Psi_{\text{in}}, \Psi_{\text{out}}, \Psi \in \mathcal{H}_{\text{scattering}}$, acted upon the same transformations

$\vec{P}' = \vec{P}\ ,\ P'^0 \neq P^0\ ,\ M'_{ij} = M_{ij}\ ,\ M'_{0i} \neq M_{0i}$ logically untenable

measurably wrong: Binding Energy weighs, Atomic Mass

Center Variables

$$P^m\,\Psi(p_1,\dots p_n) = (\sum_{i=1}^{n} p_i^m)\Psi(p_1,\dots p_n)\ ,\ M^2 = P^2\ , P^m = U^m\,M$$

$$u^m = \sum_i p_i^m / \sqrt{(\sum_j p_j)^2}\ ,\ p_i = p_{i\,\parallel} + p_{i\,\perp}\ ,\ q_i = (L_u)^{-1} p_{i\,\perp}$$

$$p_i(u,q) = \sqrt{m_i^2 + \vec{q}_i^{\,2}}\begin{pmatrix}\sqrt{1+\vec{u}^2}\\ \vec{u}\end{pmatrix} + \begin{pmatrix}\vec{u}\cdot\vec{q}_i\\ \vec{q}_i + \frac{(\vec{u}\cdot\vec{q}_i)\,\vec{u}}{1+\sqrt{1+\vec{u}^2}}\end{pmatrix}\ ,\ \sum_i \vec{q}_i = 0$$

$$(M\Psi)(u,q) = M(q)\,\Psi(u,q)\ ,\ M(q) = \sum_{i=1}^{n}\sqrt{m_i^2 + \vec{q}_i^{\,2}} \geq \sum_i m_i$$

Factorizing the Motion of the Center

$\mathcal{H} = \sum_s \mathcal{H}_s \otimes \mathcal{I}_s$

$$\big(\mathbf{P}^m \Psi\big)(u,r) = u^m\, M(r)\, \Psi(u,r)\ ,$$
$$\big(-\mathrm{i}M_{ij}\Psi\big)(u,r) = -\big(u^i\partial_{u^j} - u^j\partial_{u^i}\big)\Psi(u,r) + \Gamma_{ij}\Psi(u,r)\ ,$$
$$\big(-\mathrm{i}M_{0i}\Psi\big)(u,r) = \sqrt{1+\vec{u}^2}\,\partial_{u^i}\Psi(u,r) + \Gamma_{ij}\frac{u^j}{1+\sqrt{1+\vec{u}^2}}\Psi(u,r)$$

$$\mathbf{H} = \sqrt{1+\vec{u}^2}\Big(\sqrt{m_1^2+\vec{q}^2} + \sqrt{m_2^2+\vec{q}^2}\Big) = \sqrt{1+\vec{u}^2}\Big(m_1+m_2+\frac{\vec{z}^2}{2\mu}\Big)$$

$U_{a,\Lambda}$ irreducible on $\mathcal{H}_s$, $H' = U^0\, M'$

$$\mathbf{H}' = \mathbf{U}^0\,\mathbf{M}'\ ,\ \ \mathbf{M}' = \mathbf{M} + \mathbf{V}(\vec{x}^2)$$

More to fill a Book