LÖSUNGEN zur Klausur am 28. Januar 2000

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1) $t_1 = \frac{a}{c} + \frac{x_1}{c}$, $\begin{pmatrix} a+x_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} 3a \\ 0 \end{pmatrix}$, d.h. $\begin{array}{r} a+x_1 = 3\gamma a \\ x_1 = 3\beta\gamma a \end{array}$ $\curvearrowright$
$3(1-\beta) = \frac{1}{\gamma} = \sqrt{(1-\beta)(1+\beta)}$, $9(1-\beta) = 1+\beta$: $\beta = \frac{4}{5}$, $\gamma = \frac{5}{3}$ und $x_1 = 4a$.

2) $\vec{E}' = \begin{pmatrix} 0 \\ 0 \\ \gamma(E+\beta b) \end{pmatrix}$, $c\vec{B}' = \begin{pmatrix} 0 \\ \gamma(b+\beta E) \\ 0 \end{pmatrix}$ Na! Ich weiß doch, daß dies eine Invariante ist. Aber per Rechnung folgt es ebenfalls :
[↑ Rel 20 , (65)] $\vec{E}'^2 - c^2 \vec{B}'^2 = \gamma^2 \left[(E+\beta b)^2 - (\beta E + b)^2 \right] = E^2 - b^2$.

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3) $\left[-\left(\partial_\rho^2 + \frac{1}{\rho}\partial_\rho + \frac{1}{\rho^2}\partial_\varphi^2 + \partial_z^2 \right) + \alpha\rho^2 \right] e^{im\varphi} e^{ikz} \varphi(\rho) = E \, e^{im\varphi} e^{ikz} \varphi(\rho)$
$\curvearrowright \quad [\ \ ?\ \ ] = -\partial_\rho^2 - \frac{1}{\rho}\partial_\rho + \alpha\rho^2 + \frac{m^2}{\rho^2} + k^2$.

4) Links und rechts geradlinig , innen : $\left[-\partial_x^2 - \kappa^2 \right] \varphi = 0$, $\varphi_{\text{innen}} = A\cos(\kappa x)$,
$\varphi_{\text{rechts}} = B(b-x)$, $\begin{array}{r} A\cos(\kappa a) = B(b-a) \\ -A\kappa\sin(\kappa a) = -B \end{array}$, $\frac{\text{obere}}{-\text{untere}}$ $\curvearrowright$ $b = a + \frac{1}{\kappa}\cot(\kappa a)$.

5) $\widetilde{V} = \int d^3r \, e^{\cdots} V$: $[\,\widetilde{V}\,] = Vol \cdot E = \frac{Vol \cdot \hbar}{Zeit}$, $[\,\sigma\,] = \left(\frac{m}{\hbar} \frac{\widetilde{V}}{\hbar} \right)^2 = \left(\frac{m}{m\ell v} \ell^2 v \right)^2 = \mathit{Fläche}$.

6) $\widetilde{V} = \alpha \int_0^\infty dr \, r^2 \, \delta(r-R) \, 2\pi \int_{-1}^1 du \, e^{ikru} = 4\pi\alpha R \, \frac{\sin(kR)}{k}$.
Dies wird bei $k \to 0$ zu $4\pi\alpha R^2$ — und $\int d^3r \, \alpha \, \delta(r-R) = 4\pi\alpha R^2$ ebenfalls.

7) $\begin{pmatrix} \frac{7}{9} & \frac{4}{9} + i\frac{4}{9} \\ \frac{4}{9} - i\frac{4}{9} & -\frac{7}{9} \end{pmatrix} \begin{pmatrix} 2+2i \\ 1 \end{pmatrix} = \ldots = \begin{pmatrix} 2+2i \\ 1 \end{pmatrix}$; $\alpha = \frac{1}{3}$, $\vec{\chi}(t) = e^{-i\omega t \sigma_3}$ ·
$\cdot \left[\begin{pmatrix} \frac{2}{3}(1+i) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix} \right] = \begin{pmatrix} \frac{2}{3}(1+i)e^{-i\omega t} \\ \frac{1}{3}e^{i\omega t} \end{pmatrix}$, $P_+ = \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \vec{\chi}(t) \right|^2 = \frac{8}{9}$, $P_- = \frac{1}{9}$.

8) $H = -\frac{\hbar^2}{2\theta}\partial_\varphi^2$, $\overline{\underline{=}-}\ \overline{\underline{=}-}$, $\eta = 4$, $\varepsilon_F = \frac{\hbar^2}{2\theta} \cdot 4$, $E_0 = 0 + 4 \cdot \frac{\hbar^2}{2\theta} + \frac{\hbar^2}{2\theta} \cdot 4 = 4\,\frac{\hbar^2}{\theta}$.

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9) $p_1 = \frac{1}{2}$, $p_2 = p_4 = p_6 = \frac{1}{6}$, $S = \frac{1}{2}\ln(2) + \frac{1}{2}\ln(6)$, $\langle\, \nu \,\rangle = \frac{1}{2} \cdot 1 + \frac{1}{6}(2+4+6) = 2.5$.

10) $Z = \sum_{m=0}^\infty \eta_m \, e^{-\beta\frac{\hbar^2 m^2}{2\theta}}$ mit $\eta_0 = 2$ und $\eta_{m\geqslant 1} = 4$, $Z = 2 + 4\,e^{-\beta\hbar^2/(2\theta)}$;
$Z \to 4\int_0^\infty dm \, e^{-\beta\frac{\hbar^2 m^2}{2\theta}} = \frac{1}{\sqrt{\beta}} \cdot \text{const}_T$, $E = -\partial_\beta \frac{1}{2} \ln\left(\frac{1}{\beta}\right) = \frac{1}{2}T$ IST ja auch ein translatorischer Freiheitsgrad .

11) $E = -\partial_\beta \, 3N \ln(1/\beta) = 3NT$, $F = -T\,N\left(\ln(V) + \text{const}_V\right)$, $p = -\partial_V F = NT/V$,
$pV = NT = \frac{1}{3}E$. $\xi = \frac{1}{3}$ (statt $\frac{2}{3}$ für „klassisch–ideales Gas") ist typisch für ultrarelativistische Materie.
