Logarithmic Conformal Field Theory

Michael Flohr

22 May 2008 - NUI Maynooth

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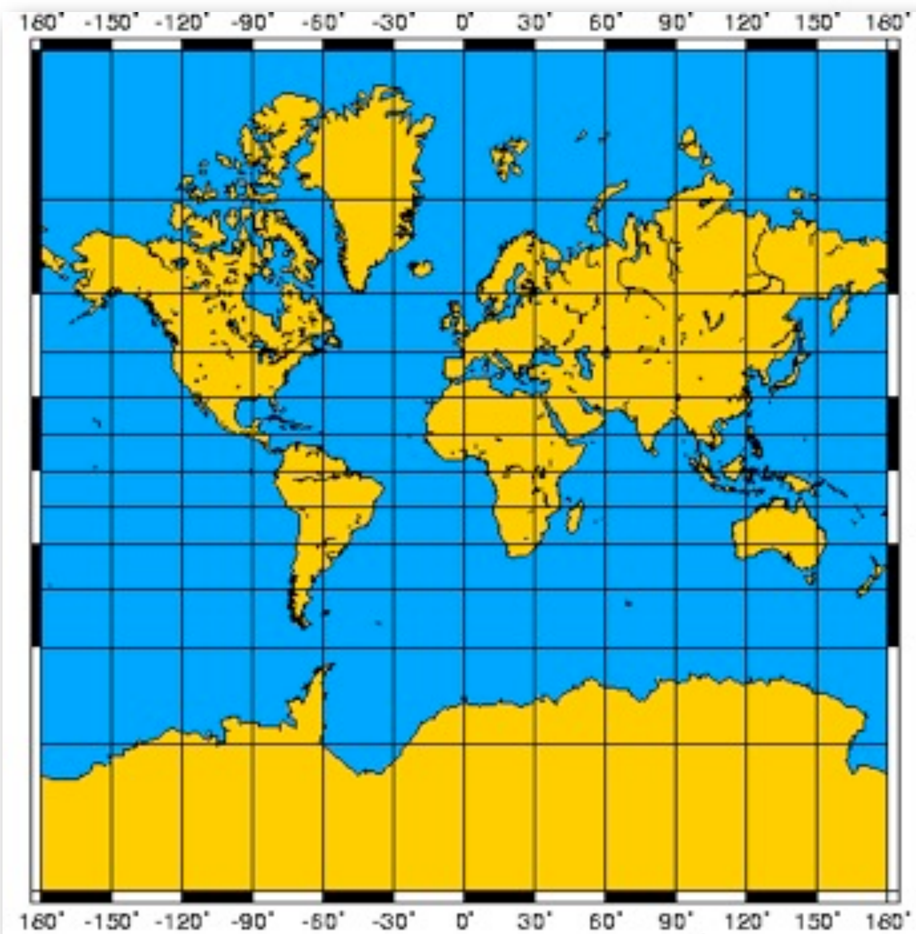

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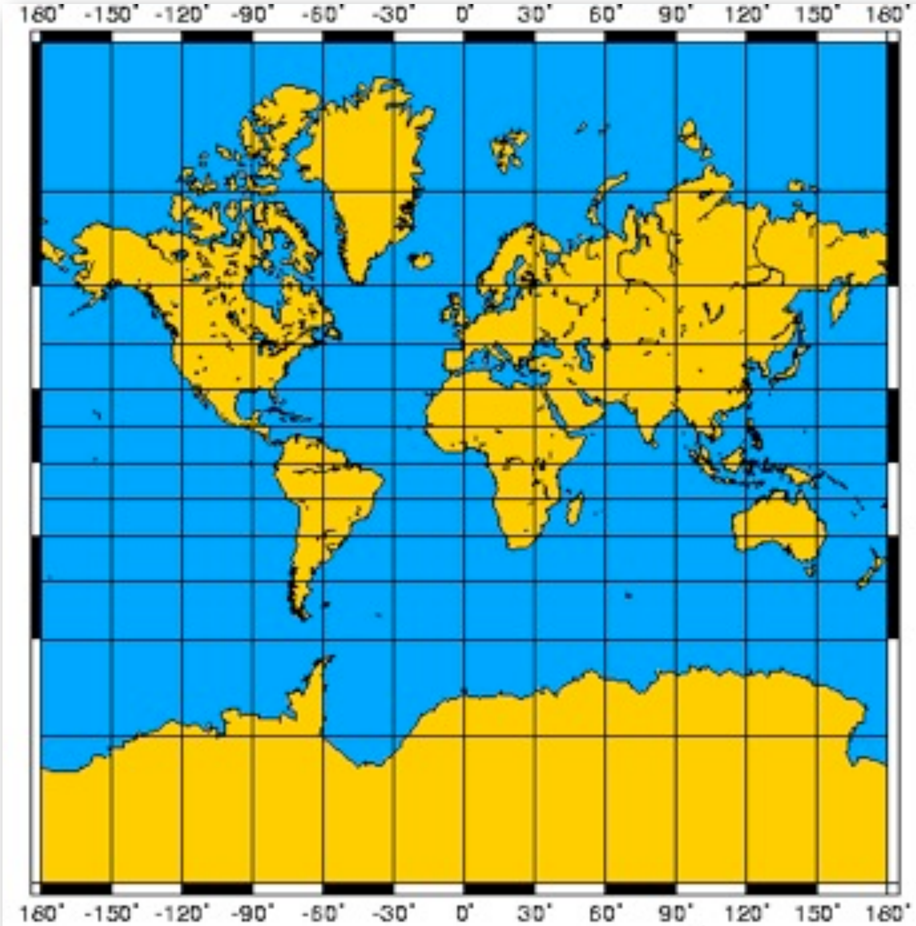

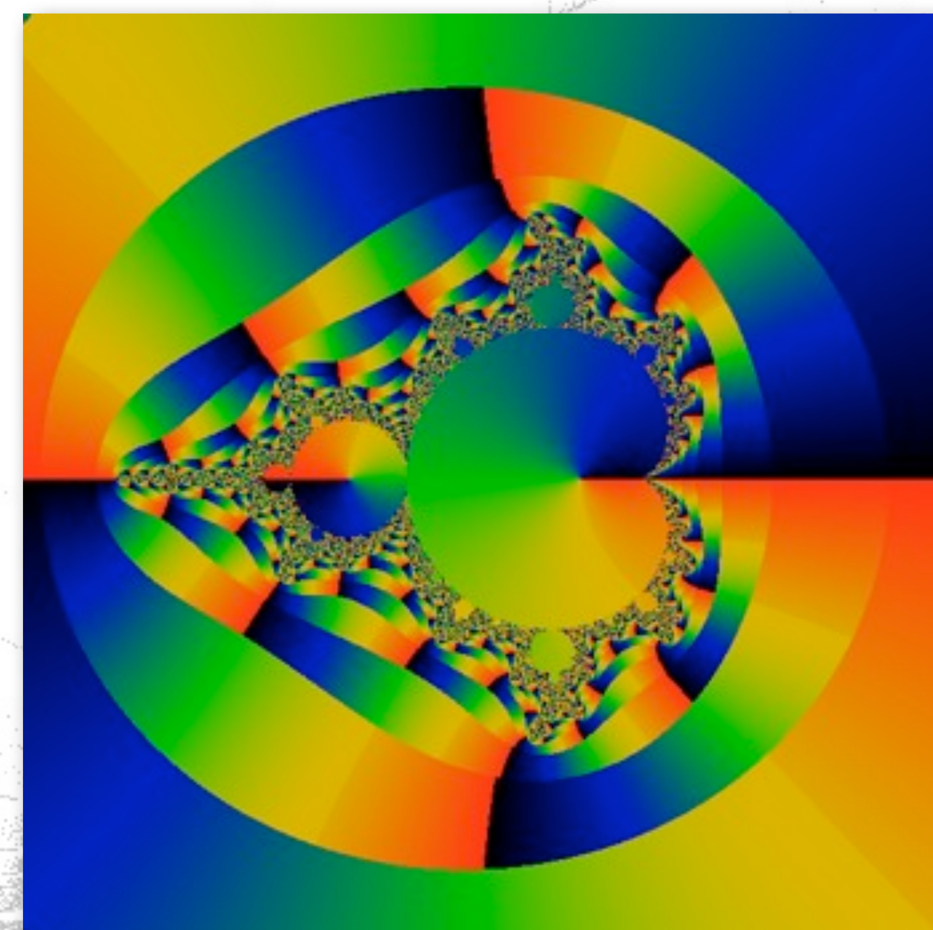

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- Important problem: classification of fusion algebras

$$[h_1, c] \star [h_2, c] = \sum_h N_{h_1 h_2}^{\ \ h}[h, c]\,, \quad N_{h_1 h_2}^{\ \ h} \in \mathbb{Z}_+$$

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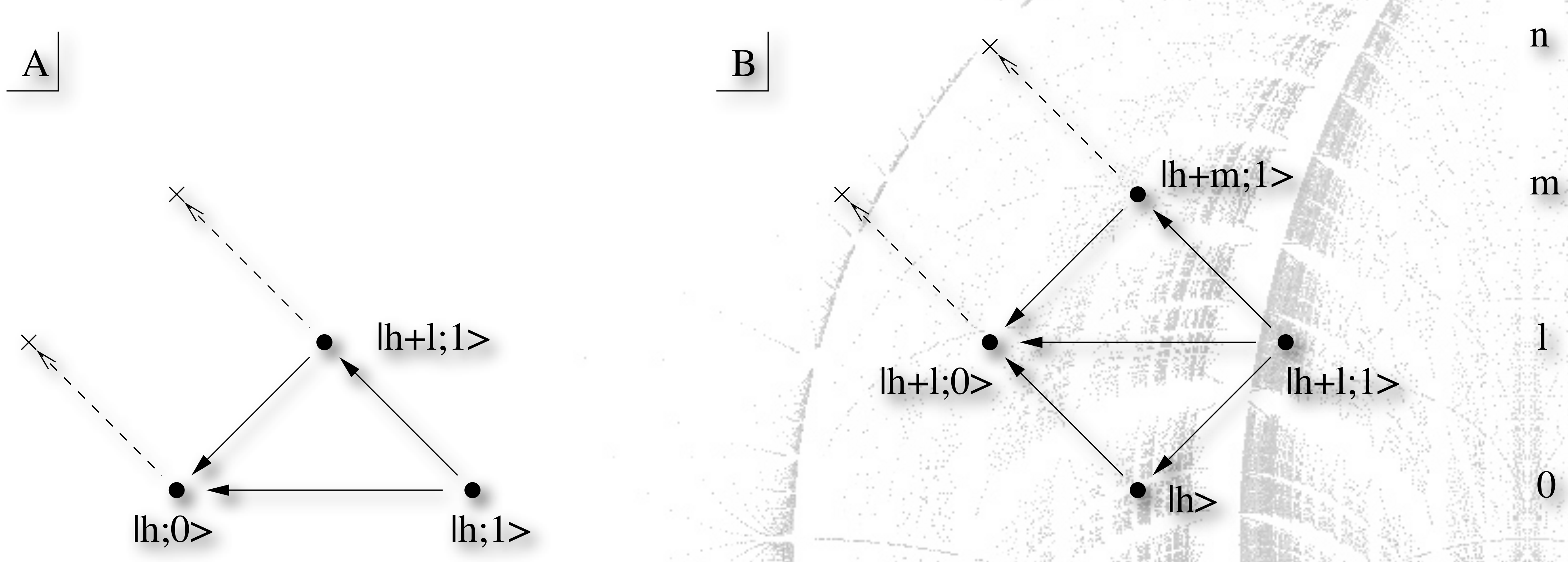

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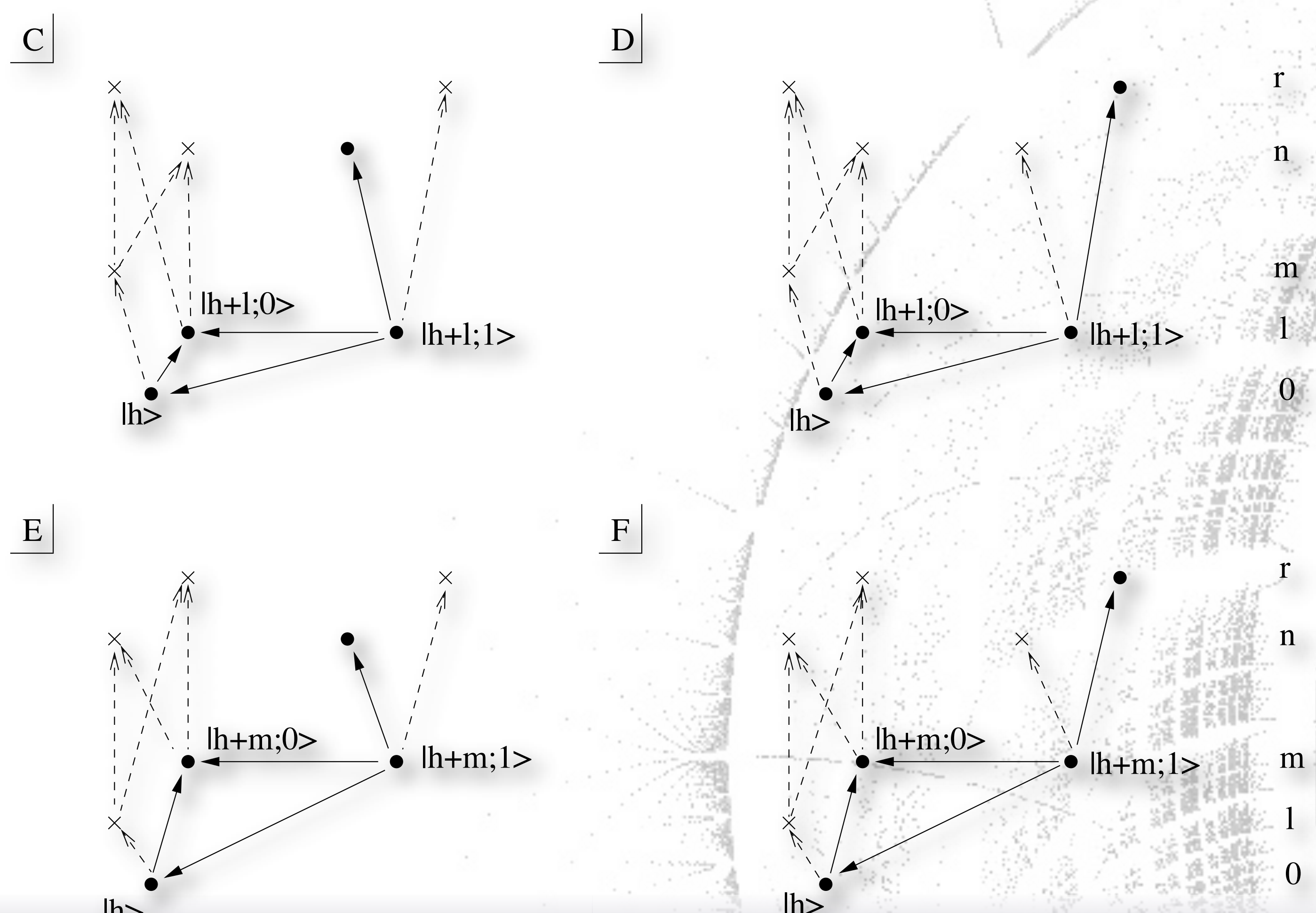

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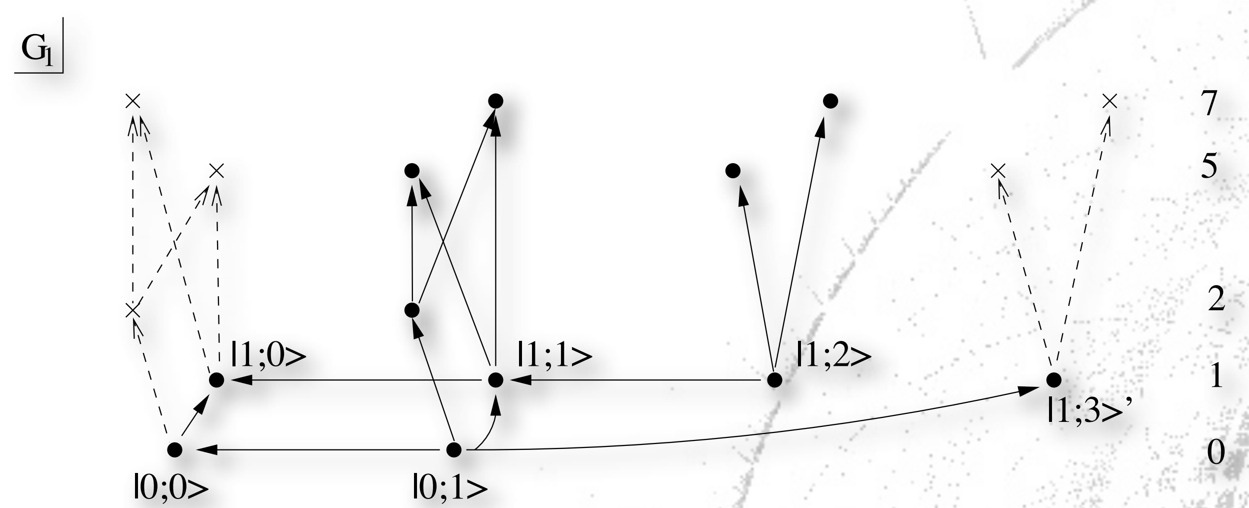

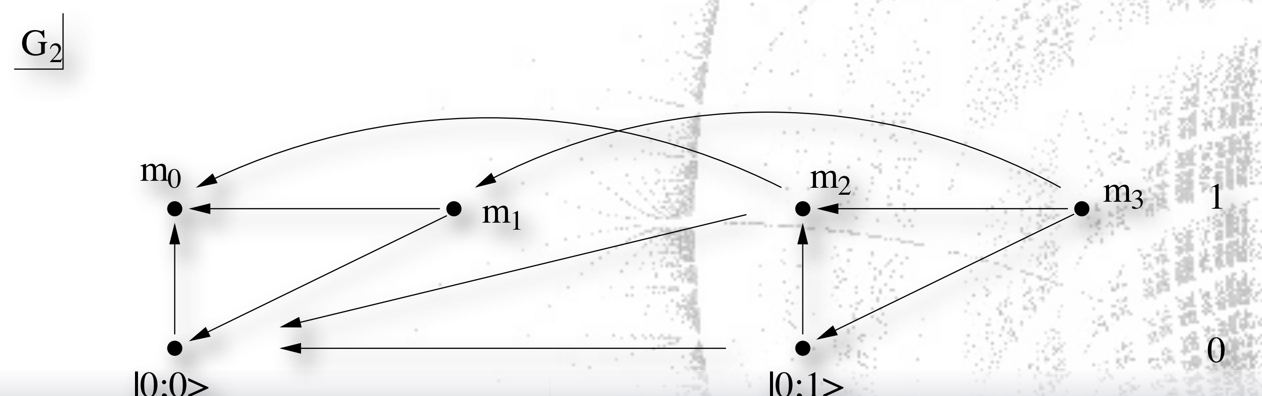

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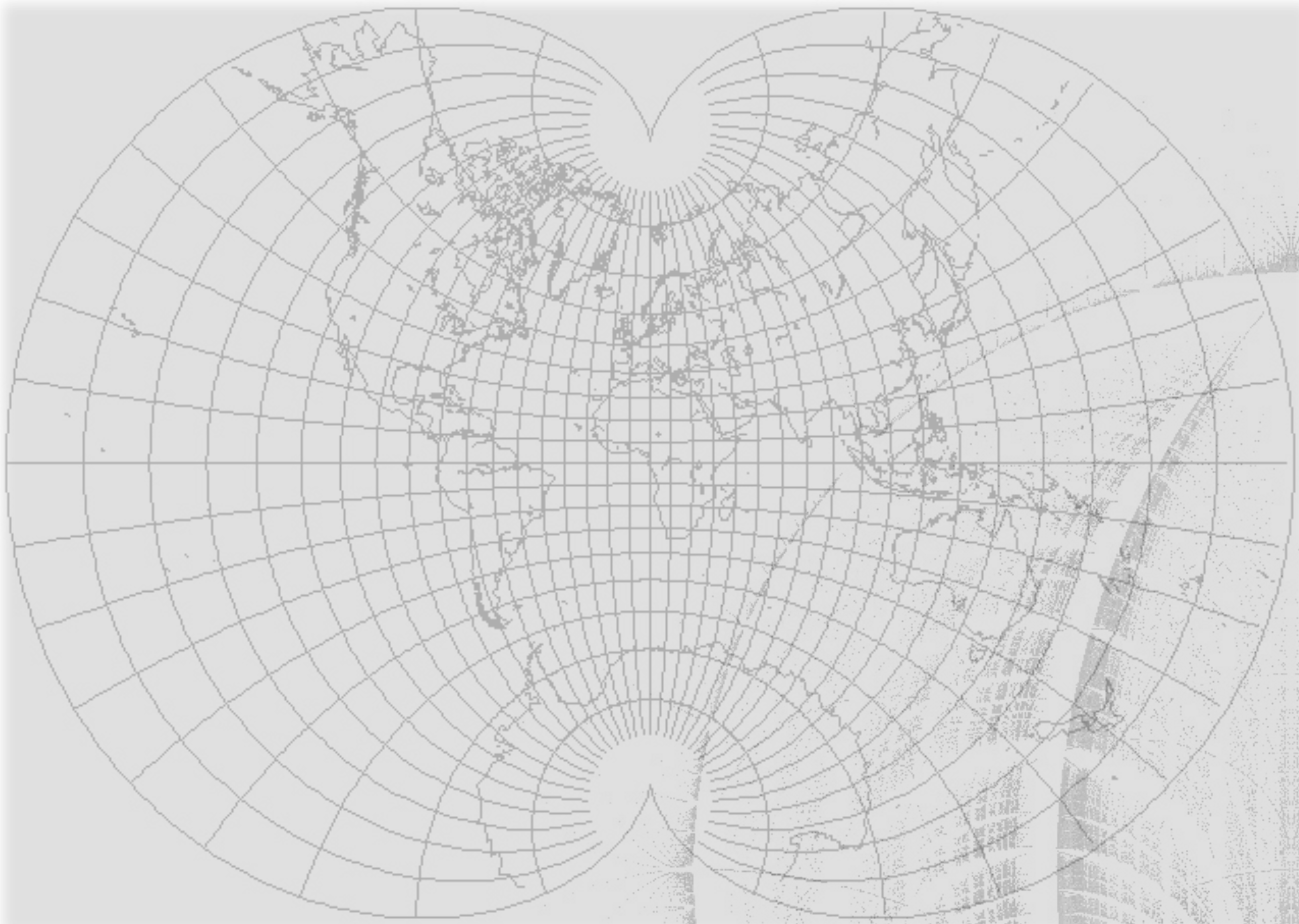

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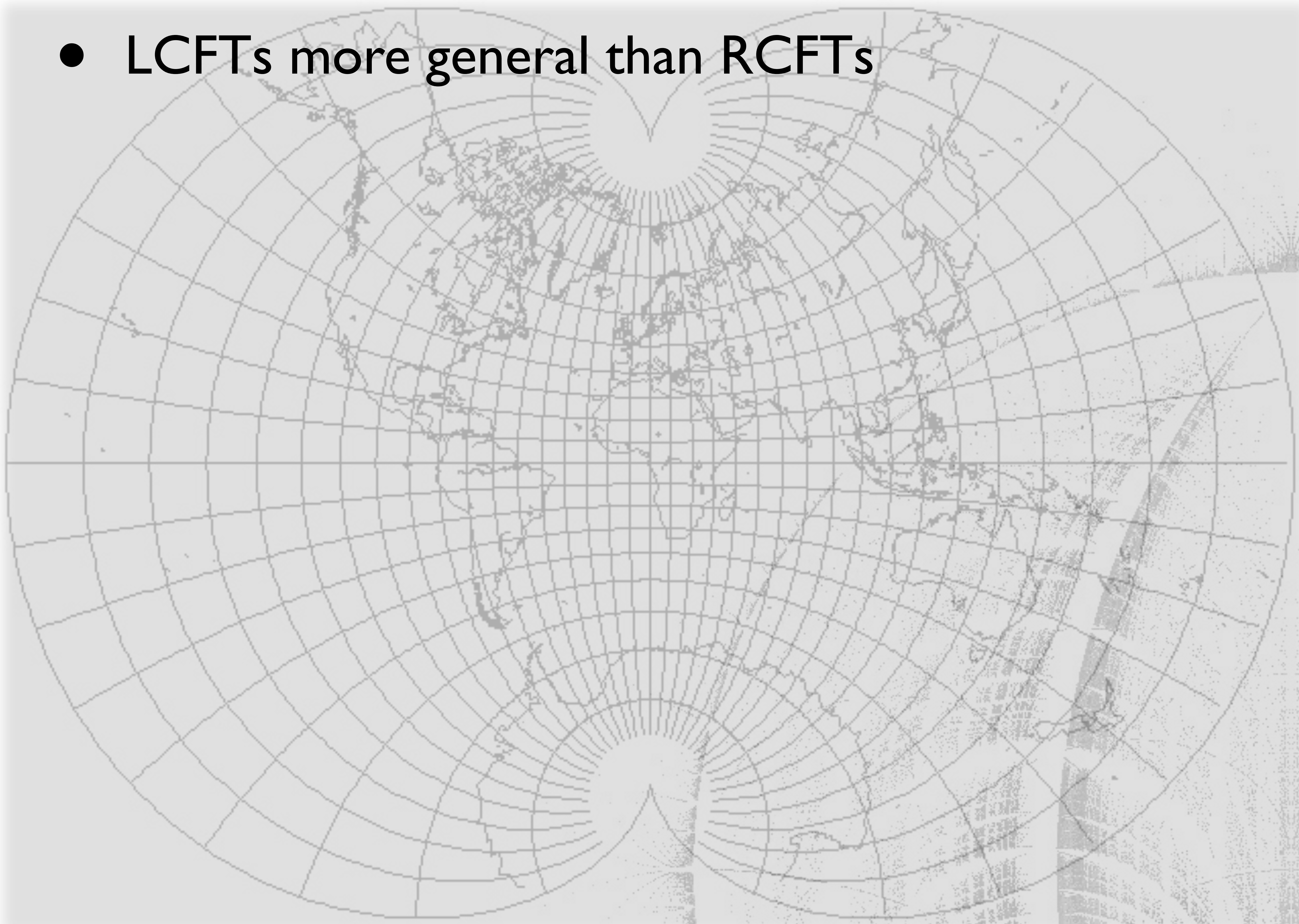

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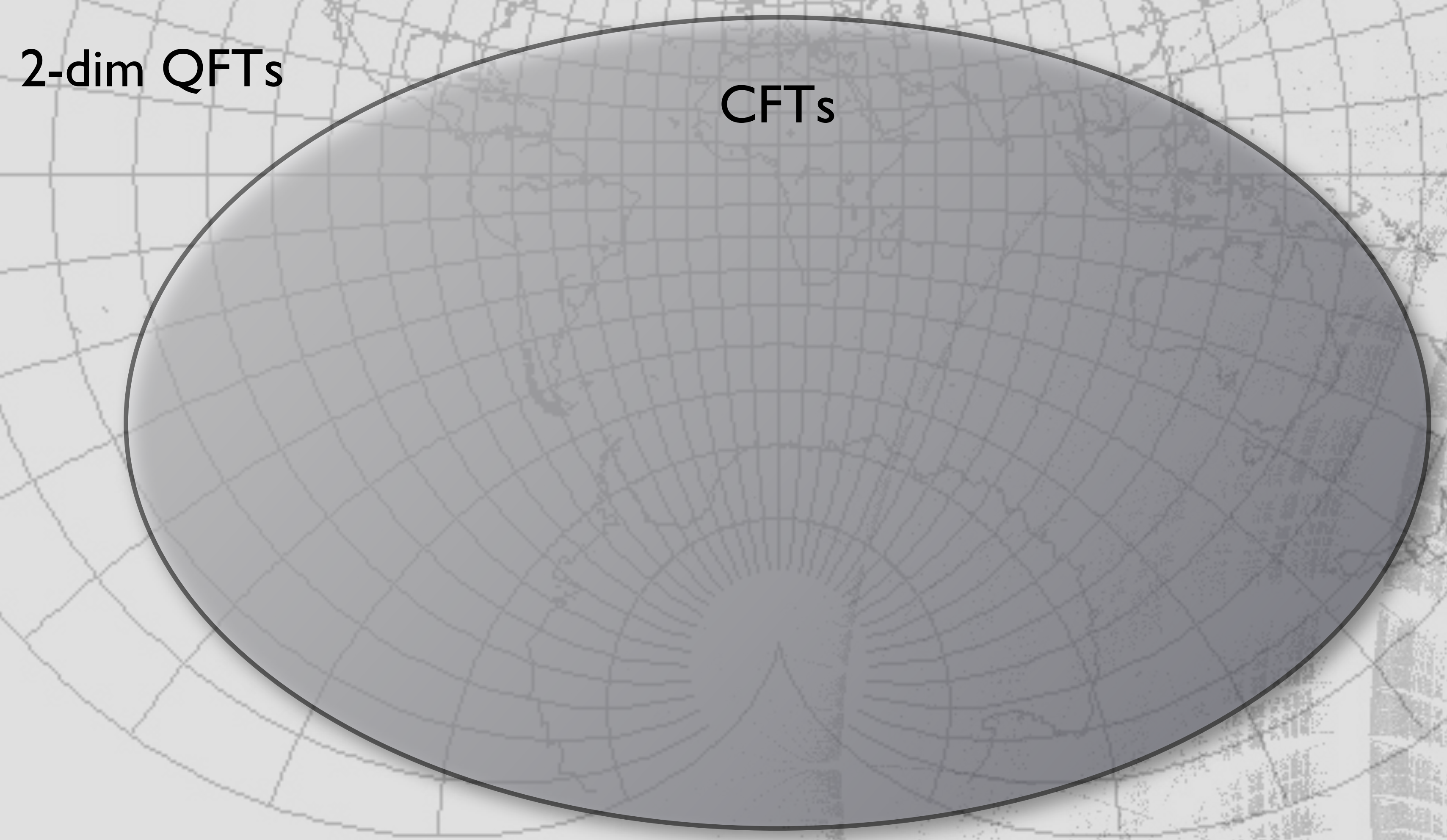

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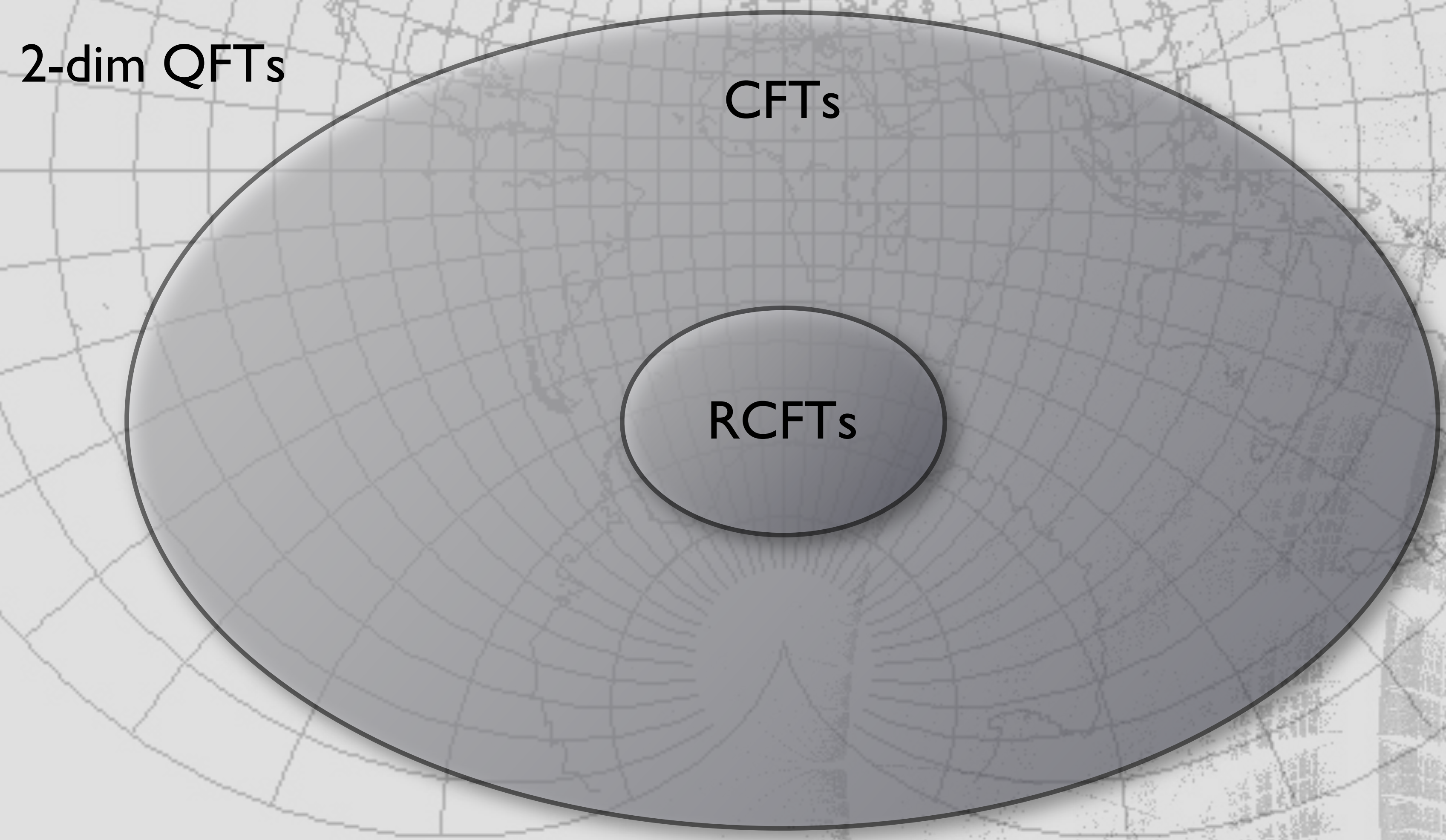

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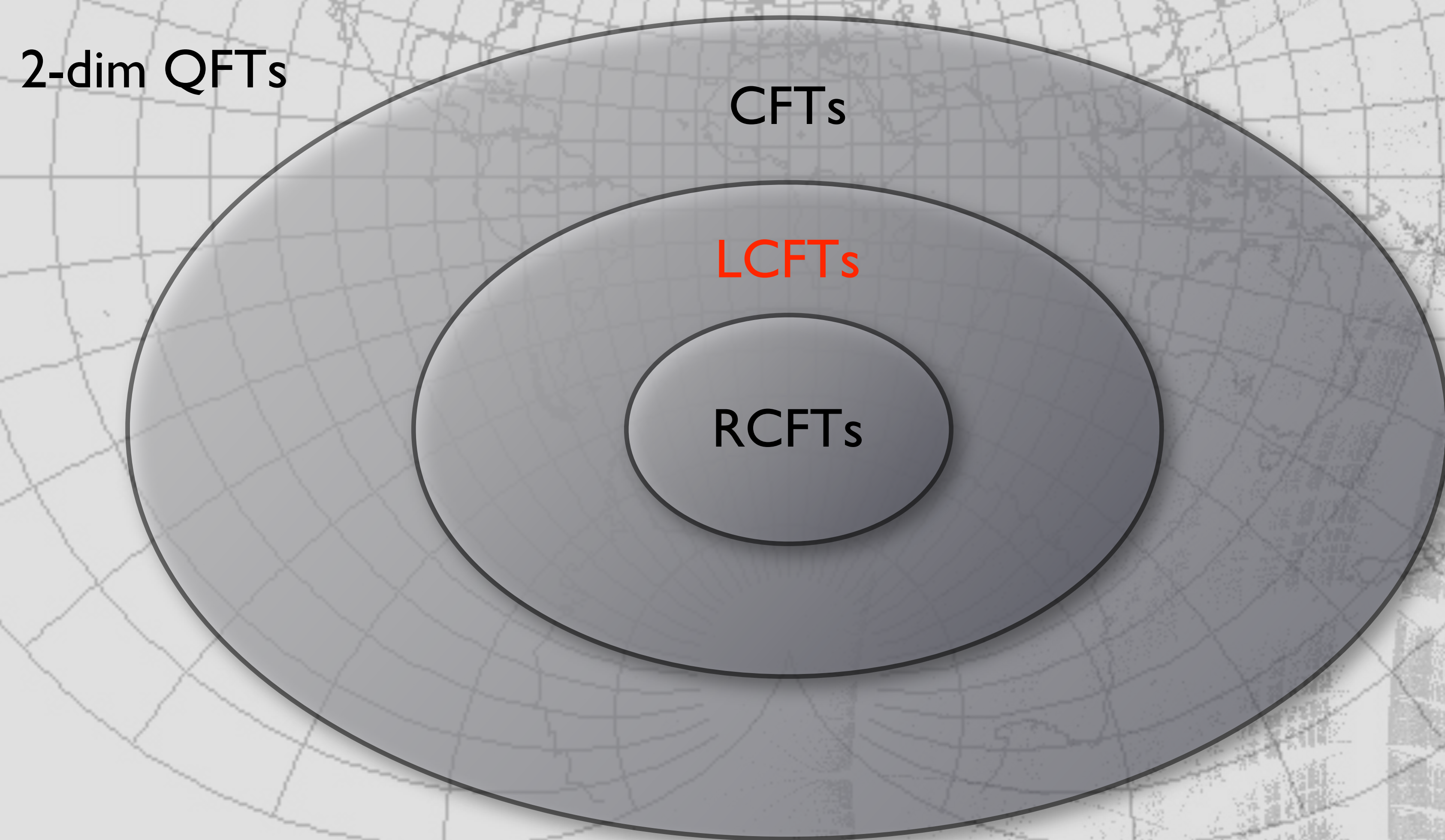

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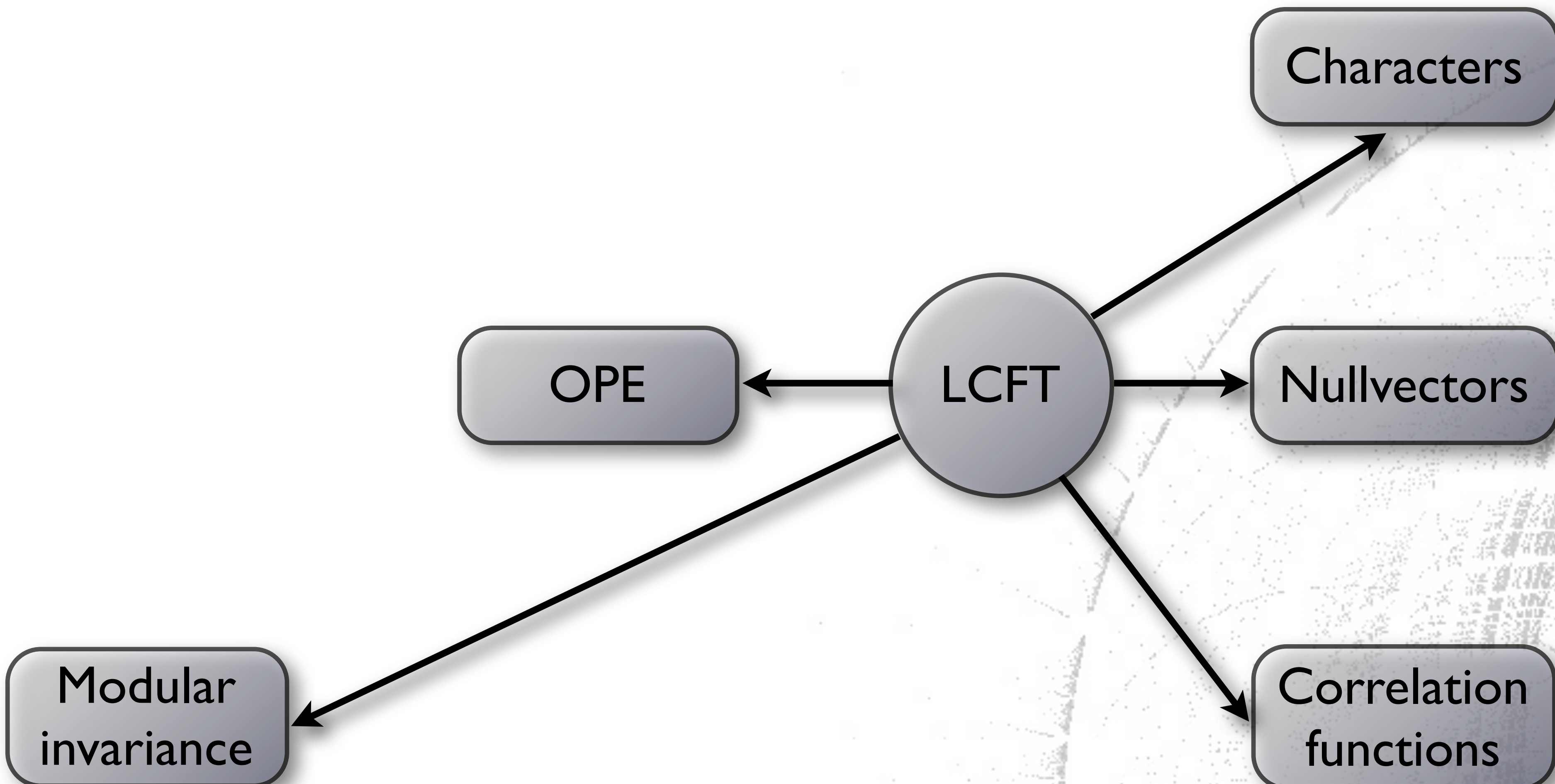

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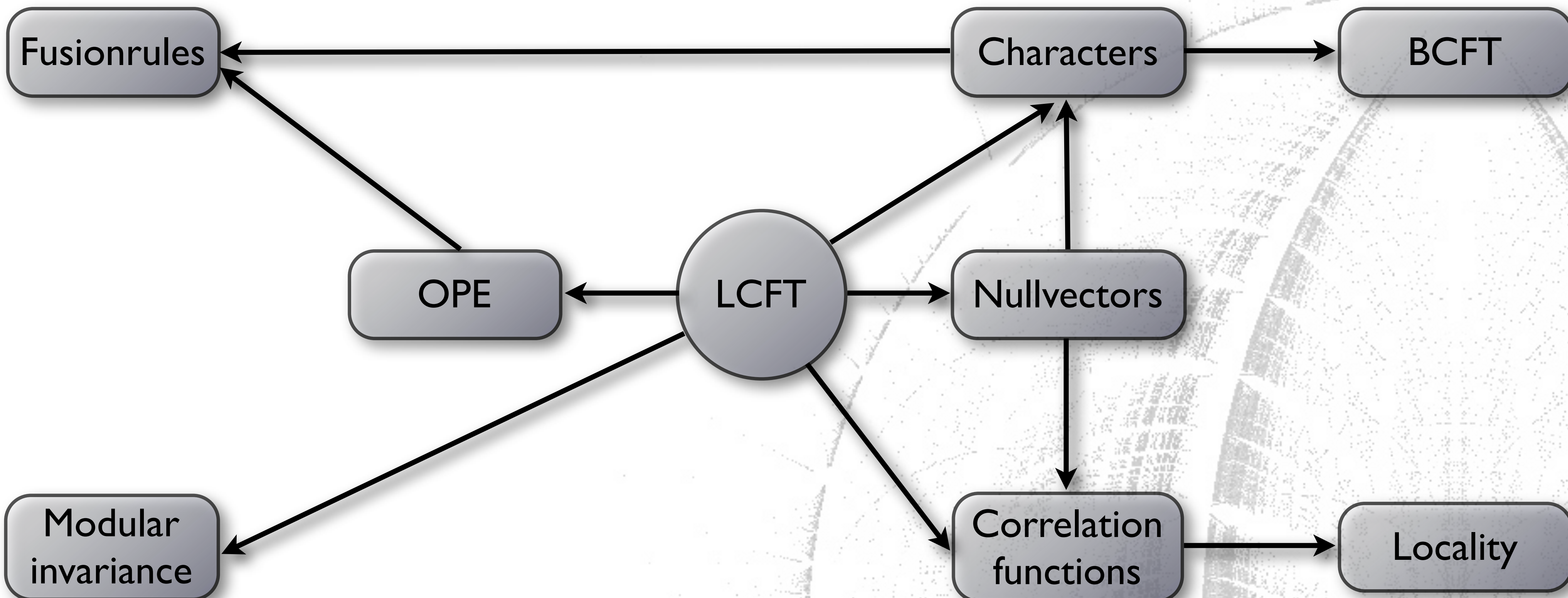

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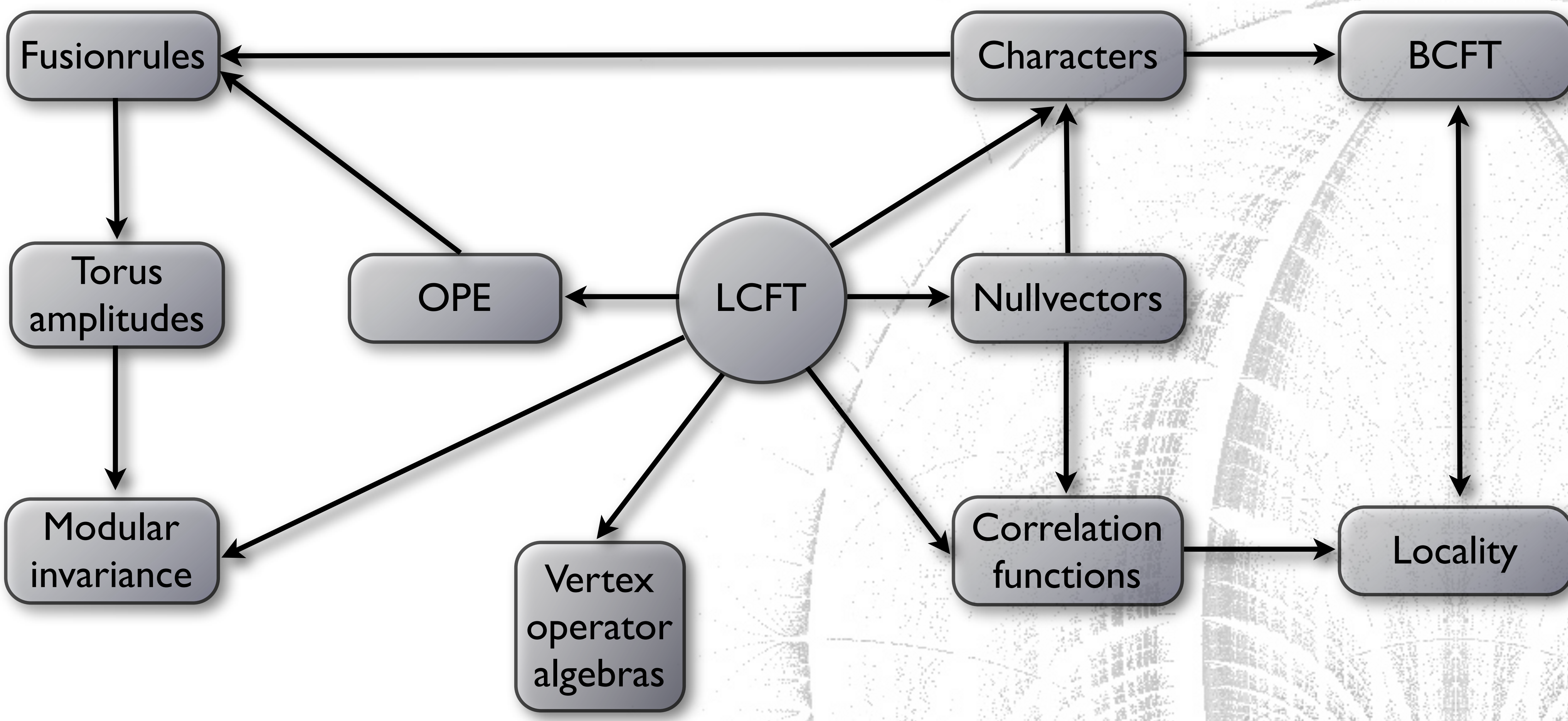

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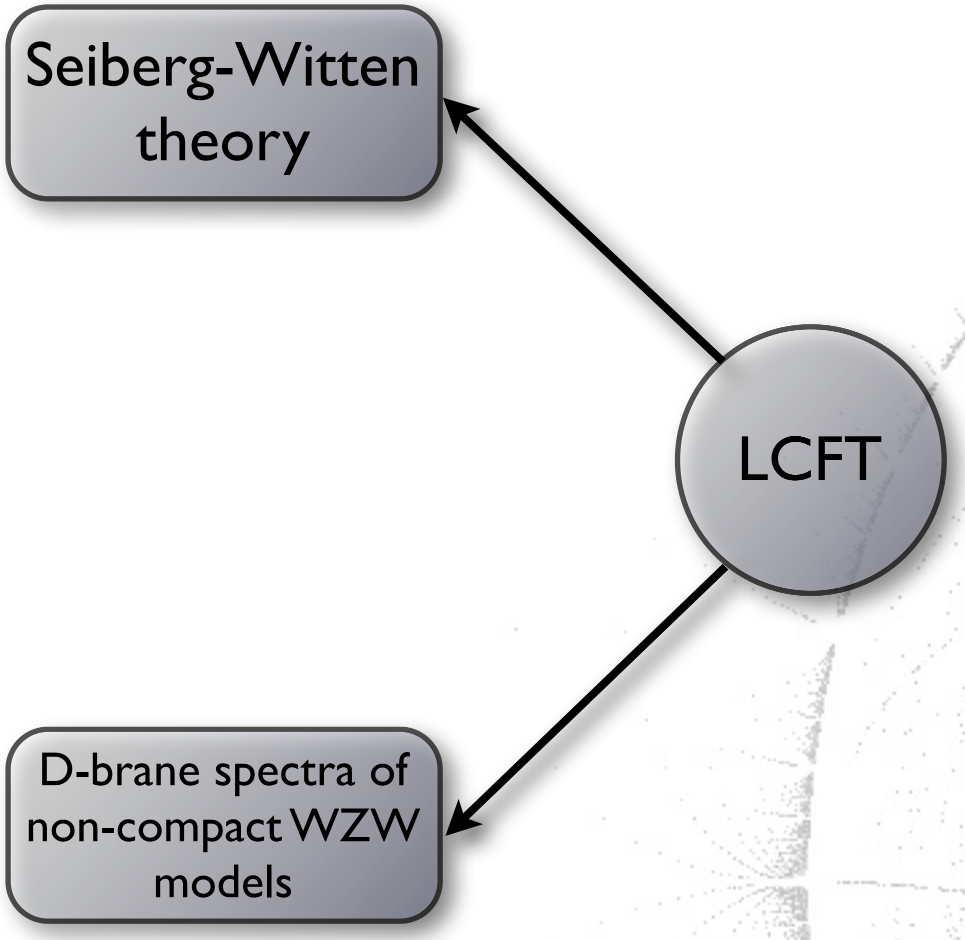

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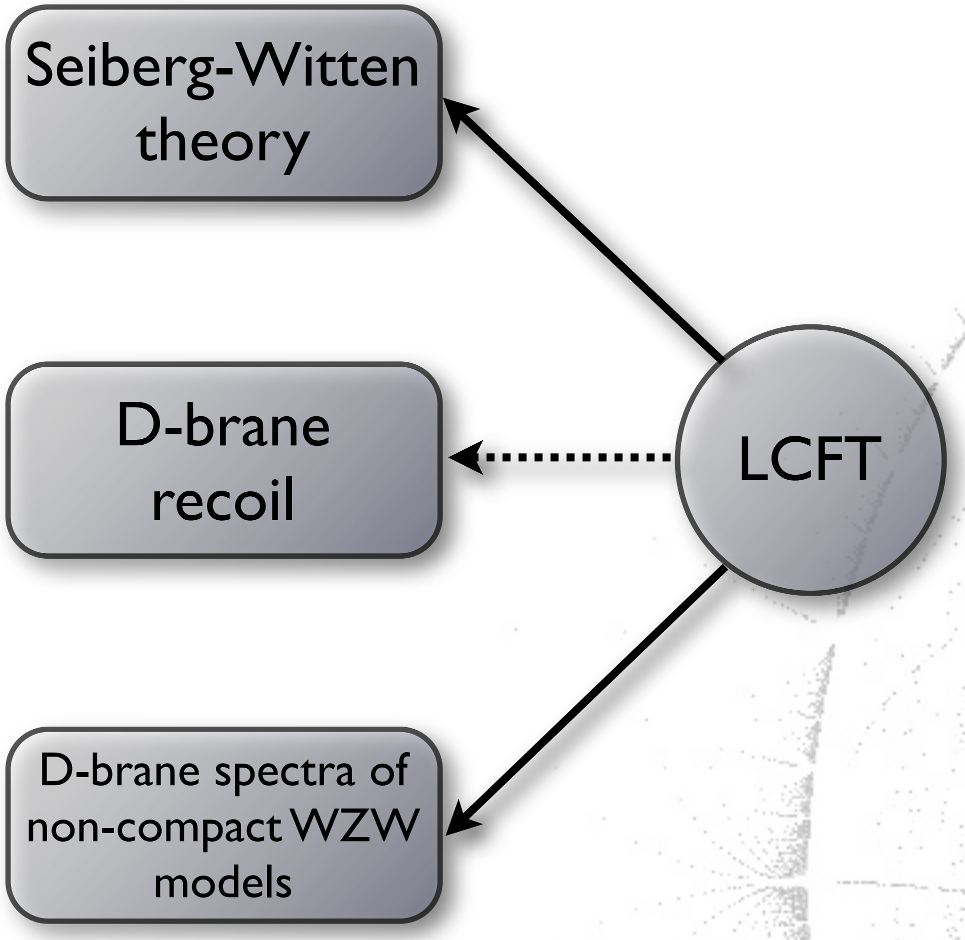

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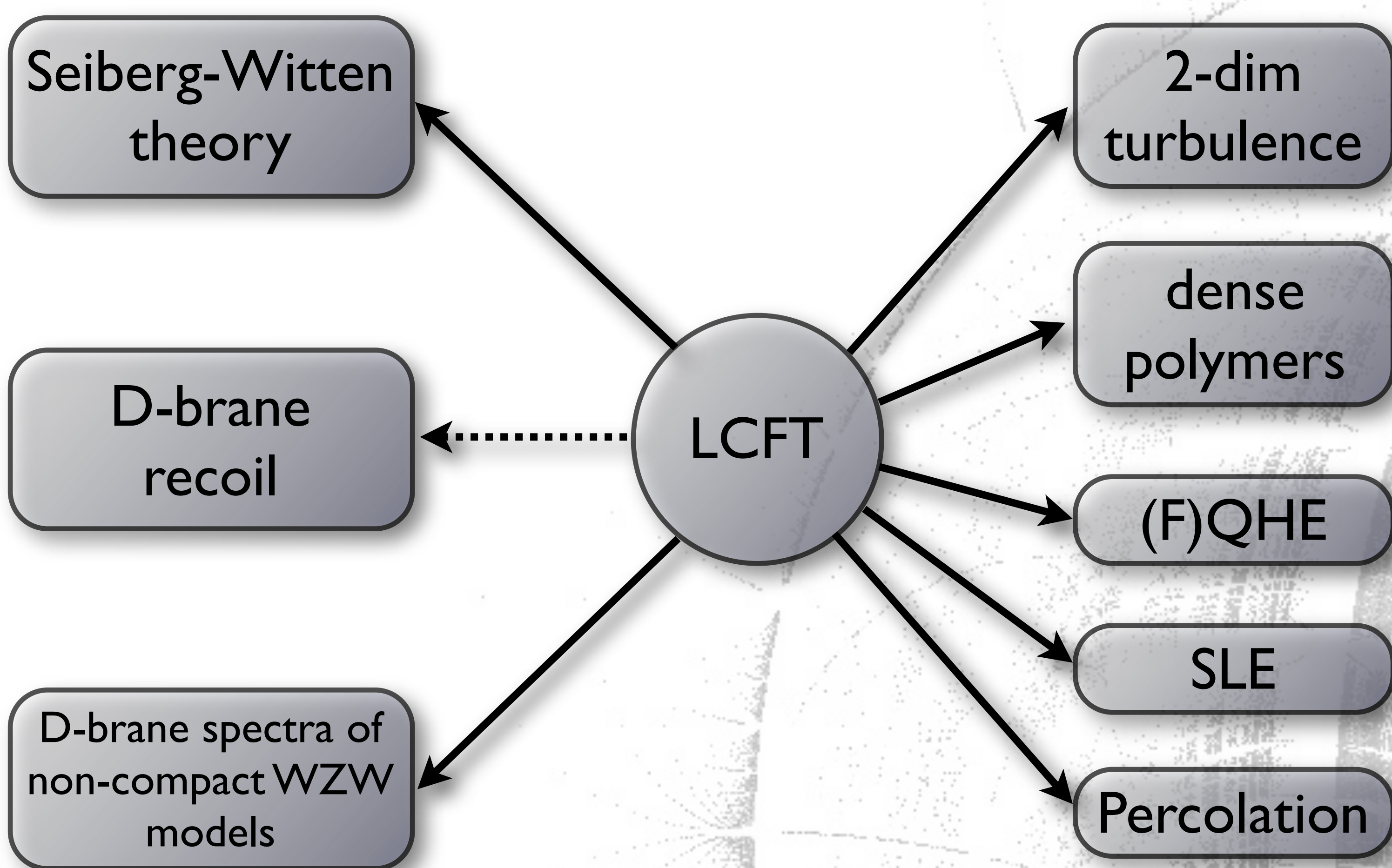

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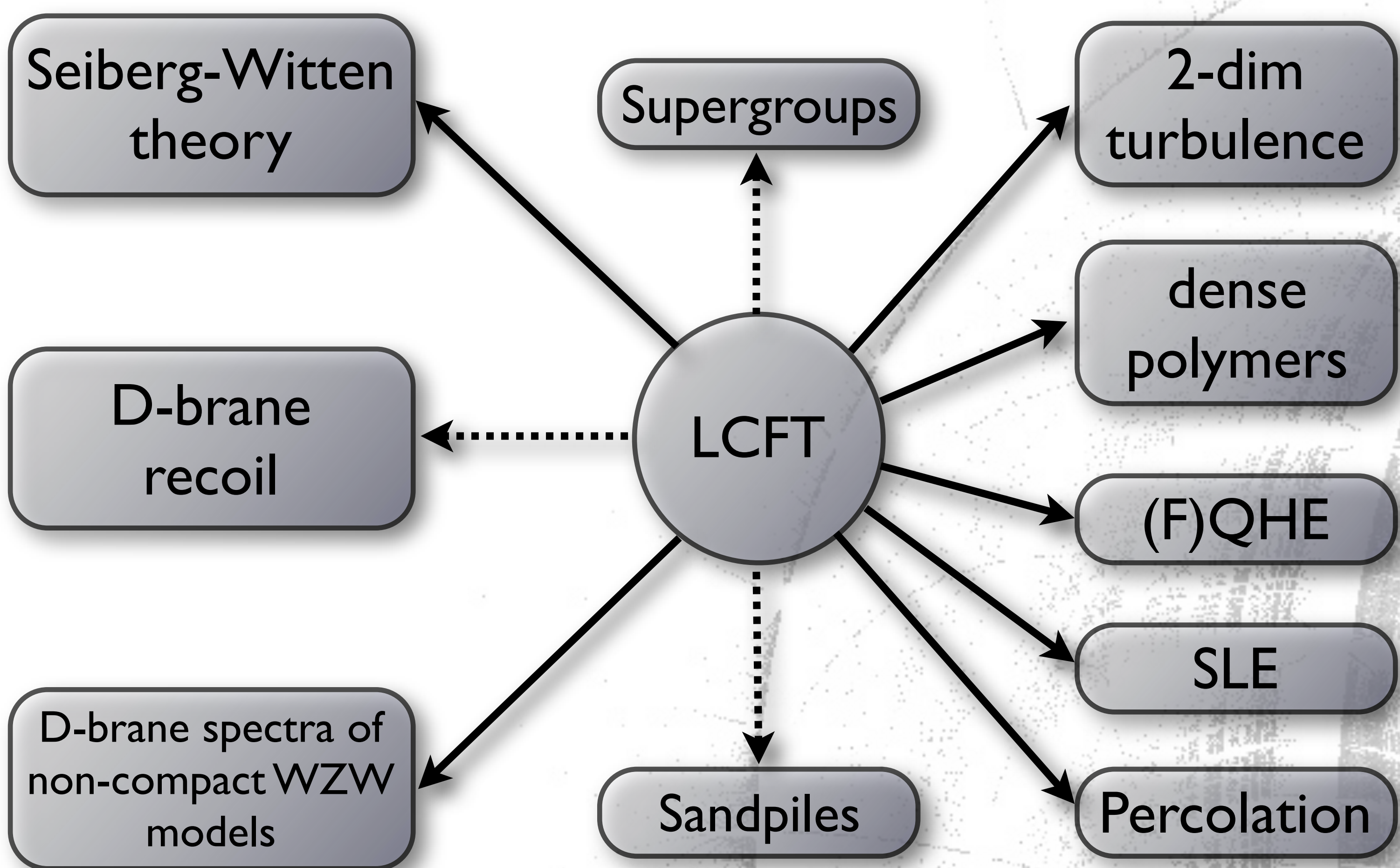

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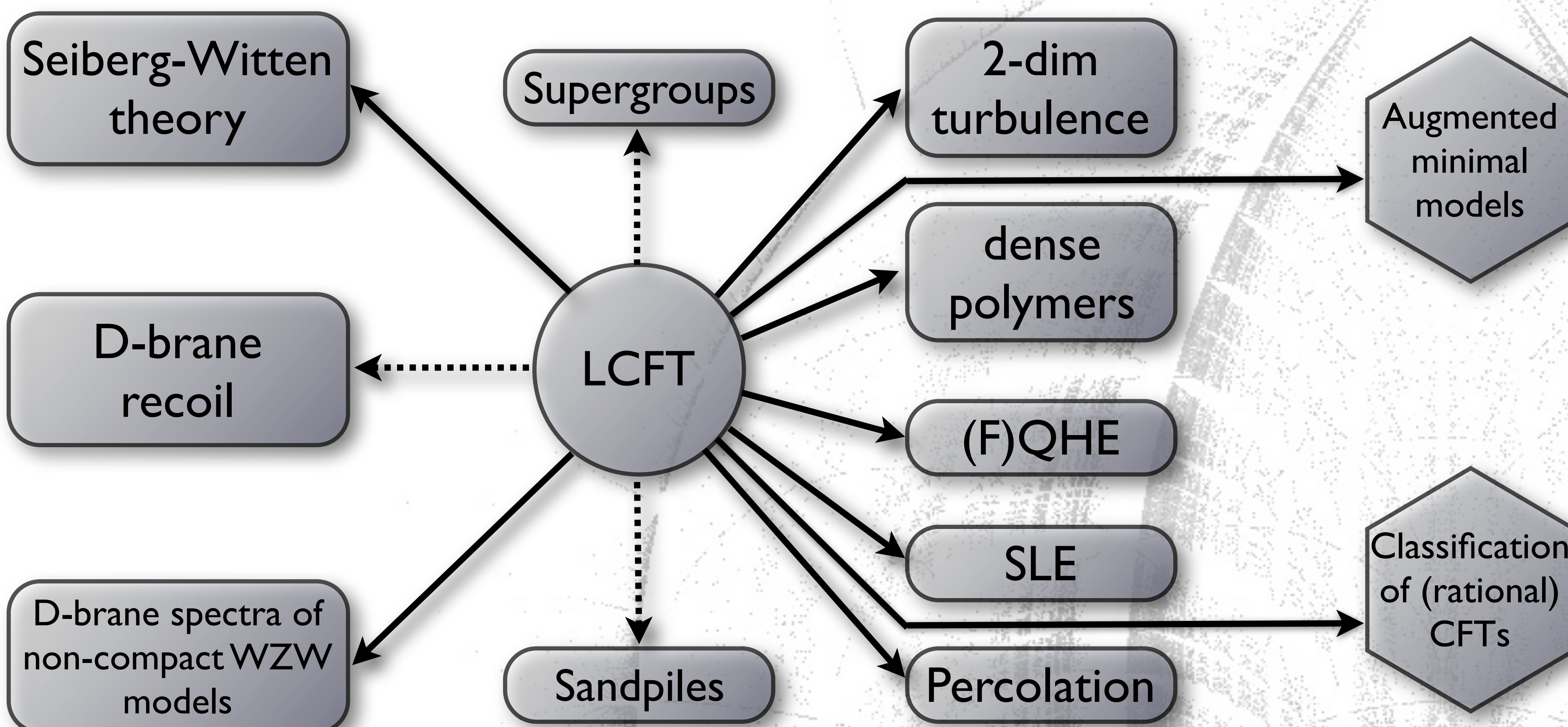

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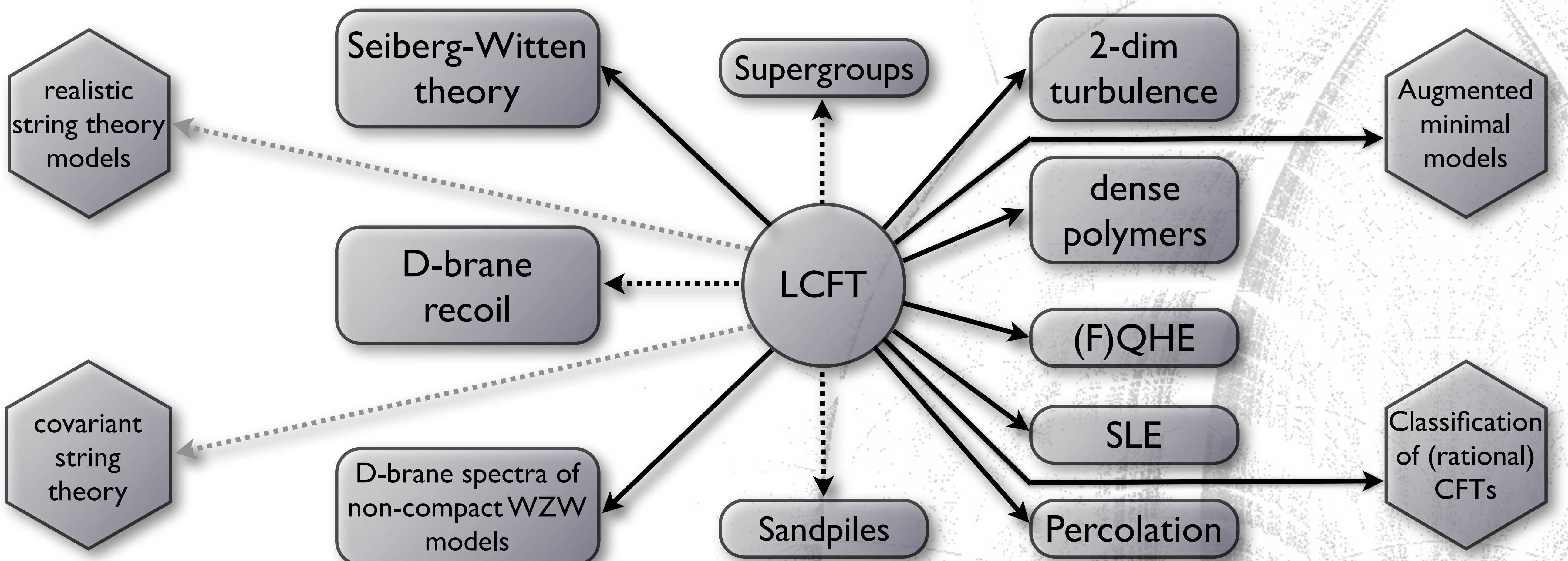
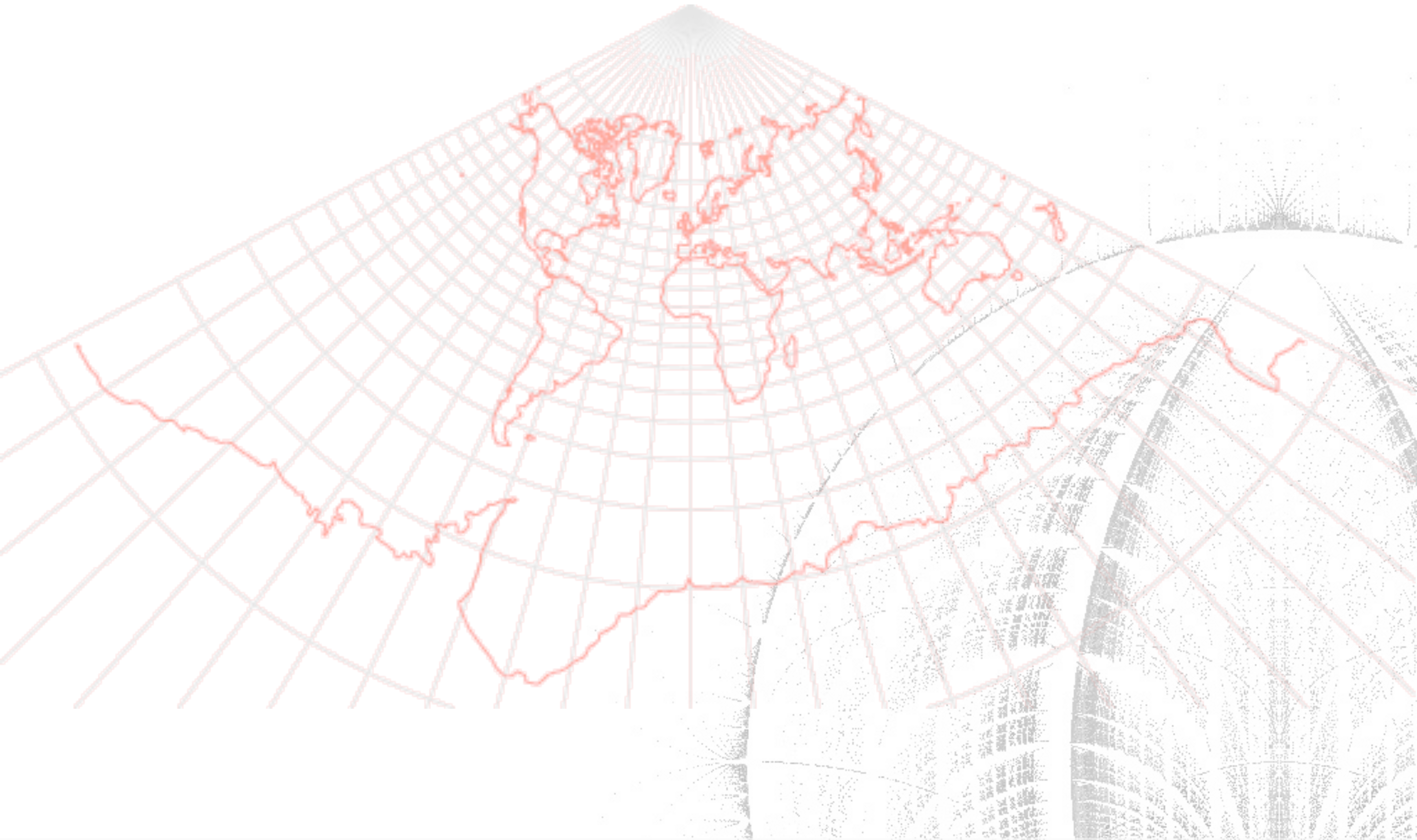


Summary & Outlook



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- I will continue to play a central role in this field