LÖSUNGEN

1) $\lambda(-b,h)+\eta(a,h)=(0,mg)\;,\quad \lambda=\frac{a}{b}\eta\;,\quad \eta\Big[\frac{a}{b}+1\Big]h=mg\;,\quad \vec{K}=\frac{mg\,b}{(a+b)h}(a,h)$

2) $\vec{r}_0=(\,R+Rc,Rs)\;,\quad \vec{r}_{\mathrm{Quelle-Spalt}}=\vec{r}_0-(a,0)=(R+Rc-a,Rs)\;,$

$$\tan(\varphi)=\frac{Rs}{R+Rc-a}=:\frac{Rs}{N}\;,\quad \varphi=\arctan\left(\frac{Rs}{N}\right)\;,\quad \dot\varphi=\frac{\frac{R\omega c}{N}-\frac{Rs}{N^2}(-R\omega s)}{1+(Rs/N)^2}$$

$$=\omega\,\frac{Rc(R+Rc-a)+R^2s^2}{(R+Rc-a)^2+R^2s^2}=\omega\,\frac{R^2(c+1)-Rac}{2R^2(c+1)-2aR(c+1)+a^2}\;;\quad a=0\;:\;\dot\varphi=\frac{\omega}{2}$$

3) $\begin{pmatrix}a\\a'\end{pmatrix}=\begin{pmatrix}\gamma&-\beta\gamma\\-\beta\gamma&\gamma\end{pmatrix}\begin{pmatrix}ct_1\\a\end{pmatrix}=\begin{pmatrix}\gamma\,ct_1-\beta\gamma a\\-\beta\gamma ct_1+\gamma a\end{pmatrix}\;,\; a=\gamma ct_1-\beta\gamma a\;,\; ct_1=a\Big(\frac{1}{\gamma}+\beta\Big)$

$$a'=-\beta\gamma a\left(\frac{1}{\gamma}+\beta\right)+\gamma a=a\left(\frac{1}{\gamma}-\beta\right)\;,\quad \gamma=\frac{5}{4}\;,\quad a'=\frac{a}{5}\;,\quad ct_1=\frac{7}{5}a$$

4) $v=Ae^{-\alpha t}+Be^{\beta t}\;,$ Bew.gl.: $B=\frac{k_0}{\alpha+\beta}\;,$ Anf.b.: $A+B=0\;,\quad v=\frac{k_0}{\alpha+\beta}(e^{\beta t}-e^{-\alpha t})$

$\Big\{$ODER: $v=e^{-\alpha t}u\,,\;\boxed{\dot u=k_0e^{(\alpha+\beta)t}\,,\;u(0)=0}\,,\;u=\frac{k_0}{\alpha+\beta}\,\left[e^{(\alpha+\beta)t}-1\right]\Big\}\;,\quad v=k_0t+O(t^2)$

5) Drehimpulserhaltung: $r_1v_1=r_0v_0=r_02v_1 \;\curvearrowright\; r_1=2r_0=2\ell\;,$ Energiesatz:

$$\frac{m}{2}v_1^2+\frac{\kappa}{2}(r_1-\ell)^2=\frac{m}{2}(2v_1)^2+\frac{\kappa}{2}\cdot 0\;,\quad 3v_1^2=\frac{\kappa}{m}\ell^2\;,\quad v_1=\ell\sqrt{\frac{\kappa}{3m}}$$

6) (a) $\vec{v}=a\omega(1+\omega t)\,(2,1,0)\;,\quad m\dot{\vec{v}}=ma\omega^2(2,1,0)\stackrel{!}{=}q(0,E,0)+qB(t)(v_2,-v_1,0)$

$=(\,qBa\omega(1+\omega t)\,,\;qE-qBa\omega 2(1+\omega t)\,)\;,\quad qB=\frac{2m\omega}{1+\omega t}\;,\quad qE=5ma\omega^2$

(b) Bew.gl. $\cdot\,\vec{v}$: $\left(\frac{m}{2}v^2\right)^\bullet=q\vec{E}\;\vec{v}=qEv_2=5m\,a^2\,\omega^3\,(1+\omega t)\;,$

Kontrolle : $\frac{m}{2}v^2=\frac{m}{2}a^2\omega^2(1+\omega t)^2\cdot 5\;,\quad \dot T=m\,a^2\omega^3(1+\omega t)\,5$ — es stimmt.

7) $(2-\lambda)(5-\lambda)-4=\lambda^2-7\lambda+6=0\;,\quad \lambda=\frac{7}{2}\pm\frac{1}{2}\sqrt{49-24}\;,\quad \lambda=1,\;6$

$$\begin{pmatrix}1&-2\\-2&4\end{pmatrix}\vec{f}_1=0\;:\;\vec{f}_1=\frac{1}{\sqrt5}\begin{pmatrix}2\\1\end{pmatrix}\;,\quad \begin{pmatrix}-4&-2\\-2&1\end{pmatrix}\vec{f}_2=0\;:\;\vec{f}_2=\frac{1}{\sqrt5}\begin{pmatrix}-1\\2\end{pmatrix}\;,$$

$$D=\frac{1}{\sqrt5}\begin{pmatrix}2&1\\-1&2\end{pmatrix}\;,\quad \vec{a}'=D\vec{a}=a\begin{pmatrix}1\\0\end{pmatrix}\;,\quad \ddot{\vec{r}}'=-\omega^2\begin{pmatrix}1&0\\0&6\end{pmatrix}\begin{pmatrix}x'\\y'\end{pmatrix}+\omega^2a\begin{pmatrix}1\\0\end{pmatrix}$$

$\ddot y'=-6\omega^2y'\;,\quad \ddot x'=-\omega^2\,(x'-a)\;,$ alle Anf.bedn. Null : $\vec{r}'(t)=\begin{pmatrix}a-a\cos(\omega t)\\0\end{pmatrix}$

8) (a) $\dot v^{(0)}+\dot v^{(1)}=g\,\left(1-\alpha v^{(0)}\right)\;,\quad \boxed{\dot v^{(0)}=g\;,\;v^{(0)}(0)=0}\;\curvearrowright\; v^{(0)}=gt\;,$

$\boxed{\dot v^{(1)}=-g\alpha v^{(0)}=-g^2\alpha t\;,\;v^{(1)}(0)=0}\;\curvearrowright\; v^{(1)}=-\frac{1}{2}g^2\alpha t^2\;,\quad v=gt-\frac{1}{2}g^2\alpha t^2$

(b) $(1+\alpha v)\dot v=(v+\frac{\alpha}{2}v^2)^\bullet=(gt)^\bullet\;,\quad v+\frac{\alpha}{2}v^2=gt+C=gt\;,\quad v^2+\frac{2}{\alpha}v-\frac{2}{\alpha}gt=0$

$v=-\frac{1}{\alpha}+\frac{1}{\alpha}\sqrt{1+2\alpha gt}=\frac{1}{\alpha}\left(-1+1+\alpha gt-\frac{1}{8}(2\alpha gt)^2\right)=$ das (a)–Resultat

9) Zähler $=x-x-\frac{x^2}{2}+\ldots\to-\frac{x^2}{2}\;,$ Nenner $=\mathrm{sh}\left(1-\left[1+\frac{x^2}{2}\right]\right)\to-\frac{x^2}{2}\;,\quad J=1$