1) $K(0)=0$ und $K(\frac{\pi}{2})=\sqrt{2}\,mg$ lt. Skizze , $\vec{0}=(0,-mg)+F(s,c)+K\,\dfrac{(-s,1-c)}{\sqrt{2-2c}}$
1. Komp.: $F=K/\sqrt{\quad}$, 2. Komp.: $mg=Fc+K(1-c)/\sqrt{\quad}$, $K=mg\sqrt{2-2c}$, $\sqrt{3}=\sqrt{2-2c_0}$, $c_0=-1/2$, $\varphi_0=2\pi/3$ **.**

2) $\vec{r}_{\text{ohne}}=R\,(\,s\,,0\,,c\,)$ mit $\omega=v_0/R$, $\vec{r}=R\,(\,s\,C\,,\,s\,S\,,\,c\,)$, $\vec{v}=R\,(\,\omega\,c\,C-\Omega\,s\,S\,,\,\omega\,c\,S+\Omega\,s\,C\,,\,-\omega\,s\,)$, $\vec{v}(\omega t=\frac{\pi}{2})=R\,(-\Omega\,S\,,\,\Omega\,C\,,\,-\omega\,)$, $v^2=R^2\,(\Omega^2+\omega^2)$ **.**

3) $y=1+\dfrac{\varepsilon}{x}$, $f(x+\varepsilon)=f(x)\left[1+2\dfrac{\varepsilon}{x}-2\dfrac{\varepsilon}{x}\right]+x^2\,2\dfrac{\varepsilon}{x}$, $f'=2x$, $f=x^2+A$
$f(1)=0 \curvearrowright A=-1$ **.**

4) $x=A\,s+B\,\text{sh}+D\,t$, $\dot{x}=A\,\omega\,c+B\,\omega\,\text{ch}+D$, $\ddot{x}=-A\,\omega^2 s+B\,\omega^2\text{sh}\overset{!}{\equiv}-\omega^2 A\,s-\omega^2 B\,\text{sh}-\omega^2 D\,t+\alpha\,\omega\,t-2\,\alpha\,\text{sh} \curvearrowright D=\dfrac{\alpha}{\omega}$, $B=-\dfrac{\alpha}{\omega^2}$,
$\dot{x}(0)=A\omega+B\omega+D\overset{!}{=}v_0 \curvearrowright A=\dfrac{v_0}{\omega}$, $x=\dfrac{v_0}{\omega}\sin(\omega t)+\dfrac{\alpha}{\omega^2}\,[\,\omega t-\text{sh}(\omega t)\,]$ **.**

5) $-\dfrac{\gamma mM}{2\,R}=-\dfrac{\gamma mM}{R}+\dfrac{\kappa}{2}R^2+\dfrac{m}{2}v^2$, $v^2=\dfrac{2}{m}\,\dfrac{\gamma mM}{R}\left(1-\dfrac{1}{2}-\dfrac{1}{2}\cdot\dfrac{1}{2}\right)$, $v=\sqrt{\dfrac{\gamma M}{2R}}$,
$V(z)=\gamma mM\left[-\dfrac{1}{z}+\dfrac{1}{4R^3}\,(2R-z)^2\right]$, $-V'(z)=\gamma mM\left[-\dfrac{1}{z^2}+\dfrac{1}{2R^3}\,(2R-z)\right]$
$-V'(R)=-\dfrac{\gamma mM}{2\,R^2}$ **.**

6) $m(t)\,\dot{\vec{v}}=q\,B\,\vec{v}\times\vec{e}_3$, Ansatz : $\vec{v}=v_0\,(-c\,,\,s\,,\,0\,)$ mit
$c:=\cos\,[\,\varphi(t)\,]$, $s:=\sin\,[\,\varphi(t)\,]$, $m(t)\,\dot{\varphi}\,(s,c,0)=qB\,(-c,s,0)\times(0,0,1)=qB\,(s,c,0)$
$\curvearrowright \dot{\varphi}=qB/m=\dfrac{qB}{m_0}\,e^{\gamma t}$ und $\varphi(t)=\dfrac{qB}{m_0\gamma}\,(e^{\gamma t}-1)$ **.**

7) $H=\begin{pmatrix}1&0&0\\0&3&2\\0&2&6\end{pmatrix}$, $\lambda_1=1$, $(3-\lambda)(6-\lambda)-4=0$, $\lambda_2=2$, $\lambda_3=7$, $\vec{f}_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$
$\begin{pmatrix}1&2\\2&4\end{pmatrix}(\,)=\vec{0}$, $\vec{f}_2=\dfrac{1}{\sqrt{5}}\begin{pmatrix}0\\2\\-1\end{pmatrix}$, $\vec{f}_3=\vec{f}_1\times\vec{f}_2$, $H'=\begin{pmatrix}1&0&0\\0&2&0\\0&0&7\end{pmatrix}$, $D=\dfrac{1}{\sqrt{5}}\begin{pmatrix}\sqrt{5}&0&0\\0&2&-1\\0&1&2\end{pmatrix}$
$D\vec{r}(0)=5a\begin{pmatrix}0\\0\\1\end{pmatrix}$, m bleibt auf z'–Achse , $V=\alpha\left(x'^2+2y'^2+7z'^2\right)$, $\omega=\sqrt{\dfrac{14\,\alpha}{m}}$ **.**

8) $f=\left(1+\dfrac{s^2}{2}\right)\Big/\sqrt{4-4s^2}=\dfrac{1}{2}\left(1+\dfrac{s^2}{2}\right)\left(1+\dfrac{s^2}{2}\right)=\text{const}+\dfrac{s^2}{2}$, $\omega=\sqrt{\kappa/m}$ **.**

9) (a) $\dot{v}^{(0)}+\dot{v}^{(1)}+\dot{v}^{(2)}=-\lambda\,\left[\,v^{(0)\,3}+3\,v^{(0)\,2}v^{(1)}\,\right]$, $\boxed{\dot{v}^{(0)}=0\,,\ v^{(0)}(0)=v_0} \curvearrowright v^{(0)}\equiv v_0$
$\boxed{\dot{v}^{(1)}=-\lambda v_0^3\,,\ v^{(1)}(0)=0} \curvearrowright v^{(1)}=-\lambda v_0^3 t$, $\boxed{\dot{v}^{(2)}=-3\lambda v_0^2\,v^{(1)}=3\lambda^2 v_0^5 t\,,\ v^{(2)}(0)=0}$
$\curvearrowright v^{(2)}=3\lambda^2 v_0^5 t^2/2$. **(b)** $-\dfrac{2}{v^3}\dot{v}=\left(\dfrac{1}{v^2}\right)^{\bullet}=2\,\lambda$, $v^2=\dfrac{1}{A+2\lambda t}$, $v=\dfrac{v_0}{\sqrt{1+2\lambda v_0^2 t}}$
$(1+x)^\lambda=1+\lambda x+\frac{\lambda(\lambda-1)}{2}x^2+\dots$, $\lambda=-\frac{1}{2}$, $v=v_0\left[\,1-\dfrac{1}{2}\,2\lambda v_0^2 t+\dfrac{3}{8}\,(2\lambda v_0^2 t)^2+\dots\,\right]$ **.**

10) $x\to x+\pi$, $J=\displaystyle\int_{-\pi}^{\pi}dx\,(-s)\left(-c-s-2\pi x-2\pi^2+x^2+2\pi x+\pi^2\right)$,
$J=\displaystyle\int_{-\pi}^{\pi}dx\,\left(s^2+\text{ungerade}\right)$, $s^2\to\dfrac{1}{2}$, $J=\pi$ **.**