The Vector-Tensor Multiplet in Harmonic Superspace

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Abstract

We describe the vector-tensor multiplet and derive its Chern-Simons coupling to the $N = 2$ Yang-Mills gauge superfield in harmonic superspace.

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The $N = 2$ vector-tensor multiplet, which was discovered many years ago by Sohnius, Stelle and West [1] and then forgotten for a while, has recently received much interest [2, 3, 4] due to the fact that it originates in the low-energy effective Lagrangian of $N = 2$ heterotic string vacua. As a representation of $N = 2$ supersymmetry, this multiplet is very similar to the massless $8 + 8$ Fayet-Sohnius hypermultiplet [5, 6] which possesses an off-shell central charge generating the equations of motion.

The vector-tensor multiplet is the only known $N = 2$, $D = 4$ supersymmetric model that has not yet been formulated in the harmonic superspace [7]. Since the harmonic superspace is believed to be a universal framework for $N = 2$ supersymmetric theories, finding a relevant formulation for the vector-tensor multiplet seems to be of principal importance. On the other hand, adequate formulations of the vector-tensor multiplet in an $N = 2$ superspace with central charges have been given in recent papers [8, 9]. Our primary goal in this letter is to show that the main results of Refs. [8, 9] have a natural origin in the harmonic superspace approach.

We start with re-formulating the Sohnius prescription of constructing supersymmetric actions [6] in harmonic superspace. The harmonic central charge superspace [7] extends the $N = 2$ central charge superspace [6], with coordinates $\{x^m,\ z,\ \theta^\alpha_i,\ \bar\theta^i_{\dot\alpha}\}$, $\overline{\theta^\alpha_i} = \bar\theta^{\dot\alpha\, i}$ (where z is the central charge real variable), by the two-sphere $S^2 = SU(2)/U(1)$ parametrized by harmonics, i.e. group elements

$$
\begin{gathered}
(u_i{}^-\ ,\ u_i{}^+) \in SU(2) \\
u^+_i = \varepsilon_{ij} u^{+j} \qquad \overline{u^{+i}} = u^-_i \qquad u^{+i} u^-_i = 1\ .
\end{gathered} \tag{1}
$$

The analytic basis of the harmonic superspace defined by

$$
\begin{aligned}
x^m_A = x^m - 2\mathrm{i}\theta^{(i}\sigma^m\bar\theta^{j)} u^+_i u^-_j \qquad & z_A = z + \mathrm{i}(\theta^{+\alpha}\theta^-_\alpha - \bar\theta^+_{\dot\alpha}\bar\theta^{-\dot\alpha}) \\
\theta^\pm_\alpha = u^\pm_i \theta^i_\alpha \qquad & \bar\theta^\pm_{\dot\alpha} = u^\pm_i \bar\theta^i_{\dot\alpha}
\end{aligned} \tag{2}
$$

is most suitable to the description of analytic superfields $\Phi(\zeta, u)$ which depends only on the variables

$$
\zeta^M \equiv \{x^m_A, z_A, \theta^{+\alpha}, \bar\theta^+_{\dot\alpha}\} \tag{3}
$$

and harmonics $u^\pm_i$ (the original basis of the harmonic superspace is called central [7]). Below we will mainly work in the analytic basis and omit the corresponding subscript "A". The explicit expressions for the covariant derivatives $D^\pm_\alpha = D^i_\alpha u^\pm_i$, $\bar D^\pm_{\dot\alpha} = \bar D^i_{\dot\alpha} u^\pm_i$ in the analytic basis can be found in Ref. [7].

The GIKOS rule [7] of constructing $N=2$ supersymmetric actions

$$\int d\zeta^{(-4)} du\, \mathcal{L}^{(4)} \qquad\qquad d\zeta^{(-4)} = d^4x d^2\theta^+ d^2\bar{\theta}^+ \tag{4}$$

involves an analytic superfield $\mathcal{L}^{(4)}(\zeta, u)$ of $U(1)$-charge $+4$ which is invariant (up to derivatives) under central charge transformations generated by $\partial_z \equiv \partial/\partial z$

$$\frac{\partial}{\partial z}\mathcal{L}^{(4)} = \frac{\partial}{\partial x^m} f^{(4)m} \ . \tag{5}$$

Here $\mathcal{L}^{(4)}$ is a function of the dynamical superfields, their covariant derivatives and, in general, of the harmonic variables.

In harmonic superspace there exists a prescription to construct invariant actions even for non-vanishing central charges. The construction makes use of a constrained analytic superfied $\mathcal{L}^{++}(\zeta, u)$. $\mathcal{L}^{++}$ is an analytic superfield of $U(1)$-charge $+2$

$$D^+_\alpha \mathcal{L}^{++} = \bar{D}^+_{\dot\alpha} \mathcal{L}^{++} = 0 \tag{6}$$

which satisfies the covariant constraint

$$D^{++}_{\rm c} \mathcal{L}^{++} = 0 \ . \tag{7}$$

$D^{++}_{\rm c}$ acts according to

$$D^{++}_{\rm c} = D^{++} + {\rm i}\,(\theta^{+\alpha}\theta^+_\alpha - \bar{\theta}^+_{\dot\alpha}\bar{\theta}^{+\dot\alpha})\frac{\partial}{\partial z} \qquad D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} - 2{\rm i}\,\theta^+\sigma^m\bar{\theta}^+\frac{\partial}{\partial x^m} \tag{8}$$

on analytic superfields. Then the action

$$S = \int d\zeta^{(-4)} du\ \left((\theta^+)^2 - (\bar{\theta}^+)^2\right) \mathcal{L}^{++} \tag{9}$$

is supersymmetric and, hence, invariant under central charge transformations. Under a supersymmetry transformation

$$\begin{aligned} \delta x^m &= -2{\rm i}\,(\epsilon^-\sigma^m\bar{\theta}^+ + \theta^+\sigma^m\bar{\epsilon}^-) \\ \delta z &= 2{\rm i}\,(\epsilon^-\theta^+ - \bar{\epsilon}^-\bar{\theta}^+) \\ \delta\theta^+_\alpha &= \epsilon^+_\alpha \qquad \delta\bar{\theta}^+_{\dot\alpha} = \bar{\epsilon}^+_{\dot\alpha} \end{aligned} \tag{10}$$

S changes by

$$\delta S = \int d\zeta^{(-4)} du \left\{ \left((\theta^+)^2 - (\bar{\theta}^+)^2\right) \delta z \frac{\partial}{\partial z}\mathcal{L}^{++} - 2\left(\epsilon^+\theta^+ - \bar{\epsilon}^+\bar{\theta}^+\right)\mathcal{L}^{++} \right\} \ . \tag{11}$$

Making use of the identity $2\left(\epsilon^+\theta^+ - \bar\epsilon^+\bar\theta^+\right) = -\mathrm{i}\, D^{++}\delta z$ and integrating by parts in (11), one arrives at

$$\delta S = -\mathrm{i}\int d\zeta^{(-4)}du\,\delta z D^{++}_{\mathrm{c}}\mathcal{L}^{++} \tag{12}$$

and this is equal to zero due to (7). The action (9) is real if $\mathcal{L}^{++}$ is imaginary

$$\breve{\mathcal{L}}^{++} = -\mathcal{L}^{++} \tag{13}$$

with respect to the analyticity preserving conjugation (smile) $\breve{} \; = \; \bar{}^{\star}$ introduced in [7], where the operation $\bar{}$ denotes the complex conjugation and the operation * is defined by $(u^+_i)^\star = u^-_i$, $(u^-_i)^\star = -u^+_i$, hence $(u^\pm_i)^{\star\star} = -u^\pm_i$.

Eq. (9) is the formulation of the Sohnius action [6] (see also [10]) in the harmonic superspace. More explicitly, in the central basis the constraint (7) means

$$\mathcal{L}^{++} = \mathcal{L}^{ij}(x,z,\theta)u^+_i u^+_j \tag{14}$$

for some u-independent superfields $\mathcal{L}^{ij}$, and the analyticity conditions (6) take the form

$$D^{(i}_\alpha\mathcal{L}^{jk)} = \bar D^{(i}_{\dot\alpha}\mathcal{L}^{jk)} = 0\ . \tag{15}$$

Since

$$d\zeta^{(-4)} = \frac{1}{16}d^4x D^{-\alpha}D^-_\alpha\bar D^-_{\dot\alpha}\bar D^{-\dot\alpha} \tag{16}$$

the action (9) turns, upon integrating over S^2, into

$$S = \frac{1}{12}\int d^4x\,\left(D^{\alpha i}D^j_\alpha - \bar D^i_{\dot\alpha}\bar D^{\dot\alpha j}\right)\mathcal{L}_{ij}\ . \tag{17}$$

It is instructive to consider two examples. The 8 + 8 Fayet-Sohnius off-shell hypermultiplet coupled to the $N = 2$ gauge multiplet is described in the $N = 2$ central charge superspace by a superfield $q_i(x,z,\theta)$ satisfying the constraints [6]

$$\mathcal{D}^{(i}_\alpha q^{j)} = \bar{\mathcal{D}}^{(i}_{\dot\alpha}q^{j)} = 0 \tag{18}$$

with $\mathcal{D}^i_\alpha$ the gauge covariant derivatives. This is equivalent to the fact that the superfield $q^+ = q_i u^{+i}$ is covariantly analytic

$$\mathcal{D}^+_\alpha q^+ = \bar{\mathcal{D}}^+_{\dot\alpha}q^+ = 0 \tag{19}$$

and satisfies the gauge covariant constarint

$$\mathcal{D}^{++}_{\mathrm{c}}q^+ = 0\ . \tag{20}$$

In the analytic basis $\mathcal{D}_{\rm c}^{++}$ is

$$\mathcal{D}_{\rm c}^{++} = \mathcal{D}^{++} + {\rm i}\,(\theta^{+\alpha}\theta^{+}_{\alpha} - \bar\theta^{+}_{\dot\alpha}\bar\theta^{+\dot\alpha})\frac{\partial}{\partial z} \qquad \mathcal{D}^{++} = D^{++} + {\rm i}\,V^{++} \tag{21}$$

where V^{++} is the analytic Yang-Mills gauge prepotential [7]. Therefore, the gauge invariant superfield

$$\mathcal{L}^{++}_{\rm FS} = \frac{\rm i}{2}\left(\breve{q}^{+}\partial_z q^{+} - \partial_z \breve{q}^{+} q^{+}\right) + m\,\breve{q}^{+}q^{+} \tag{22}$$

meets the requirements (6) and (7). Because of (20), the corresponding action can be rewritten in the following form

$$S_{\rm FS} = \int d\zeta^{(-4)}du\,\left\{-\breve{q}^{+}\mathcal{D}^{++}q^{+} + m\,\breve{q}^{+}q^{+}\left((\theta^{+})^2 - (\bar\theta^{+})^2\right)\right\} \tag{23}$$

which is very similar to the action functional of the infinite-component q-hypermultiplet [7]. Another non-trivial example is the effective action of the $N = 2$ super Yang-Mills theory [11, 12] (supersymmetry without central charges)

$$S_{\rm SYM} = {\rm tr}\int d^4x d^4\theta \mathcal{F}(W) + {\rm tr}\int d^4x d^4\bar\theta \bar{\mathcal{F}}(\bar W) \tag{24}$$

where W is the covariantly chiral field strength of the $N = 2$ gauge superfield [13]. $S_{\rm SYM}$ can be represented as follows

$$\begin{aligned} S_{\rm SYM} &= \frac{1}{4}\,{\rm tr}\int d\zeta^{(-4)}du\,\left((\theta^{+})^2 - (\bar\theta^{+})^2\right)\mathcal{L}^{++}_{\rm SYM} \\ \mathcal{L}^{++}_{\rm SYM} &= \left(\mathcal{D}^{+}\right)^2\mathcal{F}(W) - \left(\bar{\mathcal{D}}^{+}\right)^2\bar{\mathcal{F}}(\bar W)\,. \end{aligned} \tag{25}$$

It is obvious that $\mathcal{L}^{++}_{\rm SYM}$ satisfies the requirements (6) and (7).

A free vector-tensor multiplet can be described in the harmonic superspace by an analytic spinor superfield $\Psi^{+}_{\alpha}(\zeta, u)$

$$D^{+}_{\alpha}\Psi^{+}_{\beta} = \bar D^{+}_{\dot\alpha}\Psi^{+}_{\beta} = 0 \tag{26}$$

subject to the constraints

$$D^{++}_{\rm c}\Psi^{+}_{\alpha} = 0 \tag{27}$$

$$D^{-\alpha}\Psi^{+}_{\alpha} = \bar D^{-\dot\alpha}\breve\Psi^{+}_{\dot\alpha} \tag{28}$$

with $\breve\Psi^{+}_{\dot\alpha}$ the smile-conjugate of Ψ^{+}_{α}. Eq. (27) implies that in the central basis Ψ^{+}_{α} reads

$$\Psi^{+}_{\alpha} = \Psi_{\alpha i}(x, z, \theta)u^{+i} \qquad \breve\Psi^{+}_{\dot\alpha} = -\bar\Psi^{i}_{\dot\alpha}(x, z, \theta)u^{+}_{i} \tag{29}$$

for some u-independent superfields $\Psi_{\alpha i}$ and its complex conjugate $\bar{\Psi}^{i}_{\dot\alpha}$. Then, the analyticity conditions (26) are equivalent to

$$D^{(i}_{\alpha}\Psi^{j)}_{\beta} = \bar{D}^{(i}_{\dot\alpha}\Psi^{j)}_{\beta} = 0 \tag{30}$$

and the reality condition (28) takes the form

$$D^{\alpha i}\Psi_{\alpha i} = \bar{D}_{\dot\alpha i}\bar{\Psi}^{\dot\alpha i} \ . \tag{31}$$

Eqs. (30) and (31) constitute the constraints defining the field strengths of the free vector-tensor multiplet [8].

Using the anticommutation relations

$$\begin{gathered}\{D^+_\alpha, D^-_\beta\} = 2\mathrm{i}\,\varepsilon_{\alpha\beta}\partial_z \qquad \{\bar{D}^+_{\dot\alpha}, \bar{D}^-_{\dot\beta}\} = 2\mathrm{i}\,\varepsilon_{\dot\alpha\dot\beta}\partial_z \\ \{D^+_\alpha, \bar{D}^-_{\dot\beta}\} = -\{D^-_\alpha, \bar{D}^+_{\dot\beta}\} = -2\mathrm{i}\,\partial_{\alpha\dot\beta}\end{gathered} \tag{32}$$

one immediately deduces from (26) and (28) generalized Dirac equations

$$\partial_z\Psi^+_\alpha = -\partial_{\alpha\dot\beta}\breve{\Psi}^{+\dot\beta} \qquad \partial_z\breve{\Psi}^+_{\dot\alpha} = \partial_{\beta\dot\alpha}\Psi^{+\beta} \tag{33}$$

and hence

$$\partial_z^2\Psi^+_\alpha = \Box\Psi^+_\alpha \ . \tag{34}$$

The last relation can be also obtained from (26) and (27), in complete analogy to the Fayet-Sohnius hypermultiplet. We read eqs. (33) and (34) as a definition of the central charge. If one had not allowed for a central charge then the constraints (26)–(28) would have restricted the multiplet to be on-shell.

The super Lagrangian associated with the vector-tensor multiplet reads

$$\mathcal{L}^{++}_{\mathrm{vt,free}} = -\frac{1}{4}\left(\Psi^{+\alpha}\Psi^+_\alpha - \breve{\Psi}^+_{\dot\alpha}\breve{\Psi}^{+\dot\alpha}\right) \ . \tag{35}$$

Under central charge transformations it changes by derivatives

$$\partial_z\mathcal{L}^{++}_{\mathrm{vt,free}} = -\frac{1}{2}\partial_{\alpha\dot\alpha}\left(\Psi^{+\alpha}\breve{\Psi}^{+\dot\alpha}\right) \ . \tag{36}$$

The functional

$$S_{\mathrm{vt,free}} = \int d\zeta^{(-4)}du\ \left((\theta^+)^2 - (\bar\theta^+)^2\right)\mathcal{L}^{++}_{\mathrm{vt,free}} \tag{37}$$

can be seen to coincide with the action given in [8]. Another possible structure

$$\mathcal{L}^{++}_{\mathrm{der,free}} = -\frac{\mathrm{i}}{4}\left(\Psi^{+\alpha}\Psi^+_\alpha + \breve{\Psi}^+_{\dot\alpha}\breve{\Psi}^{+\dot\alpha}\right) \tag{38}$$

produces a total derivative when integrated over the superspace.

The constraints (26)–(28) can be partially solved in terms of a real u-independent potential $L(x,z,\theta)$

$$\Psi^+_\alpha = \mathrm{i}\, D^+_\alpha L \qquad\qquad \bar{L} = L \tag{39}$$

which is still restricted by

$$D^+_\alpha D^+_\beta L = D^+_\alpha \bar{D}^+_{\dot\beta} L = 0\ . \tag{40}$$

If one includes coupling to the $N=2$ Yang-Mills gauge superfield, described by the covariantly chiral strength W and its conjugate $\bar{W}$ [13], the constraints (40) can be consistently deformed as follows [9]

$$\mathcal{D}^{+\alpha}\mathcal{D}^+_\alpha \mathbb{L} = \kappa\, \mathrm{tr}\, \left(\bar{\mathcal{D}}^+_{\dot\alpha}\bar{W}\cdot \bar{\mathcal{D}}^{+\dot\alpha}\bar{W}\right) \tag{41}$$

$$\mathcal{D}^+_\alpha \bar{\mathcal{D}}^+_{\dot\beta}\mathbb{L} = -\kappa\, \mathrm{tr}\, \left(\mathcal{D}^+_\alpha W \cdot \bar{\mathcal{D}}^+_{\dot\beta}\bar{W}\right) \tag{42}$$

where $\mathbb{L}$ is a real u-independent gauge invariant superfield while W is invariant under the central charge. Such a deformation corresponds in particular to the Chern-Simons coupling of the antisymmetric tensor field, contained in the vector-tensor multiplet, to the Yang-Mills gauge field.

The independent components of the vector-tensor multiplet can be chosen as

$$\begin{gathered}
\Phi = \mathbb{L}\,| \qquad D = \partial_z \mathbb{L}\,| \\
\psi^i_\alpha = \mathcal{D}^i_\alpha \mathbb{L}\,| \qquad \bar\psi_{\dot\alpha i} = \bar{\mathcal{D}}_{\dot\alpha i}\mathbb{L}\,| \\
G_{\alpha\beta} = \tfrac{1}{2}[\mathcal{D}_{\alpha i}\,, \mathcal{D}^i_\beta]\,\mathbb{L}\,| \qquad \bar{G}_{\dot\alpha\dot\beta} = -\tfrac{1}{2}[\bar{\mathcal{D}}_{\dot\alpha i}\,, \bar{\mathcal{D}}^i_{\dot\beta}]\,\mathbb{L}\,| \\
H_{\alpha\dot\alpha} = \bar{H}_{\alpha\dot\alpha} = -\tfrac{1}{2}[\mathcal{D}^i_\alpha\,,\, \bar{\mathcal{D}}_{\dot\alpha i}]\,\mathbb{L}\,|
\end{gathered} \tag{43}$$

while the components of the vector multiplet are

$$\begin{gathered}
X = W\,| \qquad \bar{X} = \bar{W}\,| \\
\lambda^i_\alpha = \mathcal{D}^i_\alpha W\,| \qquad \bar\lambda_{\dot\alpha i} = \bar{\mathcal{D}}_{\dot\alpha i}\bar{W}\,| \\
F_{\alpha\beta} = -\tfrac{1}{4}[\mathcal{D}_{\alpha i}\,, \mathcal{D}^i_\beta]W\,| \qquad \bar{F}_{\dot\alpha\dot\beta} = \tfrac{1}{4}[\bar{\mathcal{D}}_{\dot\alpha i}\,, \bar{\mathcal{D}}^i_{\dot\beta}]\bar{W}\,| \\
Y^{ij} = -\tfrac{\mathrm{i}}{4}(\mathcal{D}^{\alpha i}\mathcal{D}^j_\alpha W + \bar{\mathcal{D}}^i_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha j}\bar{W})\,|
\end{gathered} \tag{44}$$

with F_{mn} the field strength associated with the Yang-Mills gauge field A_m. The fields H_m and G_{mn} are subject to the constraints

$$\begin{aligned}
\partial_m H^m &= \kappa\, \mathrm{tr}\,\{F^{mn}\tilde{F}_{mn} - \frac{1}{2}\partial_m(\lambda^i\sigma^m\bar\lambda_i)\} \\
\partial_m \tilde{G}^{mn} &= 2\kappa\, \partial_m\, \mathrm{tr}\,\{(X+\bar{X})\tilde{F}^{mn} + \frac{\mathrm{i}}{4}\lambda^i\sigma^{mn}\lambda_i + \frac{\mathrm{i}}{4}\bar\lambda_i\bar\sigma^{mn}\bar\lambda^i\}
\end{aligned} \tag{45}$$

which can be solved in terms of an antisymmetric tensor B_{mn} and a vector V_m

$$
\begin{aligned}
H^m &= \varepsilon^{mnkl}\partial_n B_{kl} + \kappa\,\mathrm{tr}\,\{\varepsilon^{mnkl}(A_n F_{kl} - \frac{2}{3}A_n A_k A_l) - \frac{1}{2}\lambda^i\sigma^m\bar{\lambda}_i\} \\
G_{mn} &= \partial_m V_n - \partial_n V_m + 2\kappa\,\mathrm{tr}\,\{(X+\bar{X})F_{mn} + \frac{1}{4}\lambda^i\sigma_{mn}\lambda_i - \frac{1}{4}\bar{\lambda}_i\bar{\sigma}_{mn}\bar{\lambda}^i\}\ . \qquad (46)
\end{aligned}
$$

Because of the constraints (41) and (42), the superfield $\mathcal{D}^+_\alpha\mathbb{L}$ is no longer analytic. But also the Lagrangians (35) and (38) can be deformed to obtain supersymmetric actions with Chern-Simons interactions. Similarly to Ref. [9], let us introduce the following real superfield

$$
\Sigma = \mathbb{L} - \frac{\kappa}{2}\,\mathrm{tr}\,\left(W - \bar{W}\right)^2\ . \qquad (47)
$$

Using the Bianchi identities [13, 14]

$$
\bar{\mathcal{D}}^+_{\dot\alpha}W = 0 \qquad (\mathcal{D}^+)^2 W = (\bar{\mathcal{D}}^+)^2\bar{W} \qquad (48)
$$

one can prove the important identities

$$
\begin{aligned}
\mathcal{D}^+_\alpha\bar{\mathcal{D}}^+_{\dot\beta}\Sigma &= 0 \qquad (49)\\
(\mathcal{D}^+)^2\Sigma &= -(\bar{\mathcal{D}}^+)^2\Sigma \\
&= \frac{\kappa}{2}\left\{(\bar{\mathcal{D}}^+)^2\mathrm{tr}\,\left(\bar{W}^2\right) - (\mathcal{D}^+)^2\mathrm{tr}\,\left(W^2\right)\right\}\ . \qquad (50)
\end{aligned}
$$

Therefore, the imaginary superfield

$$
\mathcal{L}^{++}_{\rm vt} = \frac{1}{4}\left\{\mathcal{D}^{+\alpha}\Sigma\mathcal{D}^+_\alpha\Sigma + \Sigma(\mathcal{D}^+)^2\Sigma - \bar{\mathcal{D}}^+_{\dot\alpha}\Sigma\bar{\mathcal{D}}^{+\dot\alpha}\Sigma\right\} \qquad (51)
$$

satisfies both the constraints (6) and (7), and therefore can be used to construct a supersymmetric action. The corresponding action functional obtained by the rule (9) describes the Chern-Simons coupling of the vector-tensor multiplet to the $N=2$ gauge multiplet. It was first derived in component approach [4] and then in $N=2$ superspace [9]. We give only the bosonic part of the component Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{\rm vt} &= \frac{1}{2}\partial^m\Phi\,\partial_m\Phi - \frac{1}{2}H^m H_m - \frac{1}{4}G^{mn}G_{mn} + \frac{1}{2}D^2 \\
&\quad + \mathrm{i}\kappa\, G^{mn}\,\mathrm{tr}\,\{(X-\bar{X})\tilde{F}_{mn}\} - \mathrm{i}\kappa\, H^m\,\mathrm{tr}\,\{(X-\bar{X})\mathcal{D}_m(X+\bar{X})\} \\
&\quad - 2\kappa\left(\Phi - \frac{\kappa}{2}\mathrm{tr}\,(X-\bar{X})^2\right)\mathrm{tr}\,\{\mathcal{D}^m\bar{X}\,\mathcal{D}_m X - \frac{1}{2}F^{mn}F_{mn} - \frac{1}{2}Y^{ij}Y_{ij} + \frac{1}{4}[X,\bar{X}]^2\} \\
&\quad + 2\kappa^2\,\mathrm{tr}\,\{(X-\bar{X})\mathcal{D}^m\bar{X}\}\,\mathrm{tr}\,\{(X-\bar{X})\mathcal{D}_m X\} \\
&\quad - \kappa^2\,\mathrm{tr}\,\{(X-\bar{X})F^{mn}\}\mathrm{tr}\,\{(X-\bar{X})F_{mn}\} \\
&\quad - \kappa^2\,\mathrm{tr}\,\{(X-\bar{X})Y^{ij}\}\,\mathrm{tr}\,\{(X-\bar{X})Y_{ij}\} + \text{fermionic terms}\ . \qquad (52)
\end{aligned}
$$

Now, we generalize the total derivative Lagrangian (38) (which is an $N=2$ analog of $\tilde{F}F$ or θ-term). Similar to [9], we introduce the real superfield

$$\Omega = \mathbb{L} + \frac{\kappa}{2}\,\mathrm{tr}\,\left(W+\bar{W}\right)^2 \ . \tag{53}$$

Its properties read

$$\mathcal{D}^+_\alpha \bar{\mathcal{D}}^+_{\dot\beta}\Omega = 0 \tag{54}$$

$$\begin{aligned}(\mathcal{D}^+)^2\Omega &= (\bar{\mathcal{D}}^+)^2\Omega \\ &= \frac{\kappa}{2}\left\{(\bar{\mathcal{D}}^+)^2\mathrm{tr}\,\left(\bar{W}^2\right) + (\mathcal{D}^+)^2\mathrm{tr}\,\left(W^2\right)\right\} \ .\end{aligned} \tag{55}$$

As a consequence, the imaginary superfield

$$\mathcal{L}^{++}_{\mathrm{der}} = \frac{\mathrm{i}}{4}\left\{\mathcal{D}^{+\alpha}\Omega\mathcal{D}^+_\alpha\Omega + \Omega(\mathcal{D}^+)^2\Omega + \bar{\mathcal{D}}^+_{\dot\alpha}\Omega\bar{\mathcal{D}}^{+\dot\alpha}\Omega\right\} \tag{56}$$

respects both the constraints (6) and (7) and therefore defines a supersymmetric action.

The Lagrangian (56) is the deformation of (38). It is therefore a deformation of the Chern-Simons form $\tilde{F}F$ which carries topological information. In components, the bosonic Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\mathrm{der,bos}} = \partial_m\Big[&\left(\Phi + \frac{\kappa}{2}\mathrm{tr}\,(X+\bar{X})^2\right)\left(H^m - \mathrm{i}\kappa\,\mathrm{tr}\,\{(X+\bar{X})\mathcal{D}^m(X-\bar{X})\}\right) \\ &+\frac{1}{2}\varepsilon^{mnkl}V_n\partial_k V_l\Big] \ .\end{aligned} \tag{57}$$

and contains total derivative terms only.

In summary, in the present paper we have described the vector-tensor multiplet and its Chern-Simons coupling to the $N=2$ gauge multiplet in harmonic superspace. It would be of interest to find an unconstrained prepotential superfield formulation for the vector-tensor multiplet, which may exist, similar to the Fayet-Sohnius hypermultiplet, in harmonic superspace only.

After this work had appeared on the hep-th archive we became aware of a recent paper [16] where the authors presented a two-form formulation of the vector-tensor multiplet in central charge superspace and derived its coupling to the non-Abelian supergauge multiplet via the Chern-Simons form. The later paper is a natural development of the research started in [8].

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