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LITTLE EXAM

You may consider this test as passed if you can collect about one third of the total number of points.

**Name:** \_\_\_\_\_

**Matrikelnummer:** \_\_\_\_\_

**Email:** \_\_\_\_\_

- [1] What is a symmetry? Why are symmetries useful properties of physical systems? (1+1 P)
- [2] Symmetries naturally have the mathematical structure of groups. What are the group axioms? (3 P)
- [3] What is the relation between a Lie group and its Lie algebra? (3 P)
- [4] Let a Lie group element  $g = \exp(X)$  be given in terms of a generator  $X$  of the corresponding Lie algebra. Express  $\det(g)$  in terms of  $\text{tr}(X)$ . (1 P)
- [5] Name the four types of the classical Lie groups. (4 P)
- [6] What is the Jacobi identity? What is its meaning? (1+1 P)
- [7] What is the adjoint representation of a Lie algebra? (2 P)
- [8] What is the Killing form? Which are the characterizing properties of the Killing form for a semi-simple Lie algebra and for a compact Lie algebra? (1+1+1 P)
- [9] What is the Cartan sub-algebra of a Lie algebra? Describe briefly, what the weights of a representation of a Lie algebra are. What are the roots of a Lie algebra? (1+2+1 P)
- [10] What is the master formula for a weight  $\mu$  with respect to a root  $\alpha$ ? What is its meaning? Argue, why two weights must always differ by integer valued linear combinations of roots. (2+2+2 P)
- [11] Say in words, why the angles between two roots  $\alpha, \beta$  are restricted such that  $4 \cos^2 \angle(\alpha, \beta) \in \{0, 1, 2, 3\}$ . (3 P)
- [12] What are the simple roots of a Lie algebra? (2 P)
- [13] A Lie algebra is completely characterized by the set of its simple roots. Explain briefly, what the Dynkin diagram of a Lie algebra is. (3 P)
- [14] What are the fundamental representations of a Lie algebra? How many fundamental representations does a rank  $r$  Lie algebra have? (3 P)
- [15] Name up to four physical systems or physical theories together with their symmetry groups. (4 P)

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(45 P)