D-Branes in a Non-Compact WZNW Model

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- Focus on corresponding CFT (H_3^+ Model): Interesting in its own right!
- Reason: It is a *non-rational* CFT

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- ... how power of factorization constraint is maintained.

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- Central Theme: Making use of $\mathrm{SL}(2,\mathbb{C})$ symmetry.

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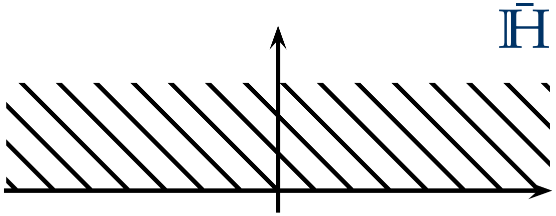

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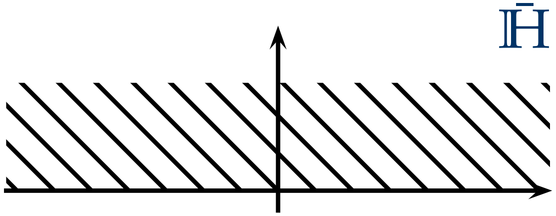

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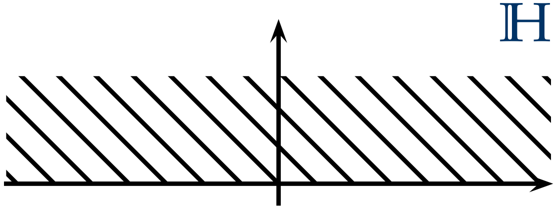

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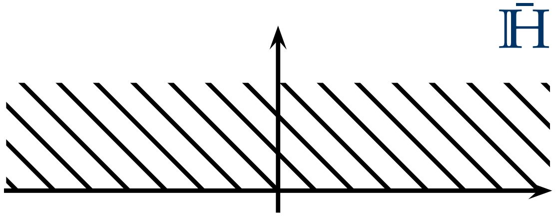

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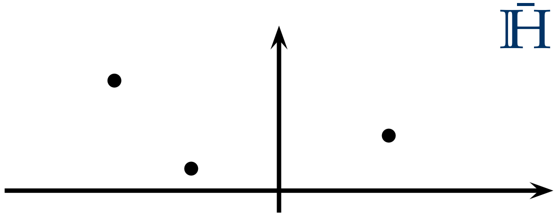

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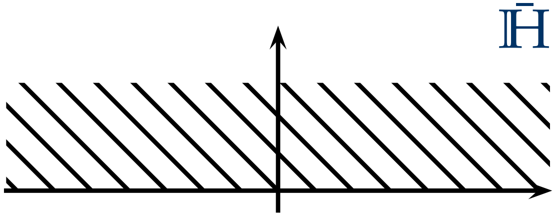

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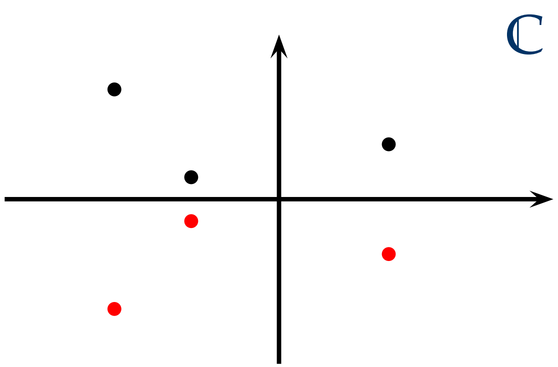

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- Describes the coupling of closed string (j) to D-brane (α).

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Cardy-Lewellen Constraints

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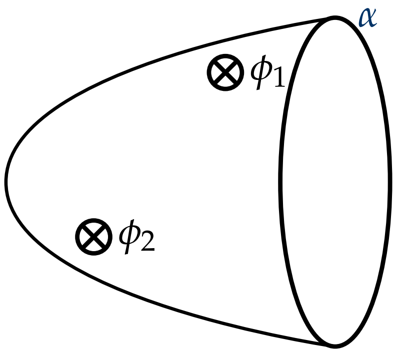

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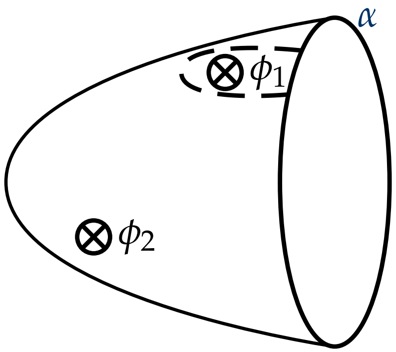

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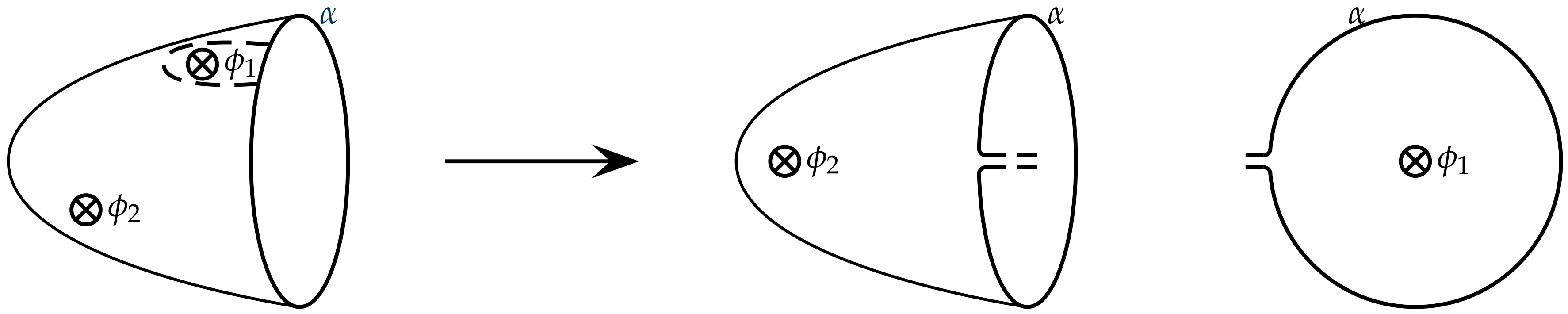

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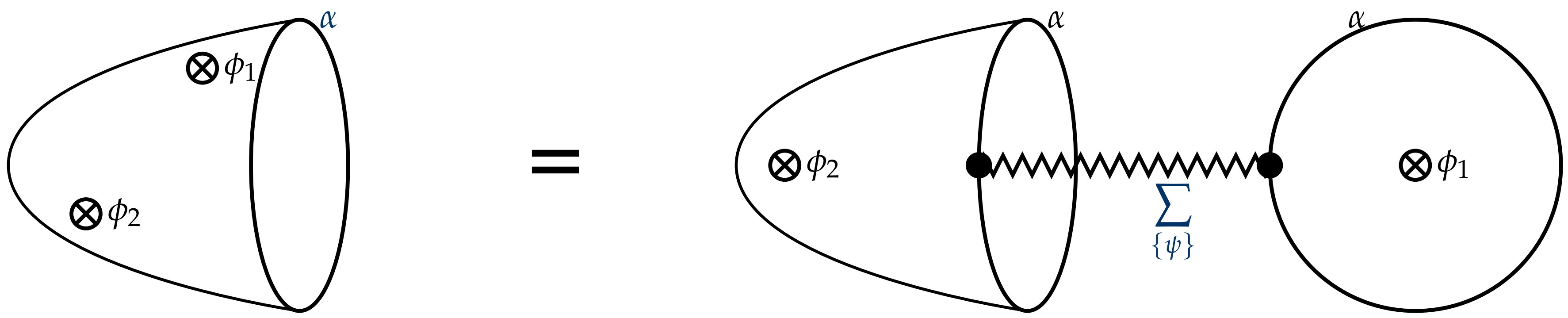

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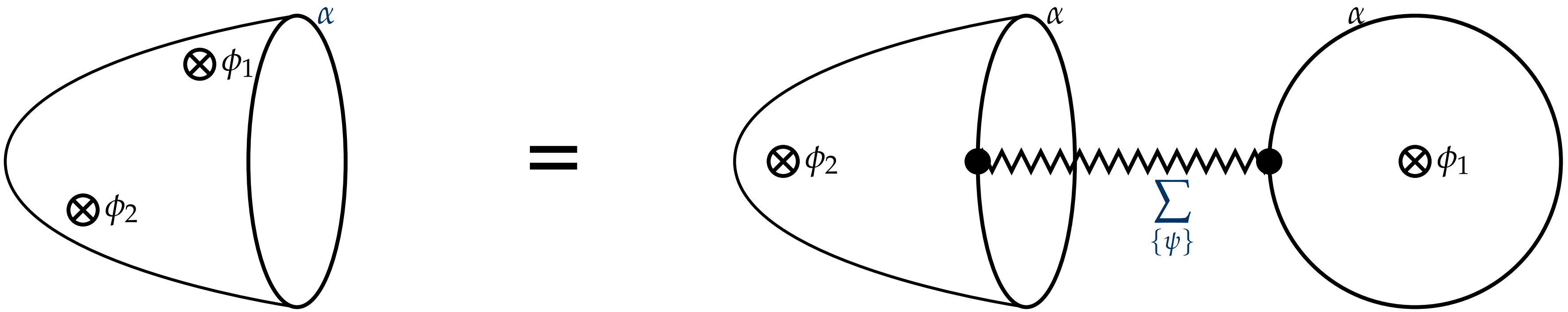

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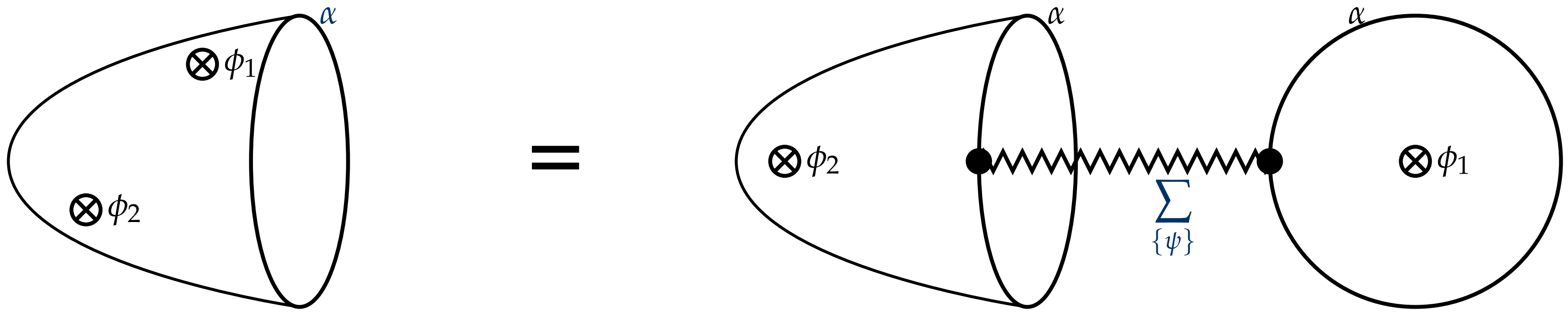

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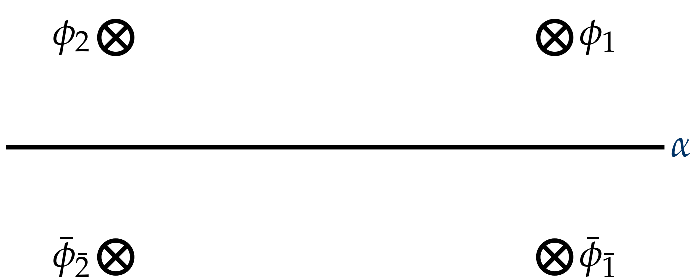

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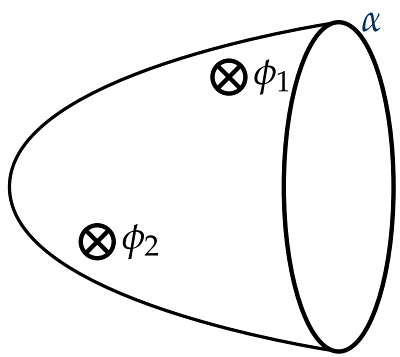

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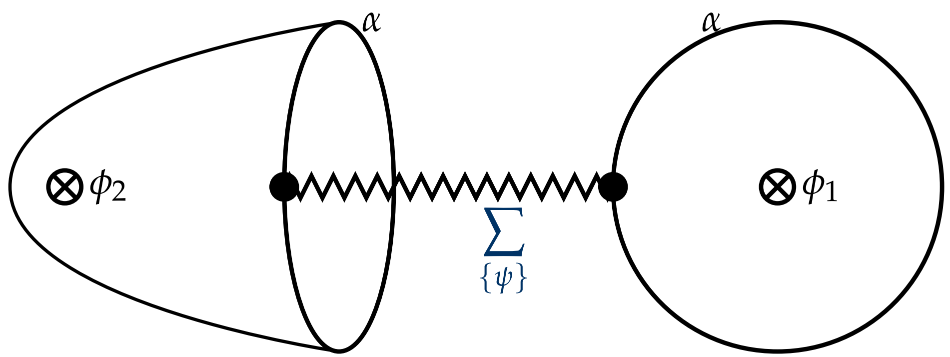

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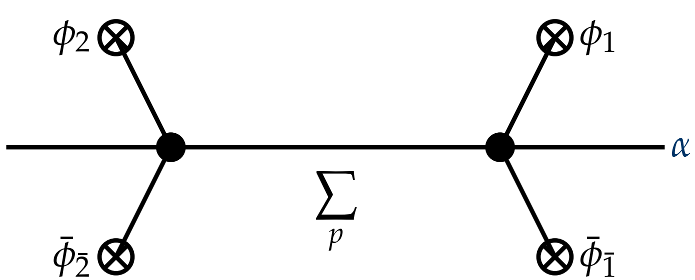

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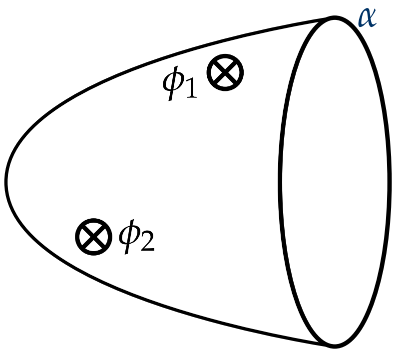

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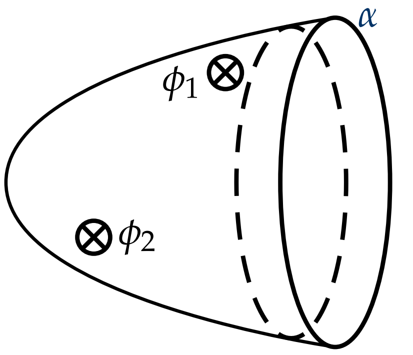

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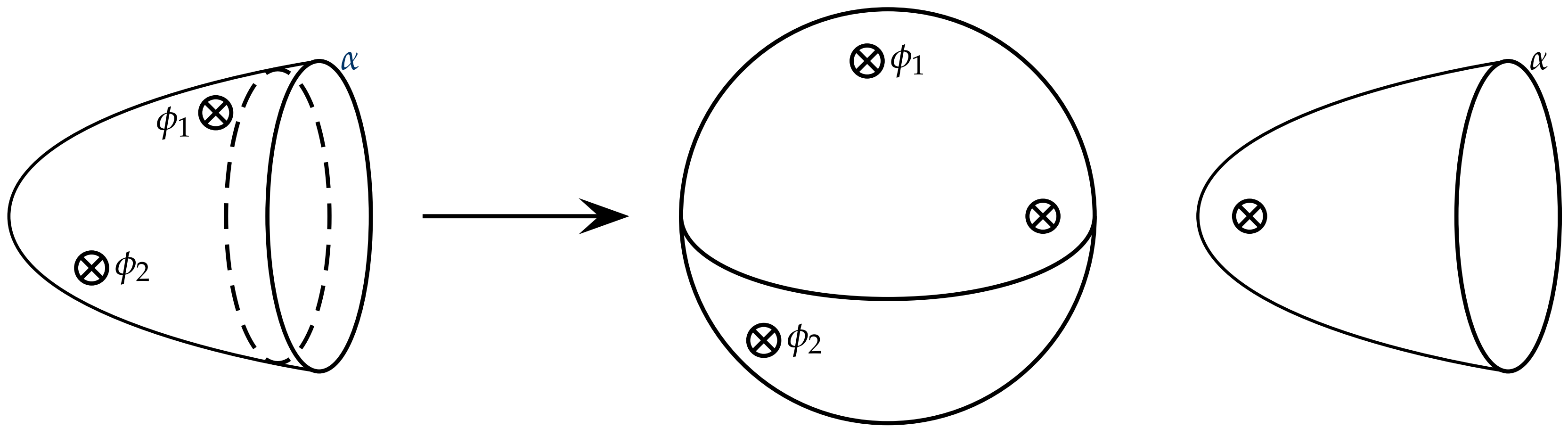

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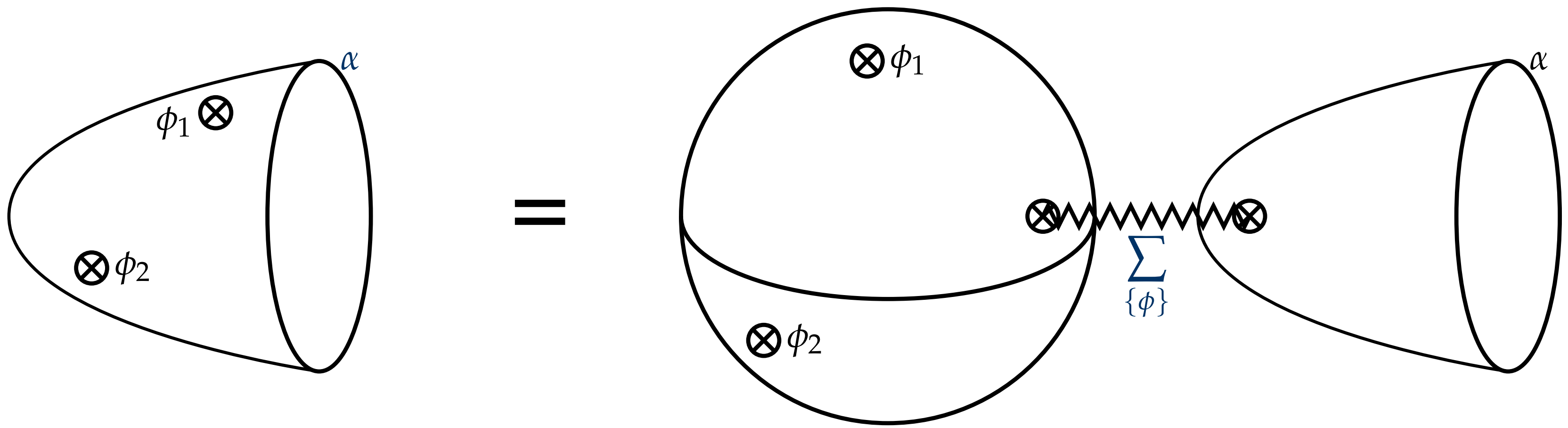

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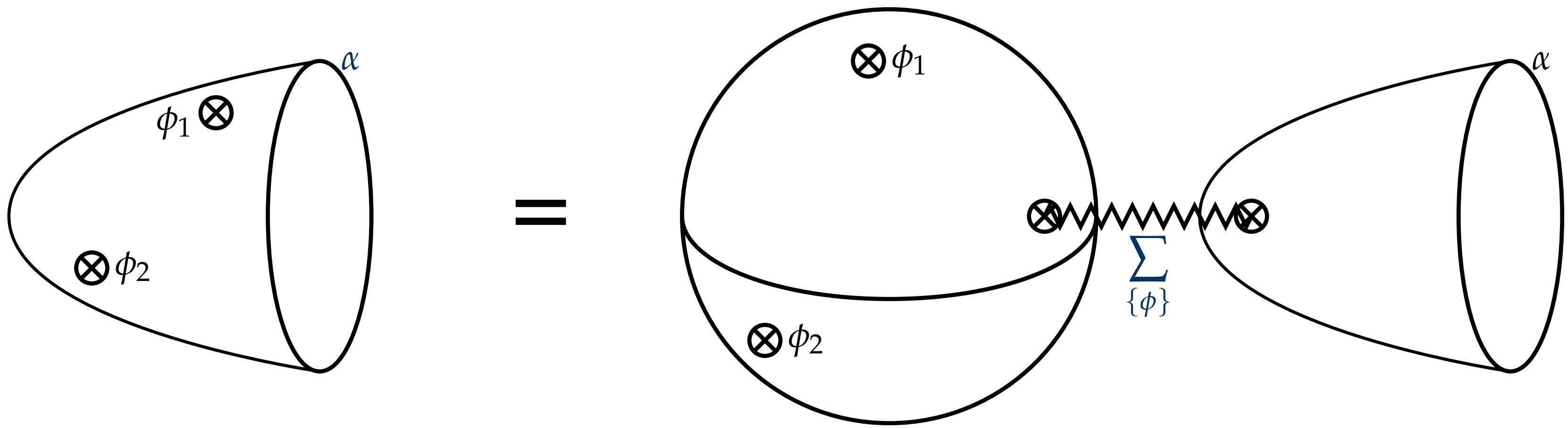

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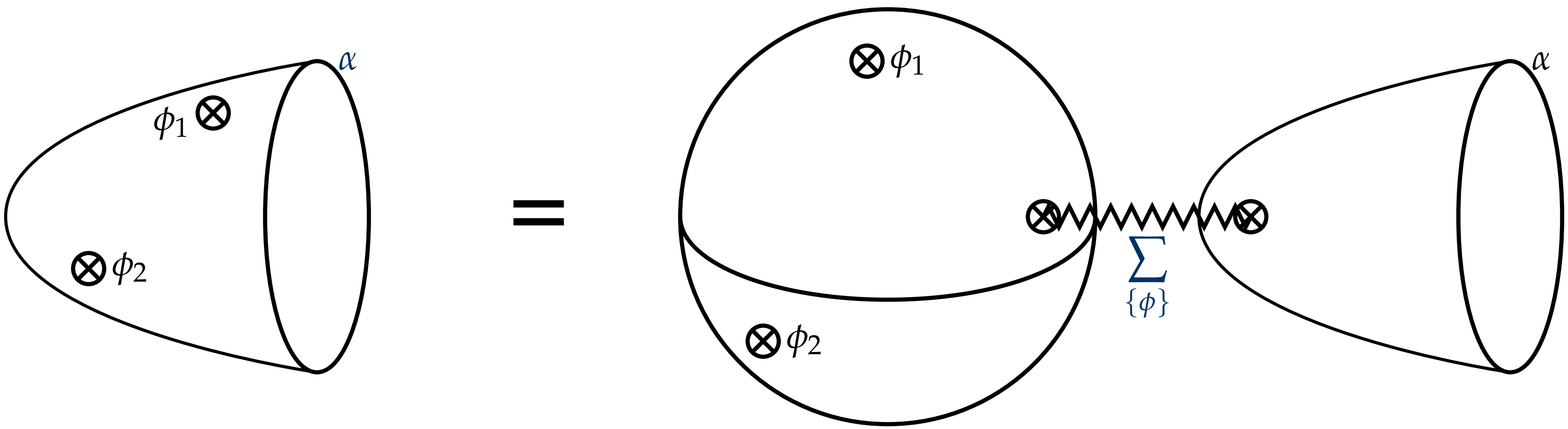

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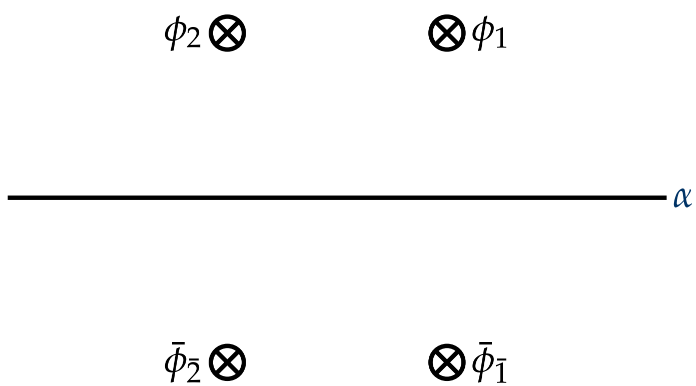

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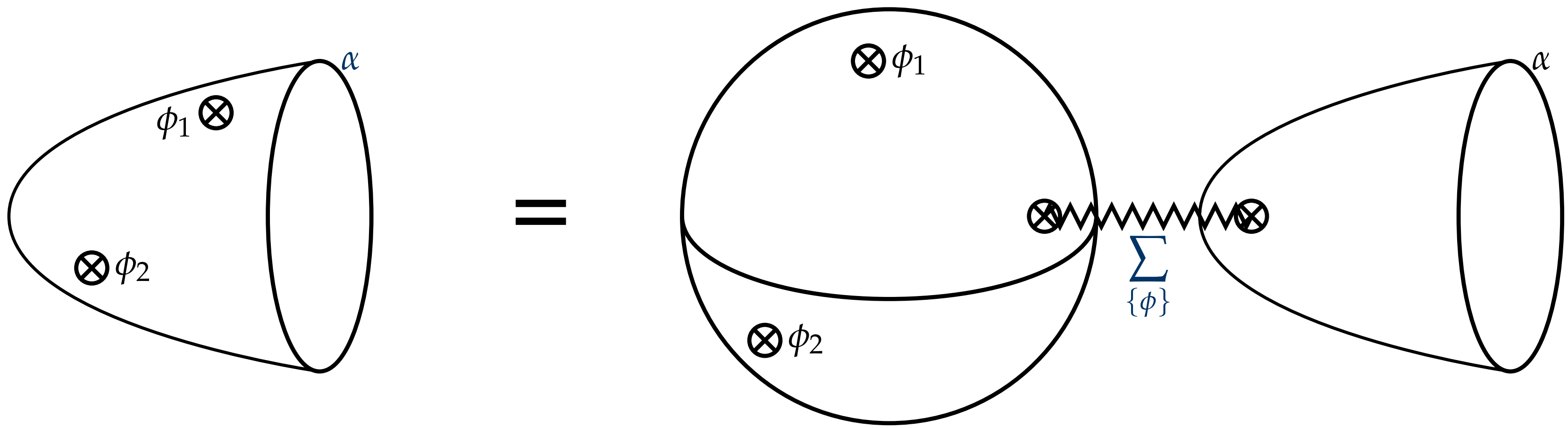

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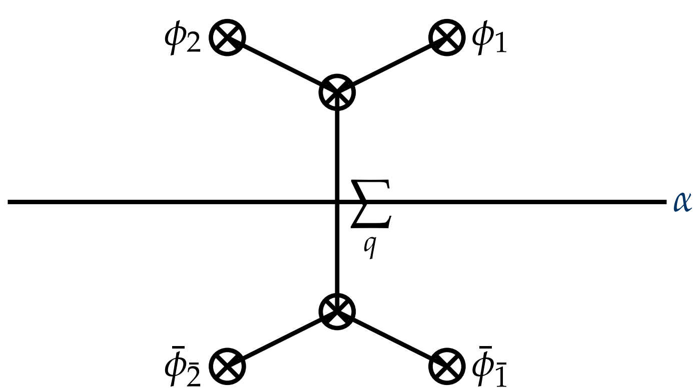

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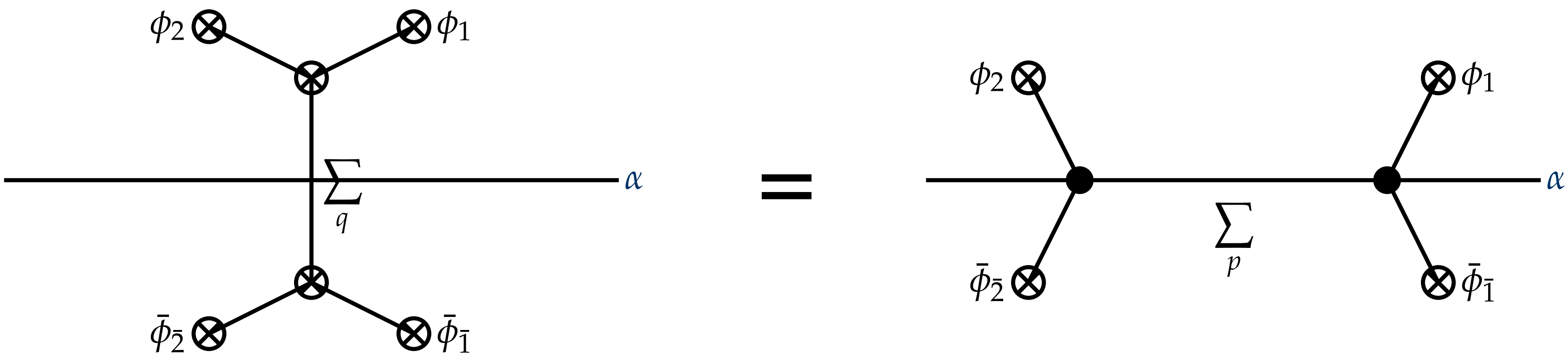

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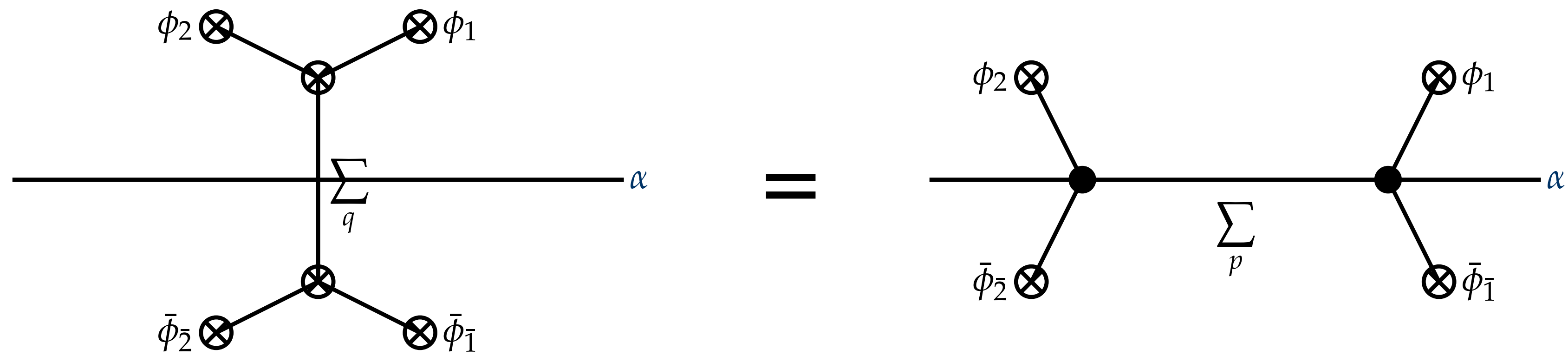

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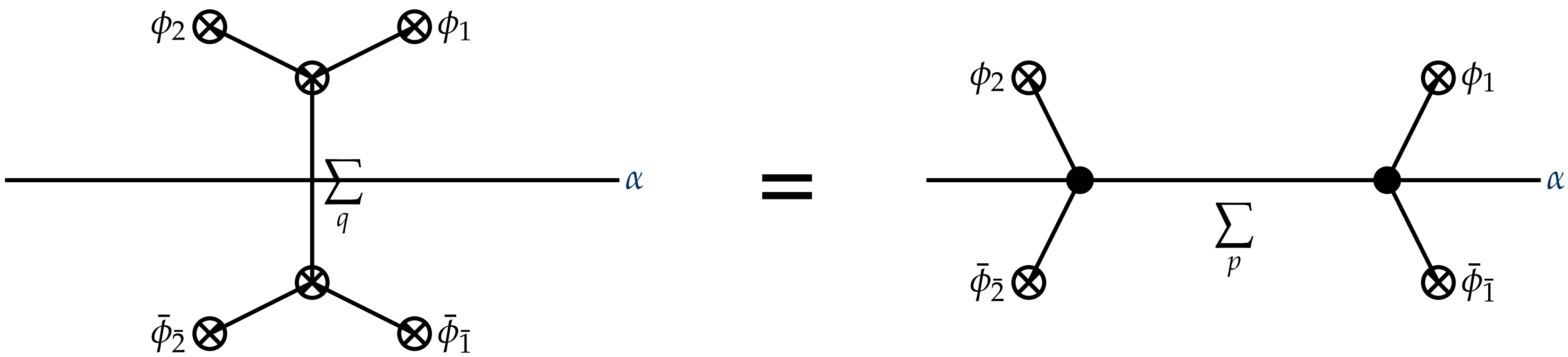

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Implementation of Factorization Constraint in H_3^+

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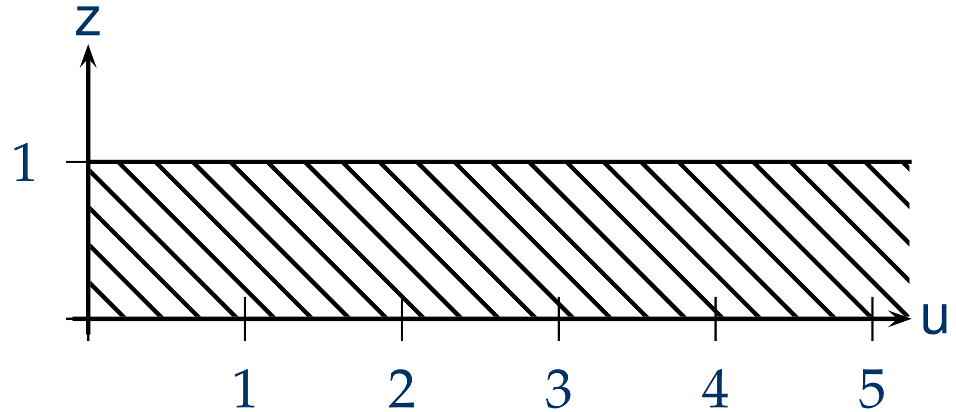

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The '$b^{-2}/2$'-Constraint

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with $C_\epsilon(j) = C(j, \frac{1}{2}, j_\epsilon)$, where

$$j_\pm = j \pm \frac{b^{-2}}{2} \qquad \text{and} \qquad j_\times = -j - 1 - \frac{b^{-2}}{2}$$

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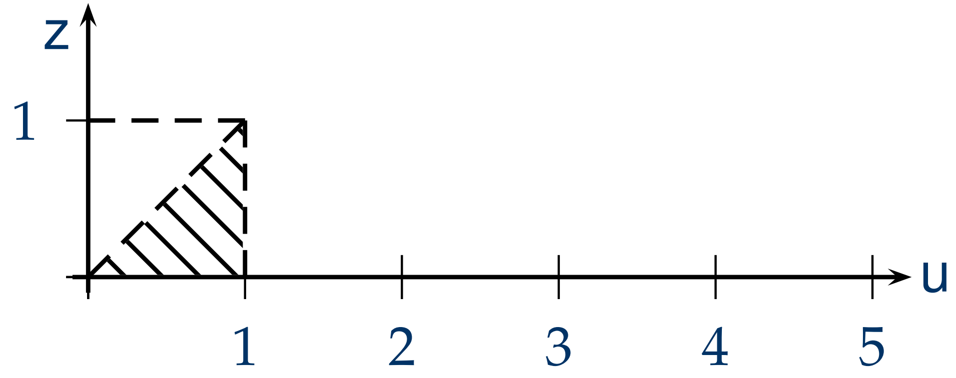

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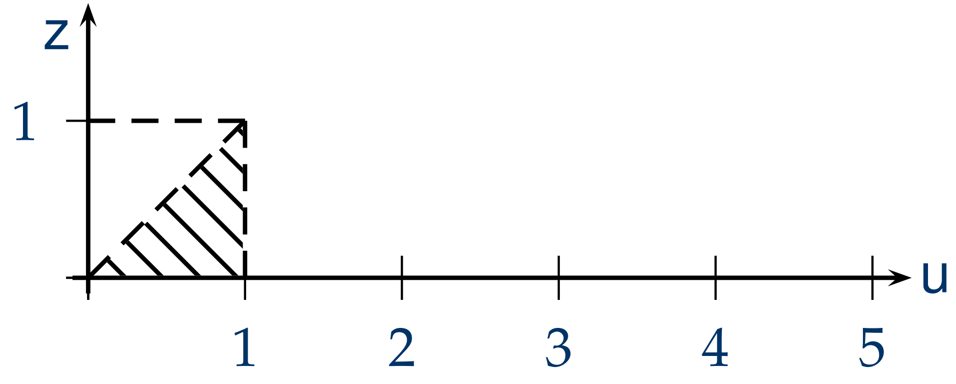
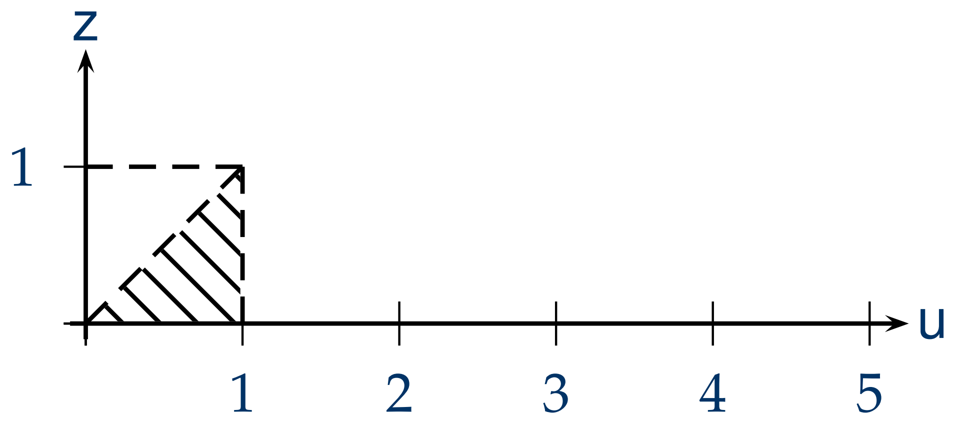


- Thus, limit $z \to 1$ cannot be taken! No constraint derivable!

Resolution



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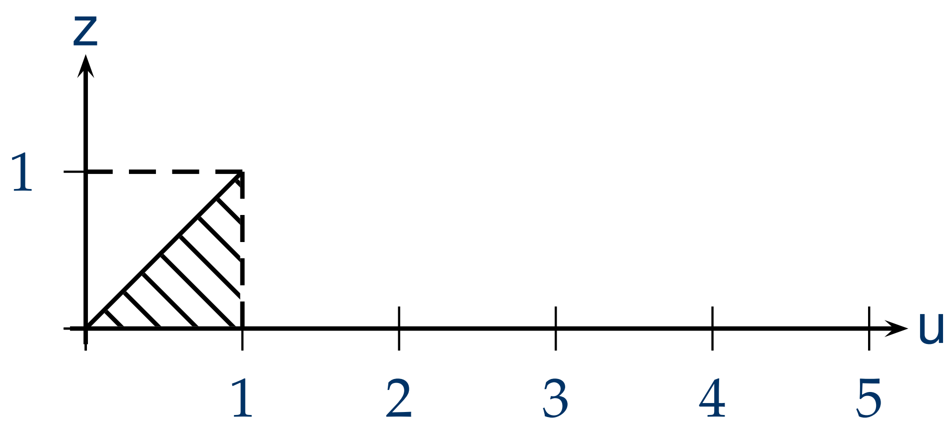

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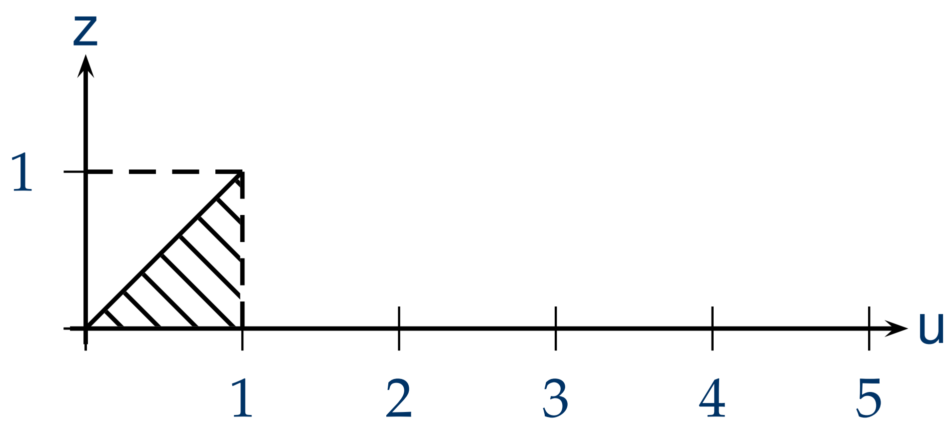

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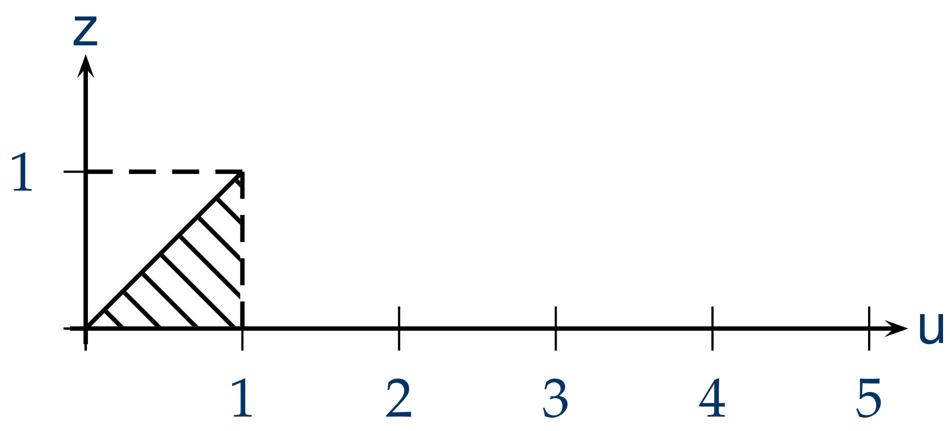

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- $u = z$ suggested to be a "new type of singularity" (*Hosomichi-Ribault proposal*)

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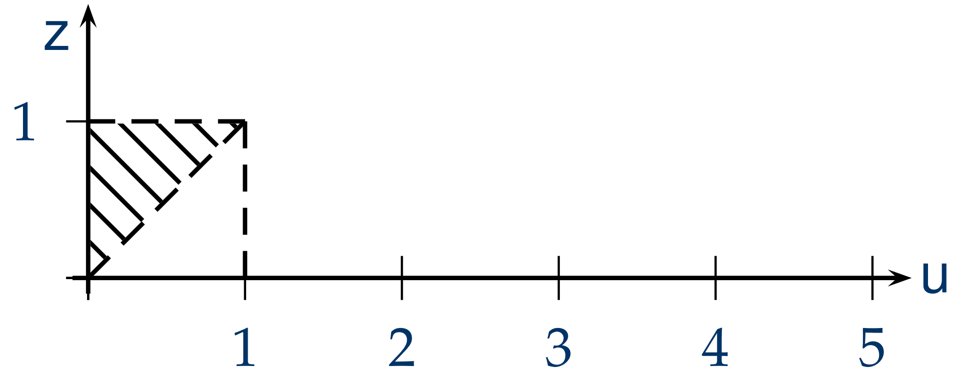

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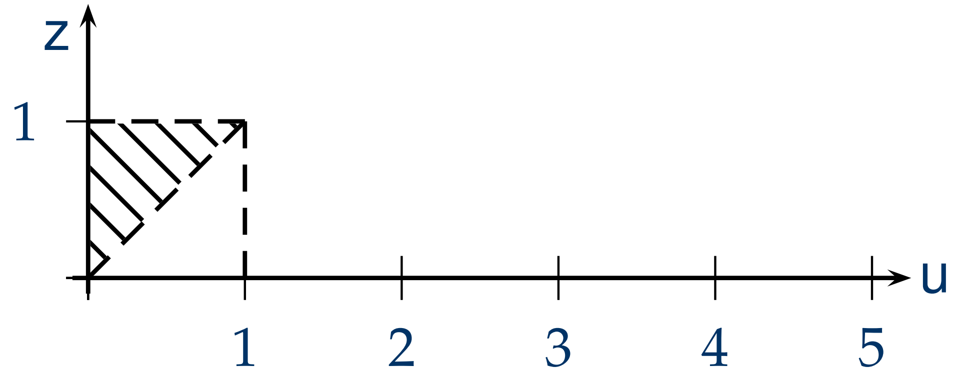

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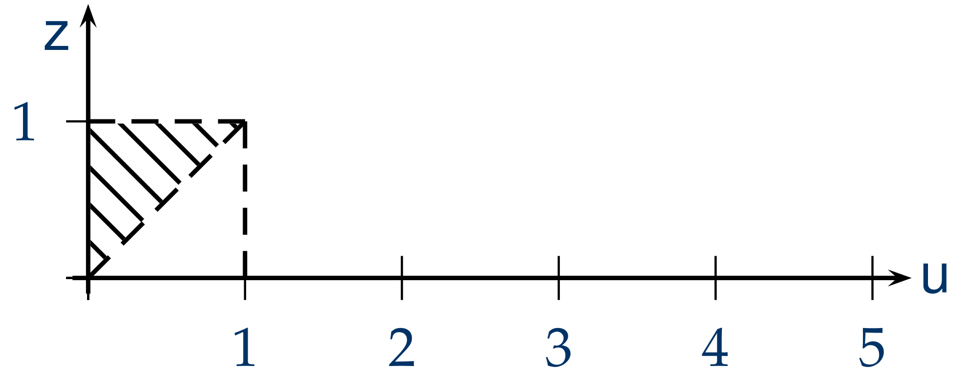

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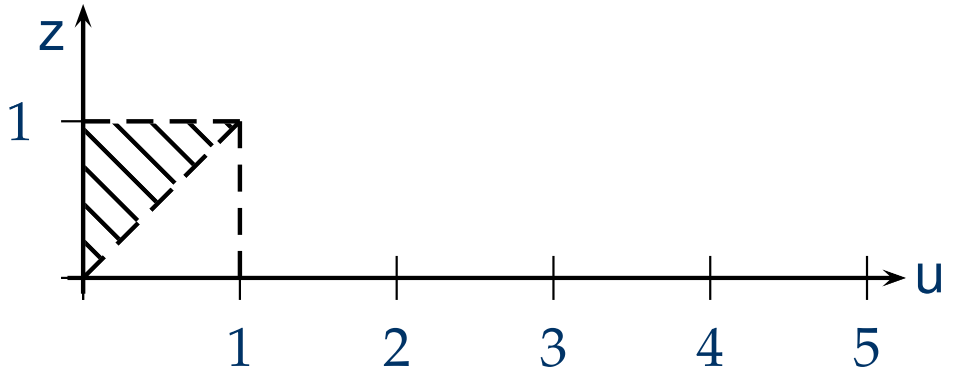

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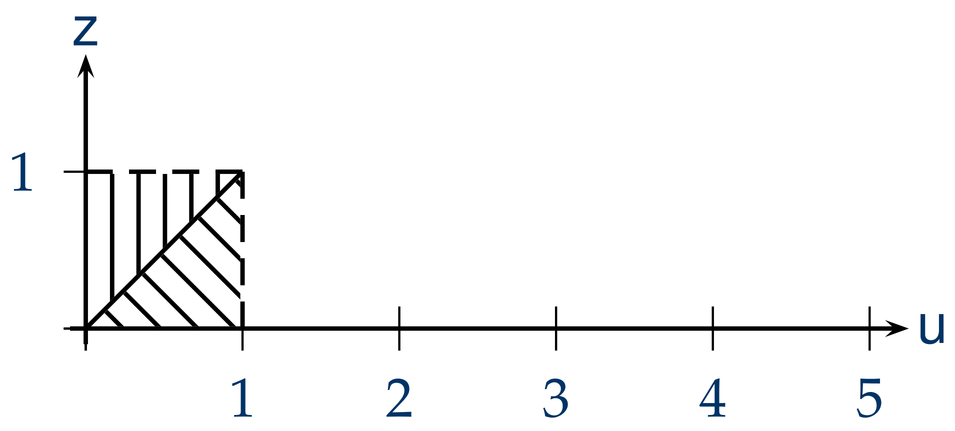

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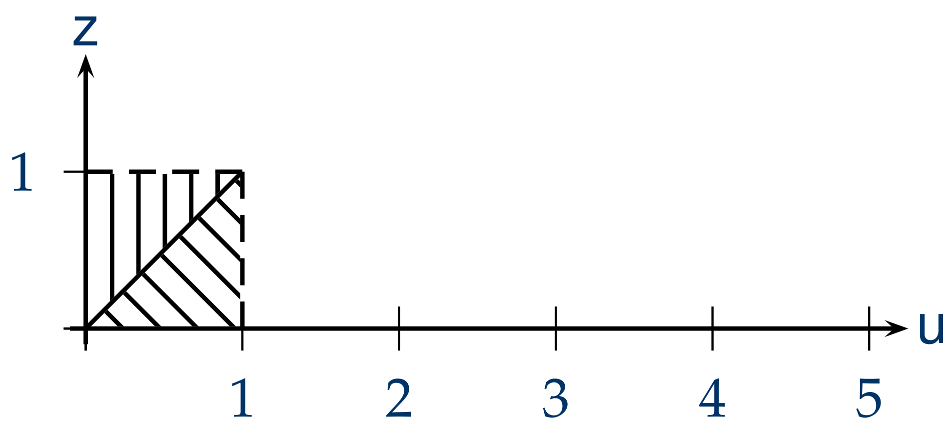

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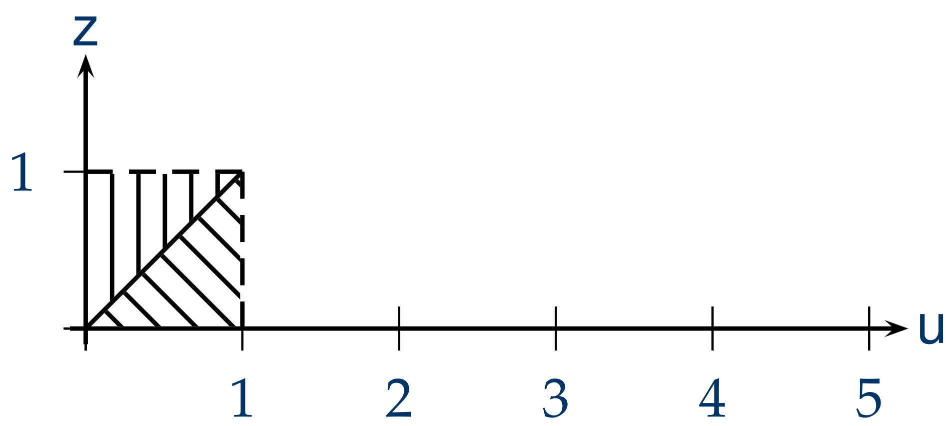

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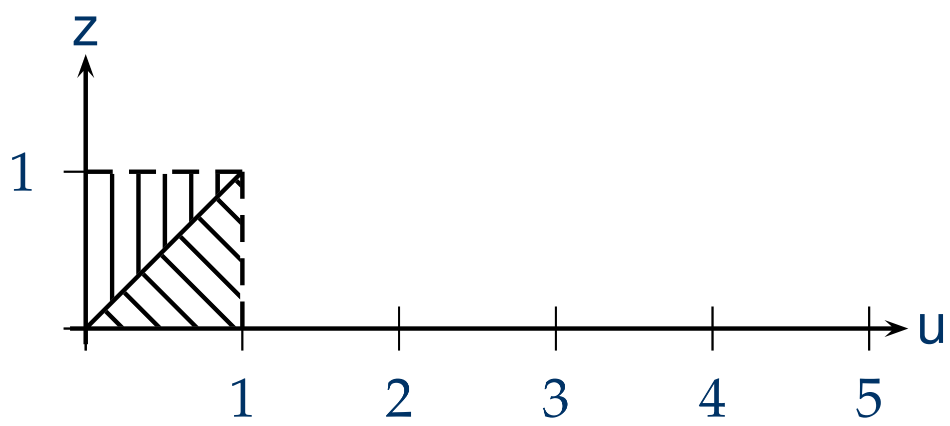

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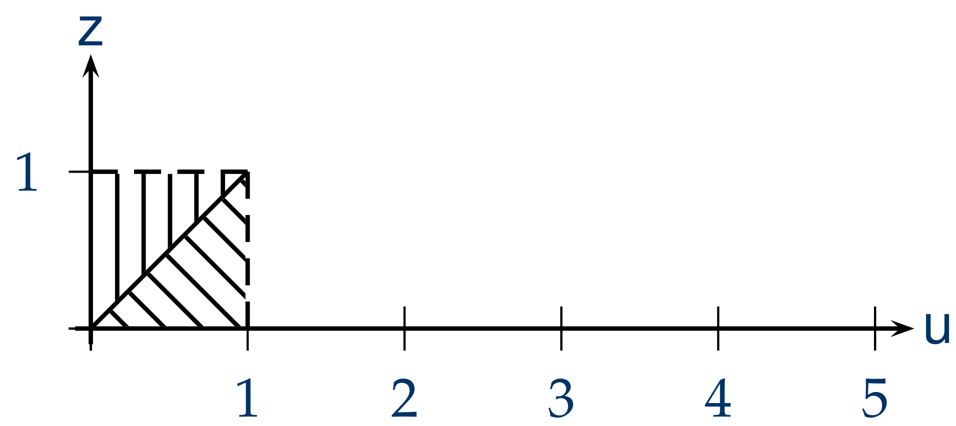

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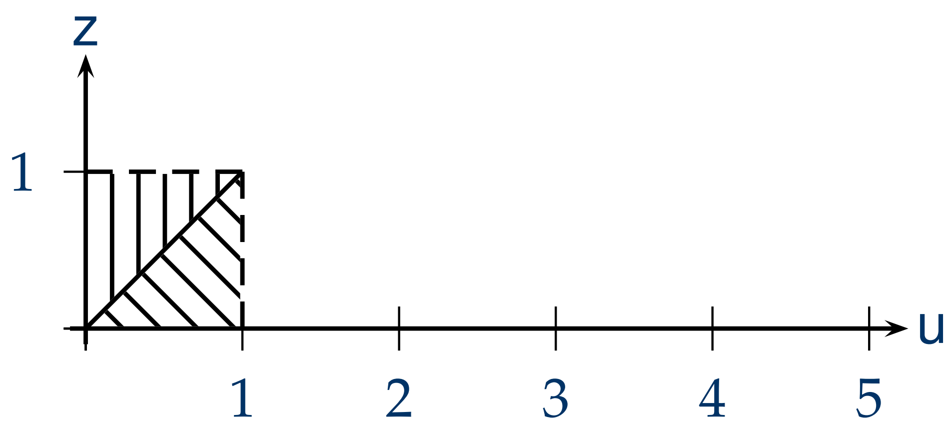

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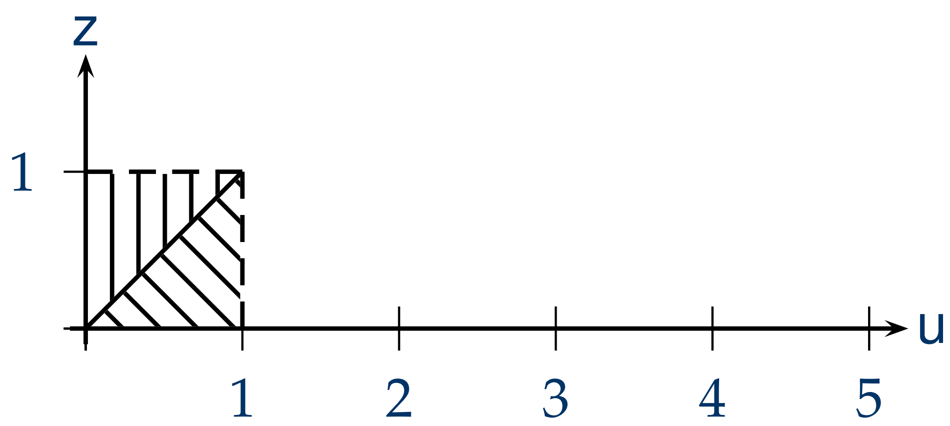

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and in the continuous case:

$$\mathrm{Res}_{j_s=b^{-2}/2}C(j_s,0|\alpha)A_\sigma(j|\alpha) = (1+(2j+1)b^2)A_\sigma(j+\frac{1}{2b^2}|\alpha)$$

Epilogue

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- Besides the AdS_2 branes, there is a second class: S^2 branes.
- What happens when studying the $b^{-2}/2$ constraint for them? (Mapping to Liouville Theory might not be possible...)

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Appendix

Digression: Duality to Liouville Theory

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$$\left\langle \Theta_{j_1}(u_1|z_1)\Theta_{j_2}(u_2|z_2) \right\rangle_{\alpha}^{(H)}$$

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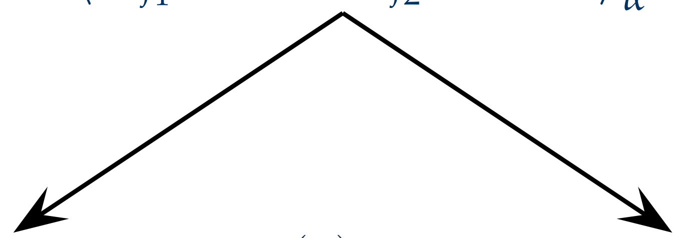

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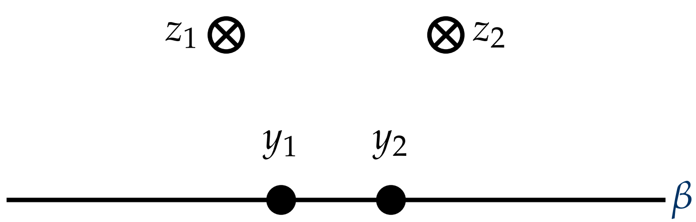

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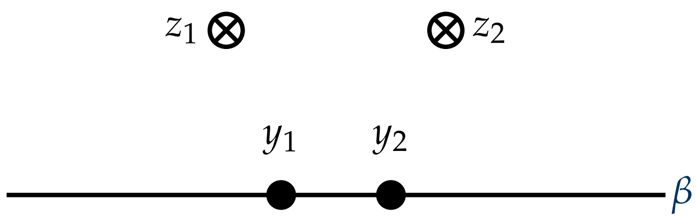
$$\left\langle V_{\gamma_1}(z_1)V_{\gamma_2}(z_2)B_{-\frac{1}{2b}}(y_1)B_{-\frac{1}{2b}}(y_2)\right\rangle_{\beta}^{(L)} \qquad \left\langle V_{\gamma_1}(z_1)V_{\gamma_2}(z_2) \qquad \right\rangle$$



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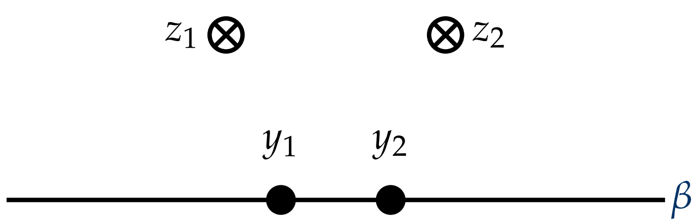

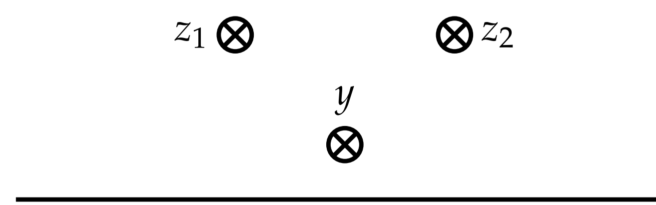

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Boundary Regime ($z \simeq 1$)

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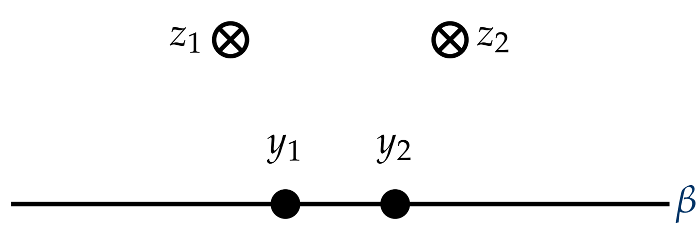

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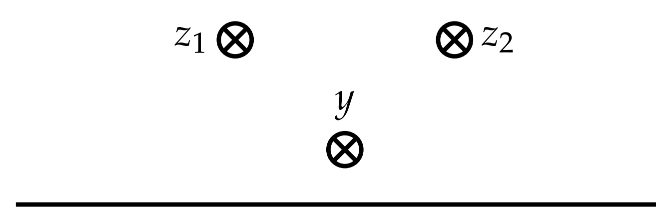

Bulk Regime ($z \simeq 0$)

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- $\Rightarrow$ Question:
- Is it still possible to derive a unique factorization constraint in this situation?