D-Branes in a Non-Compact WZNW Model

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- 3D (Euclidean) Anti-de-Sitter: Feasible Background
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- Focus on corresponding CFT (H_3^+ Model): Interesting in its own right!
- Reason: It is a *non-rational* CFT

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- ... how power of factorization constraint is maintained.

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- Central Theme: Making use of $\mathrm{SL}(2,\mathbb{C})$ symmetry.

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$$C(j_3, j_2, j_1) \propto \langle \Theta_{j_3}(u_3|z_3)\Theta_{j_2}(u_2|z_2)\Theta_{j_1}(u_1|z_1) \rangle$$

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- For $j' = 0$, i.e. $\Psi_{j'=0} = \mathbb{1}$: $C(j, 0|\alpha) = A(j|\alpha)$

Cardy-Lewellen Constraints

Factorization Constraint

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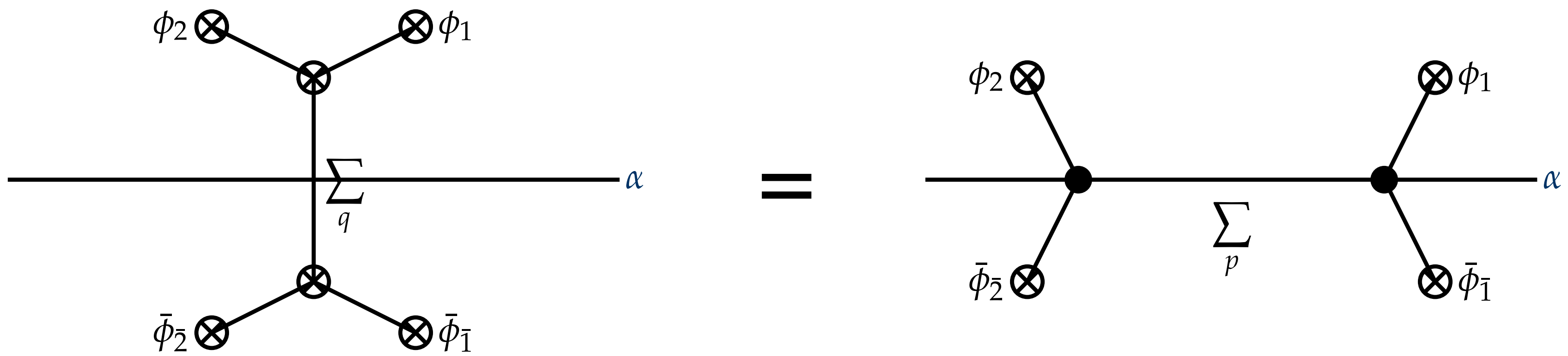

$$\sum_q C(2,1,q) F^q_{21,\bar{2}\bar{1}}(z) A(q|\alpha) \quad = \quad \sum_p C(2,p|\alpha) F^p_{2\bar{2},1\bar{1}}(1-z) C(1,p|\alpha)$$

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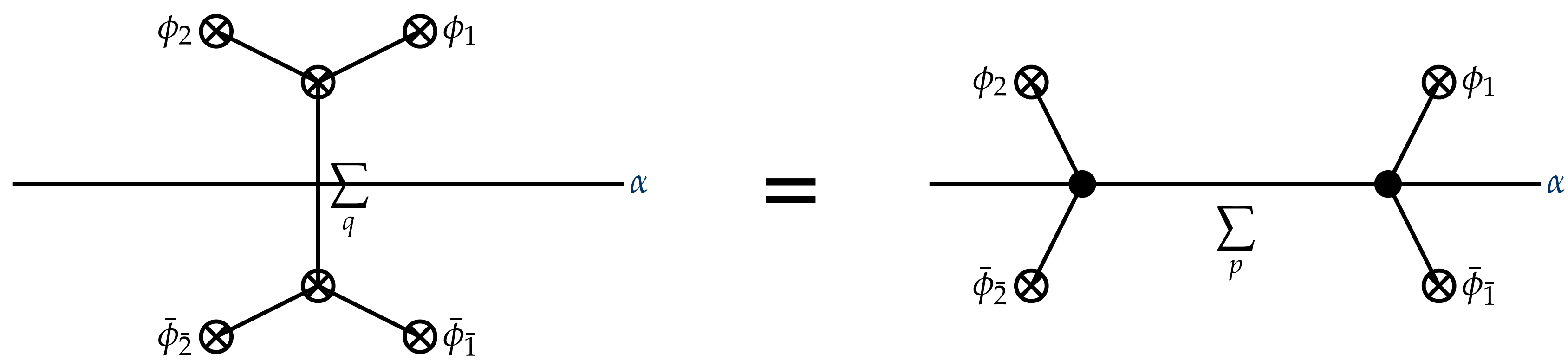

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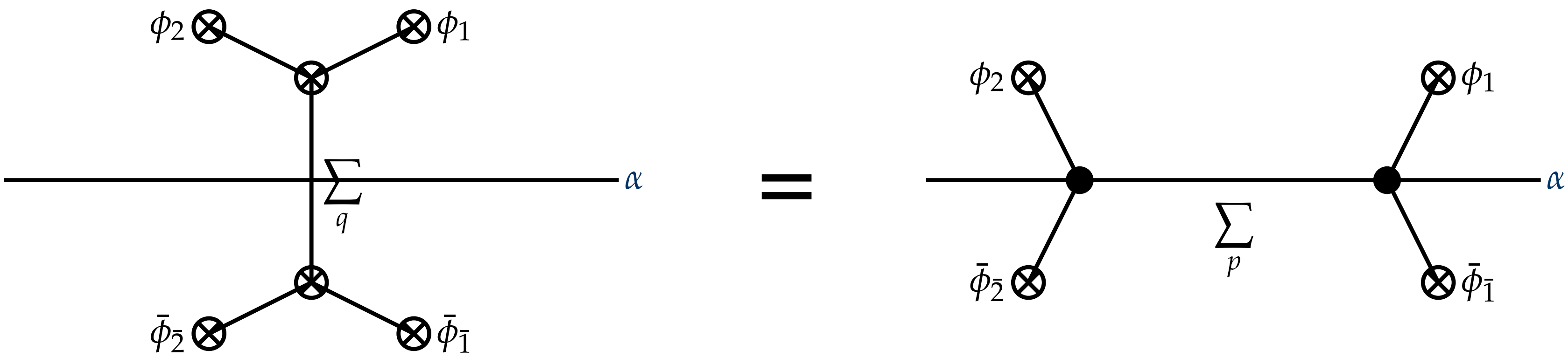
- Open string channel: Focus on contribution of identity field

Factorization Constraint



$$\sum_q C(2,1,q) F^q_{21,\bar2\bar1}(z) A(q|\alpha) = \sum_p C(2,p|\alpha) F^p_{2\bar2,1\bar1}(1-z) C(1,p|\alpha)$$

with crossing ratio $z = \frac{|z_2 - z_1|^2}{|z_2 - z_1^*|^2}$

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- ... on the LHS, the limit $z \to 1$ can be taken.

Implementation of Factorization Constraint in H_3^+

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- $\Rightarrow$ Need to distinguish two different kinds of D-branes: Continuous and Discrete.

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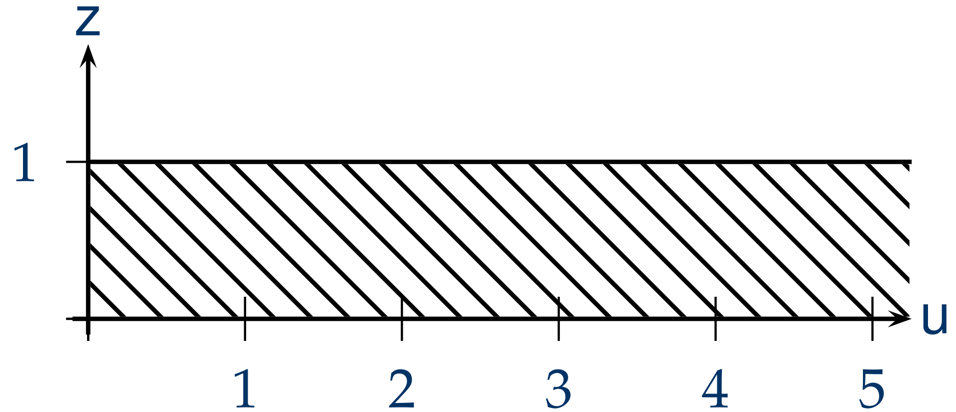

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- Need second constraint to fix leftover arbitrariness.

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with $C_\epsilon(j) = C(j, \frac{1}{2}, j_\epsilon)$, where

$$j_\pm = j \pm \frac{b^{-2}}{2} \qquad \text{and} \qquad j_\times = -j - 1 - \frac{b^{-2}}{2}$$

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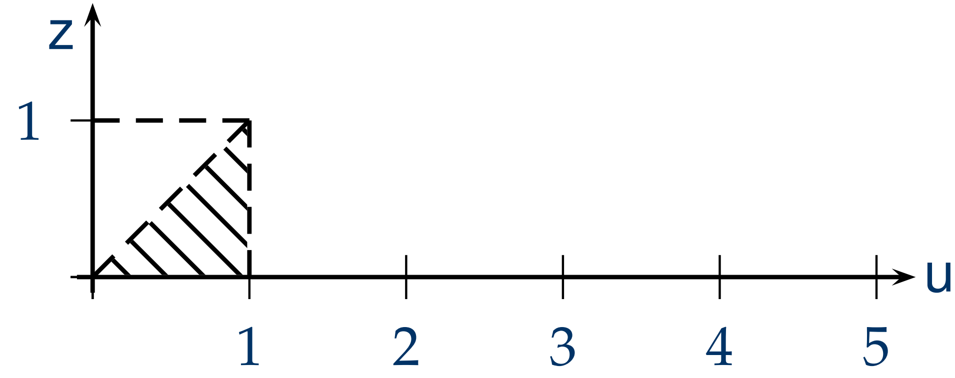

» Motivation . . .
» . . . and Outline

The $\mathbf{H}_3^+$ Model

Cardy-Lewellen Constraints

Implementation of Factorization Constraint in $\mathbf{H}_3^+$

The '$b^{-2}/2$'-Constraint
» Knizhnik-Zamolodchikov equation
» Problem
» Resolution
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Epilogue

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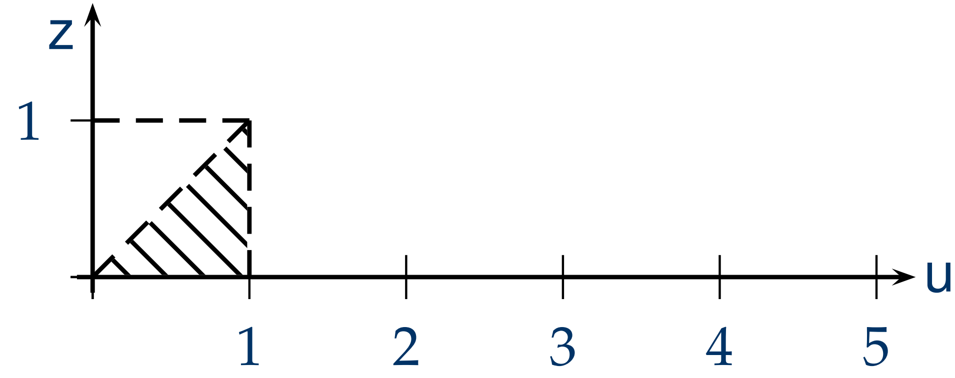
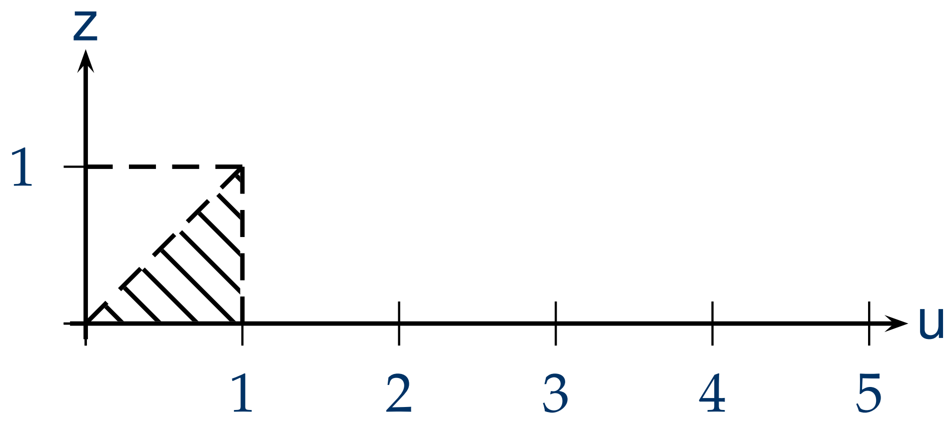


- Thus, limit $z \to 1$ cannot be taken! No constraint derivable!

Resolution



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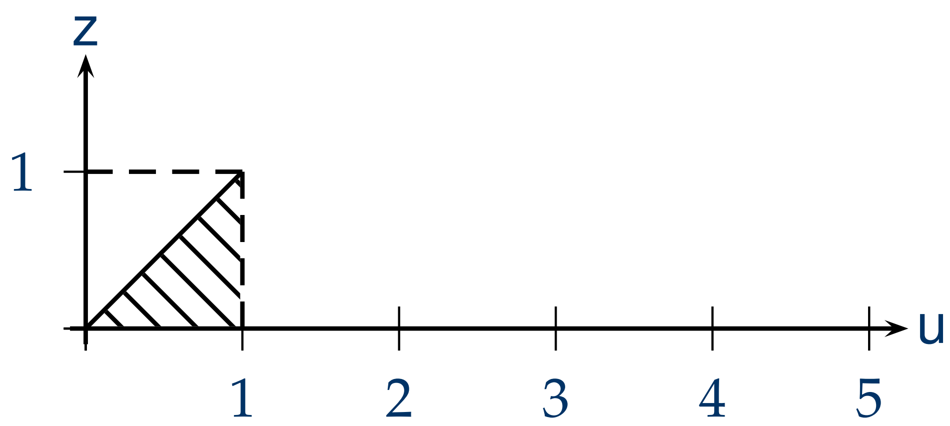

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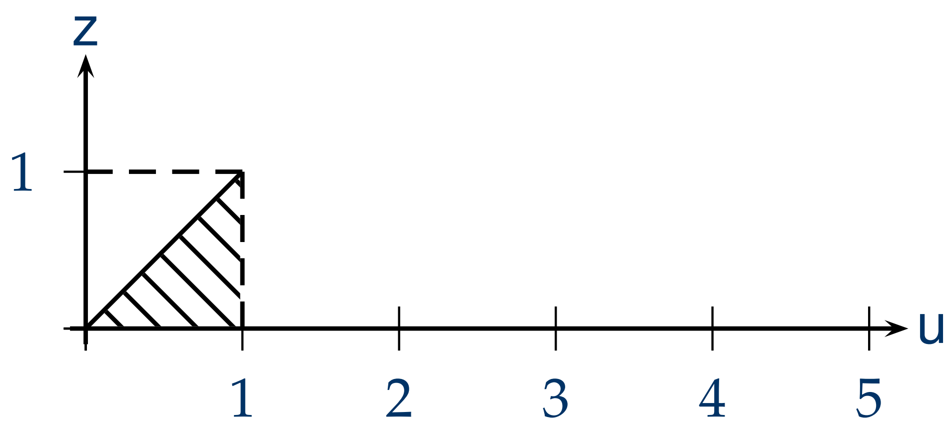

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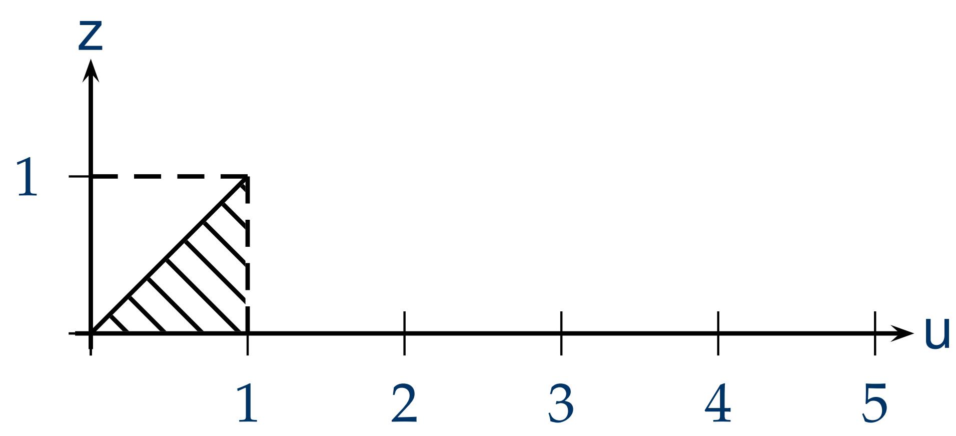

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- $u = z$ suggested to be a "new type of singularity" (*Hosomichi-Ribault proposal*)

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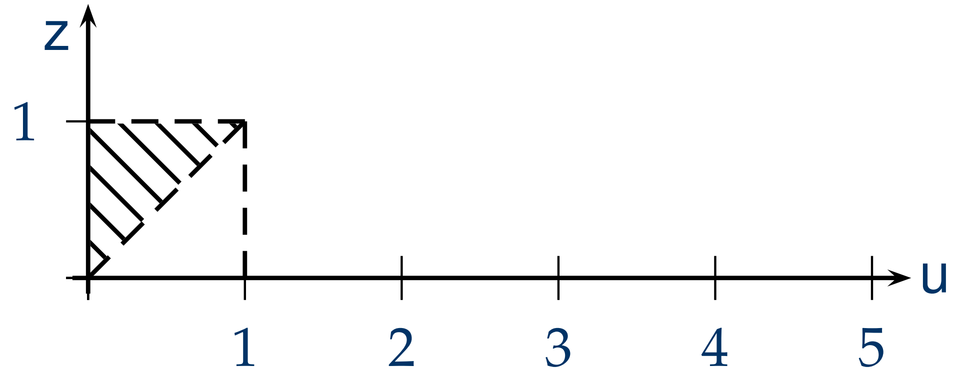

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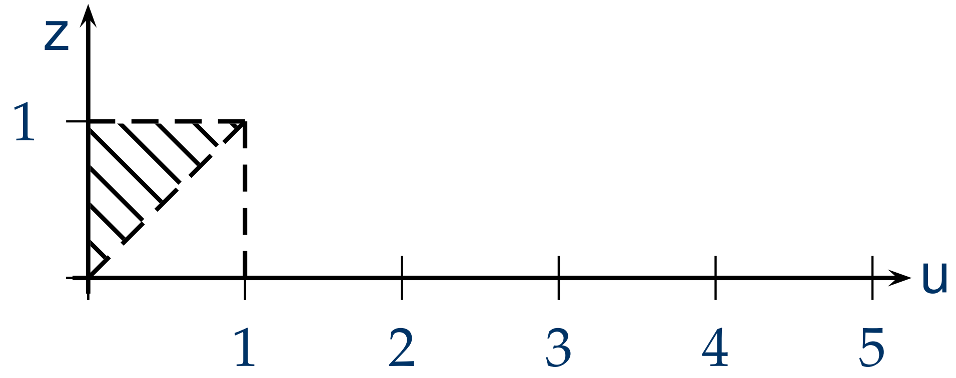

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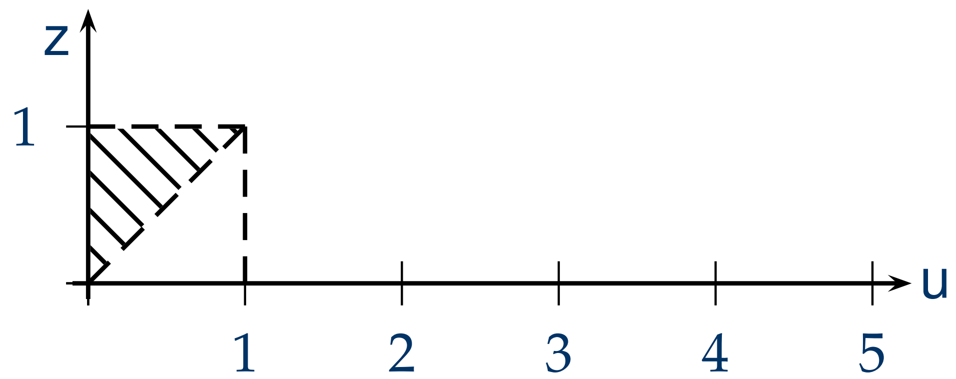

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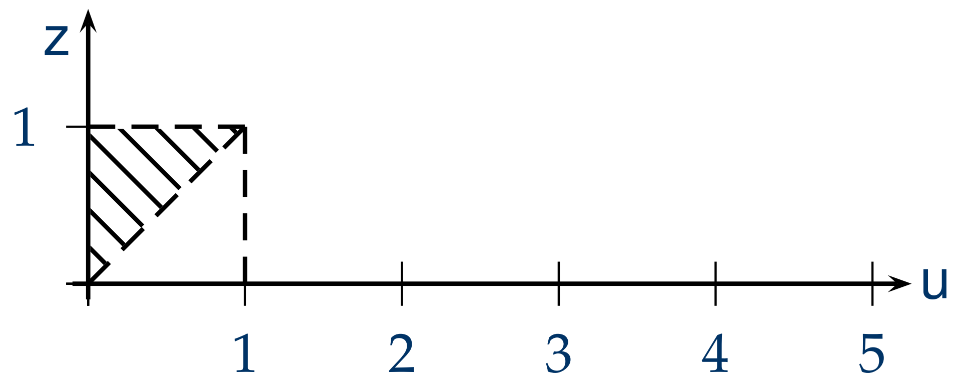
- We can also give a basis-set of solutions $\tilde{\mathcal{F}}^s_{j,+}(u|z)$, $\tilde{\mathcal{F}}^s_{j,-}(u|z)$ and $\tilde{\mathcal{F}}^s_{j,\times}(u|z)$ which are well-defined for $u < z < 1$.

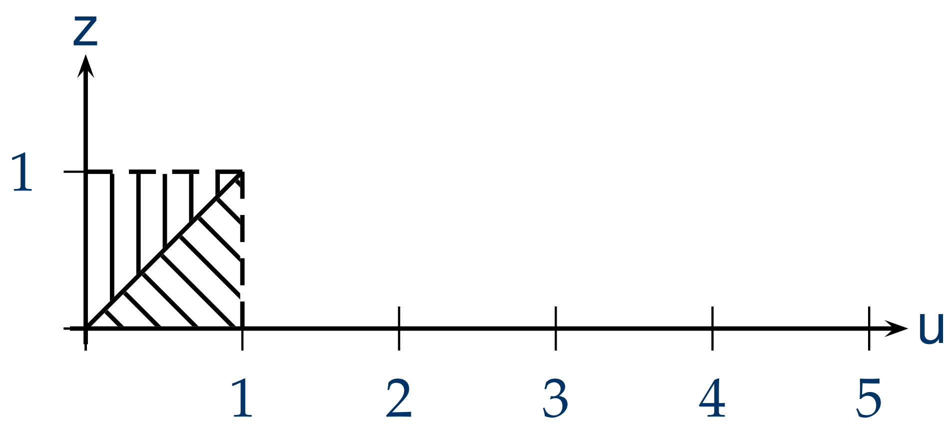


- Solution in region $u < z < 1$ is therefore expanded as

$$\tilde{G}^{(2)}_{j,\alpha}(u_i|z_i) \propto \sum_{\epsilon=+,-,\times} \tilde{a}^j_\epsilon(\alpha)\tilde{\mathcal{F}}^s_{j,\epsilon}(u|z)$$

- Again, it is finite at $u = z$.
- But also again, $\tilde{\mathcal{F}}^s_{j,-}(u|z)$ and $\tilde{\mathcal{F}}^s_{j,\times}(u|z)$ become linearly dependent at $u = z$.

Matching of Solutions



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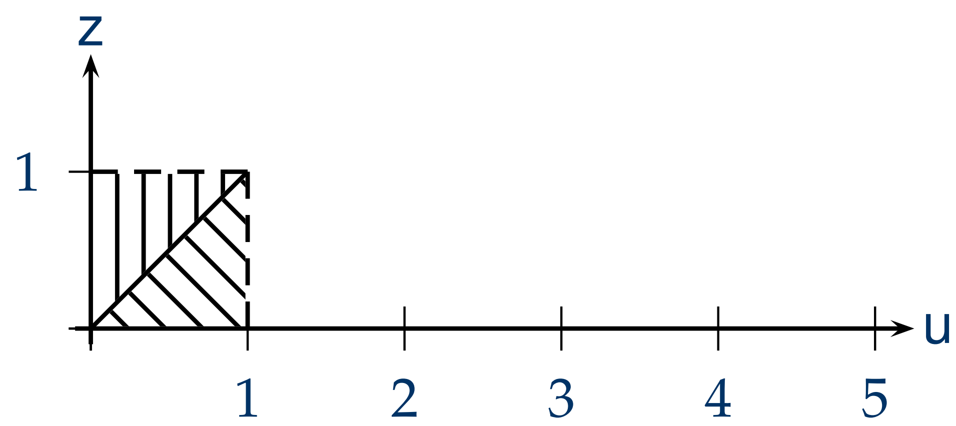

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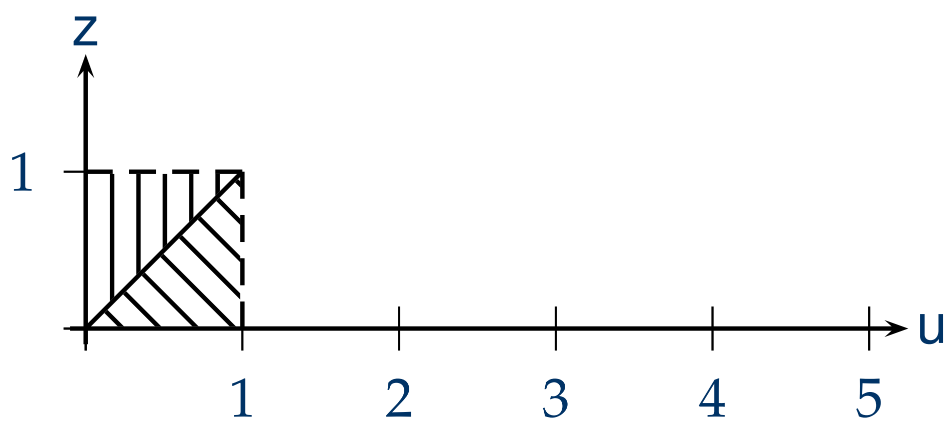

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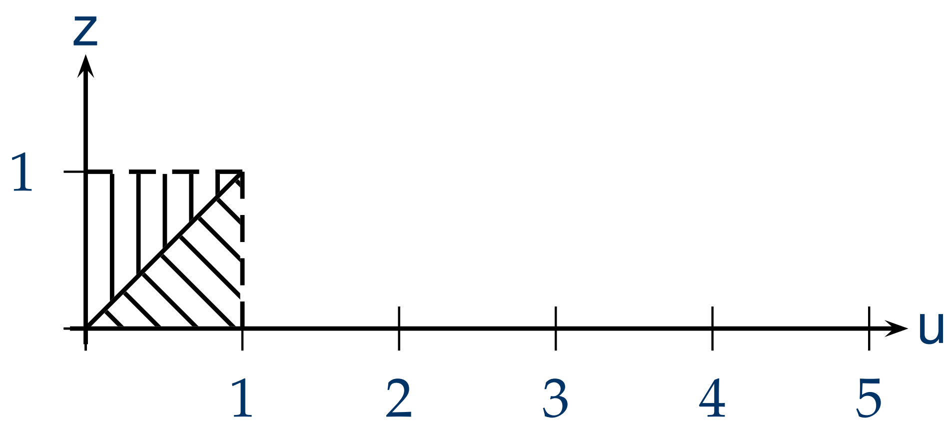

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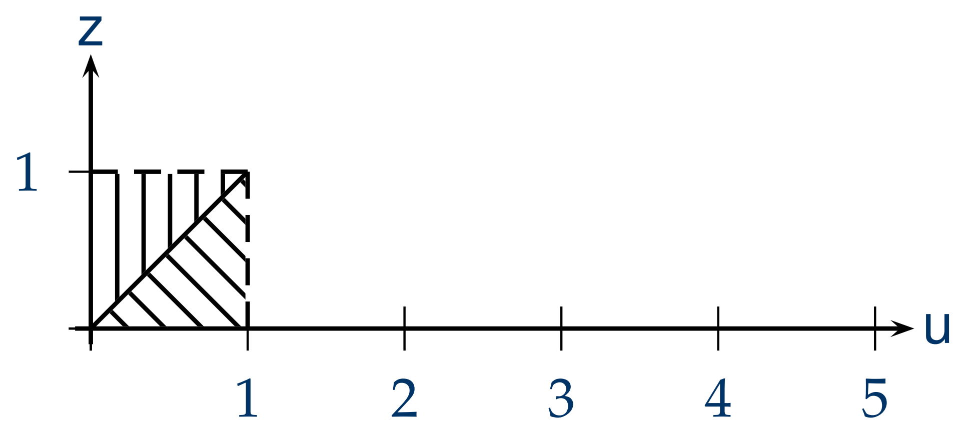

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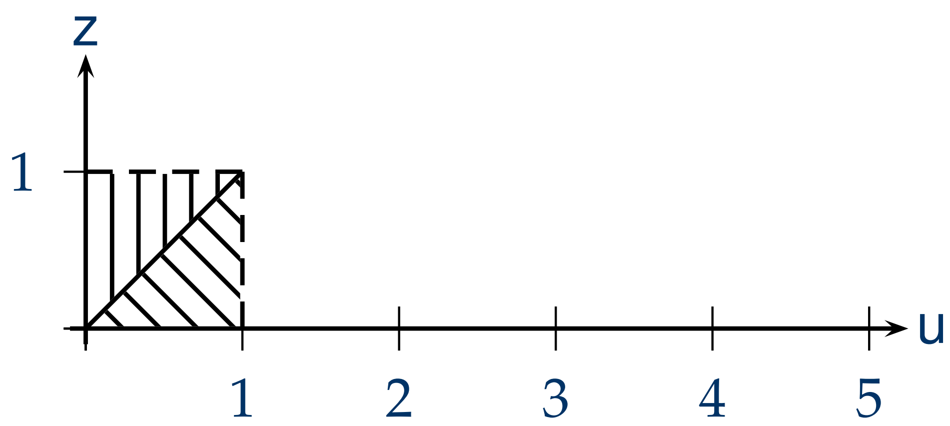

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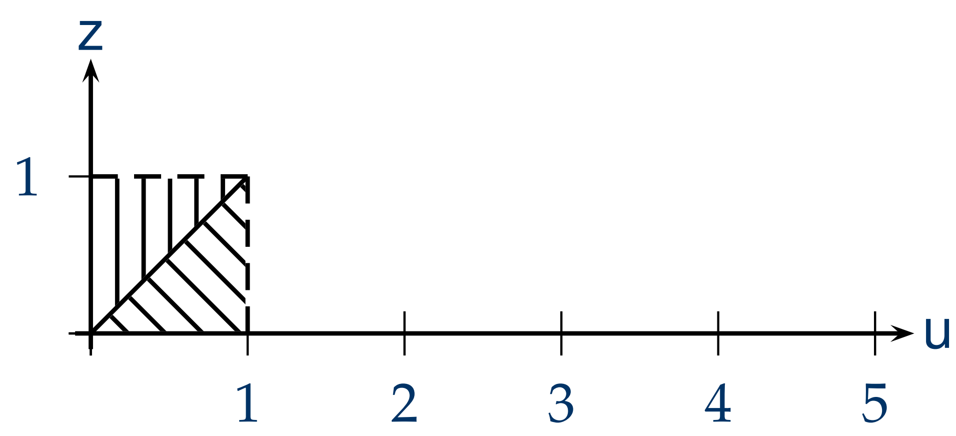

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- Henceforth, it seems that the factorization constraint looses its power.

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- New shift equations are in the discrete case:

$$A_\sigma(j|\alpha)A_\sigma(\frac{1}{2b^2}|\alpha) = \frac{1+(2j+1)b^2}{1+b^2}A_\sigma(j+\frac{1}{2b^2}|\alpha)$$

and in the continuous case:

$$\mathrm{Res}_{j_s=b^{-2}/2}C(j_s,0|\alpha)A_\sigma(j|\alpha) = (1+(2j+1)b^2)A_\sigma(j+\frac{1}{2b^2}|\alpha)$$

Epilogue

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- Its existence has been suggested by a mapping to Liouville theory (Hosomichi-Ribault proposal).

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- Besides the AdS_2 branes, there is a second class: S^2 branes.
- What happens when studying the $b^{-2}/2$ constraint for them? (Mapping to Liouville Theory might not be possible...)

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