Aspects of $(0,2)$ Orbifolds and Mirror Symmetry

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Abstract

We study orbifolds of $(0,2)$ models and their relation to $(0,2)$ mirror symmetry. In the Landau-Ginzburg phase of a $(0,2)$ model the superpotential features a whole bunch of discrete symmetries, which by quotient action lead to a variety of consistent $(0,2)$ vacua. We study a few examples in very much detail. Furthermore, we comment on the application of $(0,2)$ mirror symmetry to the calculation of Yukawa couplings in the space-time superpotential.

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1. Introduction

Despite gradual progress in revealing the existence and structure of phenomenologically promising $(0,2)$ world-sheet supersymmetric compactifications of the heterotic string [1-18], the knowledge we have is still far less compared to their more prominent left-right symmetric subset of $(2,2)$ models. Since some special elliptically fibered $(0,2)$ models in both six and four dimensions made their appearance in conjectured F-theory, heterotic string dualities [19], to have a better understanding, in particular of their moduli spaces, clearly is desirable. In the $(2,2)$ case, a combination of some non-renormalization theorems for certain couplings in the superpotential [20],[21] and mirror symmetry served as powerful tools for formulating an exact geometric description of the complex and Kähler moduli spaces [22].

In the $(0,2)$ case we are on much looser ground. On the one hand, the proof of exactness of certain Yukawa couplings in the large radius limit heavily relied on the left-moving world sheet $N=2$ supersymmetry. Furthermore, for small radius there does not even exist an algebraic distinction among the possible complex, Kähler and bundle moduli. On the other hand, $(0,2)$ mirror symmetry is still in its infancy. Even though for a special subset of $(0,2)$ models strong indications of mirror symmetry have been found in [23], we do not know whether this duality extends to more general $(0,2)$ compactifications.

With this background in mind, in this letter we investigate further the implementation of mirror symmetry in the $(0,2)$ context and its application to the calculations of certain 3-point couplings. Recently, a description of orbifolds of $(0,2)$ models in their Landau-Ginzburg phase has been presented [24]. In particular, a formula for the elliptic genus in the quotient model has been derived. In contrast to the $(2,2)$ case, a priori the $(0,2)$ superpotential has an infinite number of discrete symmetries subject to some anomaly constraints. After reviewing the basics of the orbifold construction, we systematically study $(0,2)$ orbifolds of a $(0,2)$ orbifold descendant of the $(2,2)$ quintic in very much detail. We find that even by modding out in each case only one discrete symmetry one ends up with a large number of different models showing (almost) mirror symmetry. As a by-product we find that the simple current construction given in [12] is nothing else than a Z_2 $(0,2)$ orbifold of a $(2,2)$ model. This gives a way of constructing consistent $(0,2)$ models as orbifold descendants of $(2,2)$ models. In order to see whether $(0,2)$ mirror symmetry is only an artifact for such descendants of $(2,2)$ models, we also study an example which is not supposed to be of this type.

In the last section we draw the minimal conclusion from mirror symmetry, allowing us to derive simple selection rules for some Yukawa couplings of the form $\langle 10, \overline{16}, \overline{16} \rangle$ in the case of $SO(10)$ gauge group.

2. Review of $(0,2)$ Orbifolds

We consider $(0,2)$ models described by linear $\sigma-$ models [4] which for small radius $r \ll 0$ are equivalent to $(0,2)$ Landau-Ginzburg models [1]. There is a number of chiral superfields:

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[1] The restriction to linear $\sigma-$ models might exclude a lot of the elliptically fibered models naturally arrising in recent F-theory/heterotic dualities. As shown in [11] the former models are generically not subject to world-sheet instanton corrections of the space time superpotential, whereas such a feature is expected from special divisors in F-theory [25]

$\{\Phi_i | i = 1, \ldots, N\}$ and a number of Fermi superfields: $\{\Lambda^a | a = 1, \ldots, M = N_a + N_j\}$ which are governed by a superpotential of the form

$$W = \Lambda^a F_a(\Phi_i) + \Lambda^{N_a+j} W_j(\Phi_i) \,, \tag{2.1}$$

where W_j and F_a^l are quasi-homogeneous polynomials. In the large radius limit the W_j define hypersurfaces in a weighted projective space and the F_a define a vector bundle on this space. For appropriate choices of the constraints W_j and F_a, the superpotential has an isolated singularity at the origin and is quasi-homogeneous of degree one, if one assigns charges ω_i/m to Φ_i, n_a/m to Λ_a, and $1 - d_j/m$ to Λ_{N_a+j}. Quasi-homogeneity implies the existence of a right-moving R-symmetry, and a left-moving $U(1)_L$. The associated currents are denoted by J_R and J_L, respectively. The charges of the various fields with respect to these $U(1)$ currents are summarized in the following table:

Field	ϕ_i	ψ_i	λ_a	λ_{N_a+j}
q_L	$\frac{\omega_i}{m}$	$\frac{\omega_i}{m}$	$\frac{n_a}{m} - 1$	$-\frac{d_j}{m}$
q_R	$\frac{\omega_i}{m}$	$\frac{\omega_i}{m} - 1$	$\frac{n_a}{m}$	$1 - \frac{d_j}{m}$

Table 1: *Left and right charges of the fields in the LG model.*

Of course, the fermions, ψ_i, belong to the chiral superfield, Φ_i, while the fermions, λ_a are the lowest components of the Fermi superfields Λ_a. Anomaly cancellation for these two global $U(1)$ symmetries is equivalent to the anomaly conditions expected from the large radius limit:

$$\sum \omega_i = \sum d_j \,, \qquad \sum n_a = m \,,$$
$$\sum d_j^2 - \sum w_i^2 = m - \sum n_a^2 \,. \tag{2.2}$$

For appropriate choices of the functions W_j and F_a, in general there exist a bunch of discrete symmetries of the superpotential acting on the fields as

$$\Phi_i \to e^{2\pi i q_i} \Phi_i, \Lambda_a \to e^{-2\pi i q_a} \Lambda_a \,. \tag{2.3}$$

This defines a $\mathbb{Z}_h$ action on the fields, where h is the minimal common denominator of the the charges q_i, q_a. In general one has multiple quotient actions of order $h^0, \ldots, h^{P-1}$, where the first quotient should be the GSO projection $\mathbb{Z}_m$. Then one can define the following quantities:

$$R^{\mu\nu} = \sum_{a=1}^M q_a^\mu q_a^\nu - \sum_{i=1}^N q_i^\mu q_i^\nu \,, \qquad r^\mu = \sum_{a=1}^M q_a^\mu - \sum_{i=1}^N q_i^\mu \,. \tag{2.4}$$

As was shown in [24] the orbifold partition function can be written as a sum over all twisted sectors as

$$Z_{orb}(\tau, \nu) = \frac{1}{\prod h^\mu} \sum_{\alpha^0, \beta^0 = 0}^{h^0-1} \cdots \sum_{\alpha^{P-1}, \beta^{P-1} = 0}^{h^{P-1}-1} \epsilon(\vec{\alpha}, \vec{\beta}) \, \vec{\beta} \underset{\vec{\alpha}}{\boxed{}} (\tau, \nu, 0) \,, \tag{2.5}$$

and is modular invariant only if the phase factor

$$\epsilon(\vec{\alpha}, \vec{\beta}) = e^{\pi i \vec{w}(\vec{\alpha}+\vec{\beta})} \, e^{\pi i \vec{\alpha} Q \vec{\beta}} \tag{2.6}$$

satisfies

$$Q^{\mu\nu} + Q^{\nu\mu} \in 2\mathbb{Z}\,, \qquad w^\mu + Q^{\mu\mu} \in 2\mathbb{Z}\,,$$
$$(w^\mu - r^\mu)h^\mu = 0 \mod 2\,, \qquad (Q^{\mu\nu} + R^{\mu\nu})h^\nu = 0 \mod 2\,, \tag{2.7}$$

for any $\mu, \nu \in \{0, \ldots, P-1\}$. These conditions provide constraints on r^μ and $R^{\mu\mu}$:

$$r^\mu h^\mu \in \begin{cases} 2\mathbb{Z} \text{ for } h^\mu \text{ even}\,, \\ \mathbb{Z} \text{ for } h^\mu \text{ odd}\,, \end{cases} \qquad R^{\mu\mu} h^\mu \in \begin{cases} 2\mathbb{Z} \text{ for } h^\mu \text{ even}\,, \\ \mathbb{Z} \text{ for } h^\mu \text{ odd}\,. \end{cases} \tag{2.8}$$

For the off-diagonal terms one obtains the condition

$$R^{\mu\nu} = \frac{\mathbb{Z}}{h^\mu} + \frac{\mathbb{Z}}{h^\nu}\,. \tag{2.9}$$

In order for the quotient theory to be used as the internal sector of a heterotic string theory, there are some further conditions on the charges that have to be satisfied. The difference of the left and right moving $U(1)$ charges in every single twisted sector must be an integer:

$$\sum_a \vec{q}_a - \sum_i \vec{q}_i \in \mathbb{Z}^P\,. \tag{2.10}$$

The gauginos form the untwisted sector must not be projected out

$$\vec{w} = \left(\sum_a \vec{q}_a - \sum_i \vec{q}_i \right) \mod 2\,. \tag{2.11}$$

Lastly, we want our canonical projection onto states with left-moving charge $q_L = \frac{1}{2}r^0 \mod \mathbb{Z}$. This requirement leads to the condition,

$$(Q^{\mu 0} - R^{\mu 0}) \in 2\mathbb{Z}\,. \tag{2.12}$$

This determines $Q^{\mu 0}$ in terms of $R^{\mu 0} \mod 2$.

The massless sector of the orbifold contributes only to the so-called χ_y genus defined as

$$\chi_y = \lim_{q \to 0} (i)^{N-M} q^{\frac{N-M}{12}} y^{\frac{1}{2}r^0} Z_{orb}(q, y)\,. \tag{2.13}$$

We denote the contribution to χ_y from a twisted sector $\vec{\alpha}$ by $\chi_y^{\vec{\alpha}}$. The contribution from each twisted sector is determined in terms of the function,

$$f^{\vec{\alpha}}(\vec{z}) = (-1)^{\vec{w}\vec{\alpha}} e^{2\pi i \vec{z} \vec{Q}_{\vec{\alpha}}} q^{E_{\vec{\alpha}}} \frac{\prod_a (-1)^{[\vec{\alpha}\vec{q}_a]} (1 - e^{2\pi i \vec{z}\vec{q}_a} q^{\{\vec{\alpha}\vec{q}_a\}})(1 - e^{-2\pi i \vec{z}\vec{q}_a} q^{1-\{\vec{\alpha}\vec{q}_a\}})}{\prod_i (-1)^{[\vec{\alpha}\vec{q}_i]} (1 - e^{2\pi i \vec{z}\vec{q}_i} q^{\{\vec{\alpha}\vec{q}_i\}})(1 - e^{-2\pi i \vec{z}\vec{q}_i} q^{1-\{\vec{\alpha}\vec{q}_i\}})}\,, \tag{2.14}$$

where $\chi_y^{\vec{\alpha}}$ is given by expanding $f^{\vec{\alpha}}(\vec{z})$ in powers of q, and retaining terms of the form $q^0 e^{-2\pi i \vec{z}(\vec{\sigma}+\vec{n})}$, where $\vec{n} \in \mathbb{Z}^P$ and $\vec{\sigma} = \frac{1}{2}\vec{w} + \frac{1}{2}\vec{\alpha}(Q - R)$. Finally, we set $z_1 = \ldots = z_{P-1} = 0$. Furthermore, we have used the abbreviation $\{x\} = x - [x]$ in (2.14). The fractionalized charges and energies in the twisted sectors are given by the formulae:

$$\vec{Q}_{\vec{\alpha}} = \sum_a \vec{q}_a (\vec{\alpha}\vec{q}_a - [\vec{\alpha}\vec{q}_a] - \frac{1}{2}) - \sum_i \vec{q}_i (\vec{\alpha}\vec{q}_i - [\vec{\alpha}\vec{q}_i] - \frac{1}{2})\,,$$
$$E_{\vec{\alpha}} = \frac{1}{2} \sum_a (\vec{\alpha}\vec{q}_a - [\vec{\alpha}\vec{q}_a] - 1)(\vec{\alpha}\vec{q}_a - [\vec{\alpha}\vec{q}_a]) - \frac{1}{2} \sum_i (\vec{\alpha}\vec{q}_i - [\vec{\alpha}\vec{q}_i] - 1)(\vec{\alpha}\vec{q}_i - [\vec{\alpha}\vec{q}_i])\,. \tag{2.15}$$

This is a formula which can easily be put onto a computer, making more excessive calculations feasible.

3. Some special quotients

3.1. (0,2) quotients of (2,2) models

In this section we apply the orbifolding procedure to some special models, leading to interesting aspects of $(0,2)$ models. First, we consider $(2,2)$ models which are given by a hypersurface in a weighted projective space $\mathbb{P}_{\omega_1,\ldots,\omega_5}[d]$. Let us transform such a model to a $(0,2)$ model with data

$$V(\omega_1,\ldots,\omega_5;d) \longrightarrow \mathbb{P}_{\omega_1,\ldots,\omega_5}[d]. \qquad (3.1)$$

Note that in the Landau-Ginzburg phase the superpotential is

$$W = \sum_{i=1}^{5} \Lambda_i \frac{\partial P}{\partial \phi_i} + \Lambda_6\, P. \qquad (3.2)$$

with P being a transversal polynomial of degree d. Calculating the massless spectrum of such a model gives exactly the $(2,2)$ result with extra gauginos occurring in a twisted and the untwisted sector extending the gauge group from $SO(10)$ to E_6. By decoupling the left moving fermions λ from the bosons ϕ we are free to consider also general $(0,2)$ orbifolds. If $d/\omega_1 = 2l+1$ is odd we deform the superpotential (3.2) to

$$W = \Lambda_1 \phi_1^{2l} + \sum_{i=2}^{5} \Lambda_i \frac{\partial P}{\partial \phi_i} + \Lambda_6\, \phi_1^{2l+1} \qquad (3.3)$$

and divide by the following $\mathbb{Z}_2$ action

$$J = \left(\frac{2l+1}{2}, 0, 0, 0, 0; 0, 0, 0, 0, 0, \frac{2l-3}{2} \right) \qquad (3.4)$$

which satisfies all anomaly conditions (2.8)(2.9). By calculating a few examples one finds that this orbifold corresponds exactly to the implementation of the simple current

$$(\vec{q}_i, \vec{q}_a) = (0\ 2l+1\ 1)(0\ 0\ 0)^4(1)(0) \qquad (3.5)$$

into the conformal field theory partition function introduced in [12]. Thus, following [14], the move from a $(2,2)$ model $\mathbb{P}_{\omega_1,\ldots,\omega_5}[d]$ to a $(0,2)$ model

$$V(\omega_1,\ldots,\omega_5;d) \longrightarrow \mathbb{P}_{2\omega_1,l\omega_1,\omega_2,\ldots,\omega_5}[(l+2)\omega_1, 2l\omega_1] \qquad (3.6)$$

can be described as a $(0,2)$ orbifold of the $(2,2)$ model. Analogously, one expects that $(2,2)$ models can produce different kinds of $(0,2)$ models via quotient actions. Thus, orbifolding provides a nice way of constructing $(0,2)$ descendants out of $(2,2)$ models.

3.2. Classifying all (0,2) orbifolds of the quintic

Exactly for the class of models reinterpreted as orbifolds in the last section, mirror symmetry has been investigated in [14]. If the $(2,2)$ model is of Fermat type, it has been shown by Greene and Plesser [26] that orbifolding by the maximal discrete symmetry (preserving

the left moving $N = 2$ symmetry) leads to the mirror model. Even more striking, successive orbifolding leads to a completely mirror symmetric set of vacua. We want to see whether a similar pattern also holds in the $(0, 2)$ context. In [24] some orbifolds of the $(0, 2)$ descendant of the quintic

$$V(1, 1, 1, 1, 1; 5) \longrightarrow \mathbb{P}_{1,1,1,1,2,2}[4, 4] \tag{3.7}$$

have been constructed. Successive modding by a few generating $\mathbb{Z}_5$ orbifolds and introducing non-trivial discrete torsion has lead to a mirror symmetric subset of all $(0, 2)$ orbifold models. In this section we want to be more ambitious and start a classification of all possible discrete symmetries of the superpotential

$$W = \sum_{i=1}^{4} \lambda_i \phi_i^4 + \lambda_5 \phi_5^2 + \lambda_6 \phi_6^2 + \lambda_7 \phi_5 \phi_6 \,. \tag{3.8}$$

As a first observation, the decoupling of fermionic and bosonic degrees of freedom in the superpotential allows for an infinite set of solutions of the Diophantine equations encoding the conditions for its invariance and the consistency of the model. In particular, for almost any n we have a non-empty set of solutions with $\mathbb{Z}_n$ symmetry. However, the set of different models with different spectra is finite, but much larger as in the case of $(2, 2)$ models. Due to the torsion $Q^{\mu,0}$ determined by the condition (2.12), one can restrict the search for solutions with $\mathbb{Z}_n$ symmetry to integers modulo n such that for each n there are only finitely many possibilities to check.

As a second observation, we note that e.g. all the orbifold models of the $(0, 2)$ descendant of the quintic constructed by successive modding by certain $\mathbb{Z}_5$ symmetries can also be found by modding just once with a higher symmetry, for example $\mathbb{Z}_{15}$ or $\mathbb{Z}_{25}$. Led by the theory of induced representations for poly-cyclic groups, we conjecture that *all* $(0, 2)$ orbifolds to a given basis model can be obtained by modding out just one (suitable high) symmetry $\mathbb{Z}_n$.

For the $(0, 2)$ descendant of the quintic we found all possible orbifold solutions with one $\mathbb{Z}_n$ symmetry modded out, $2 \leq n \leq 37$. This yields 107850 models, but only 180 different $SO(10)$ spectra and 91 different E_6 spectra as well as two $SO(12)$ orbifold models with $N_{32} = N_{\overline{32}} = 6$ or 4. There are non-trivial solutions for all these n except $n = 2, 4$. The set of different orbifold models obtained so far is certainly not yet complete, but it is almost. Assuming the correctness of our conjecture above, and keeping in mind that the naive symmetry of the quintic is $\mathbb{Z}_5$, we conjecture that $\mathbb{Z}_n$ orbifolds with an upper limit $n = 125 = 5^3$ would exhaust the complete set of orbifold models – which, however, is outside our computation abilities.

The main observation now is that already our yet incomplete set does enjoy mirror symmetry to a surprisingly high extent, if $SO(10)$ models are considered. The situation for E_6 models is much less clear, which mainly is due to the limited ability to read off gauginos in untwisted sectors using the method introduced in [23]. Figures 1 and 2 present our results.

3.3. Mirror symmetry for general $(0, 2)$ models

Mirror symmetry might be something to be expected for $(0, 2)$ descendants of $(2, 2)$ models. Therefore, it is natural to look for orbifolds of a general $(0, 2)$ model which is not a $(2, 2)$
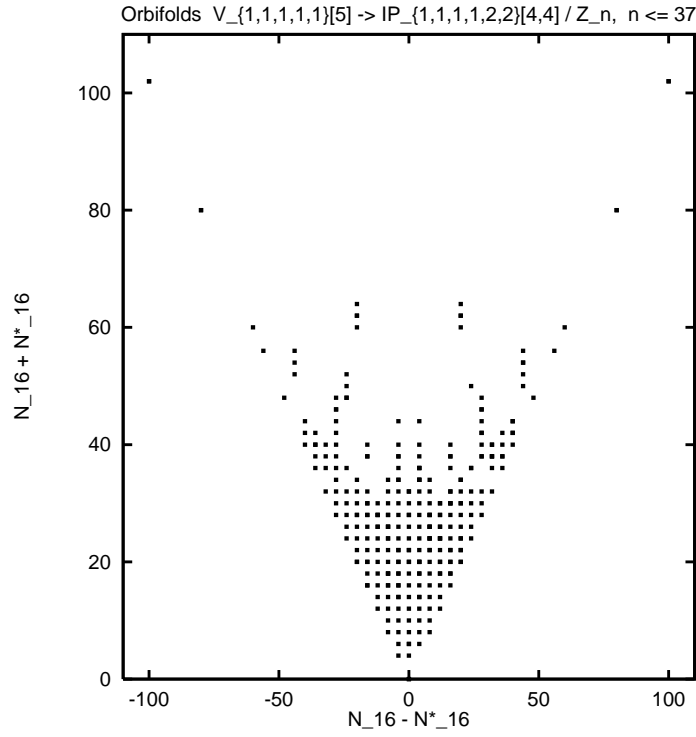
Figure 1: *The almost complete set of orbifolds for the (0, 2) descendant of the quintic.*
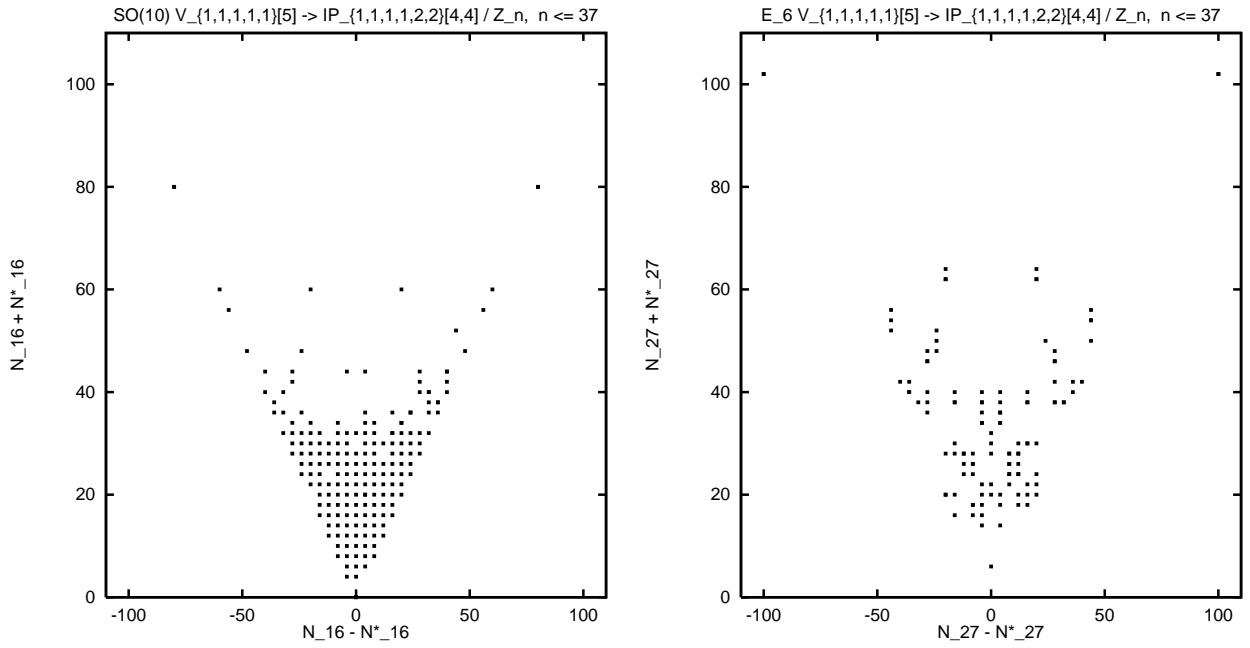
Figure 2: *Orbifold models of the (0, 2) quintic split into $SO(10)$ and E_6 gauge groups.*

descendant. As an example for this, let us take the $(N_{16}, N_{\overline{16}}) = (75, 1)$ model

$$V(1, 1, 1, 1, 4; 8) \longrightarrow \mathbb{P}_{1,1,2,2,3,3}[6, 6] \tag{3.9}$$

with the following choice for the superpotential:

$$W = \lambda_1 \phi_1^7 + \lambda_2 \phi_2^7 + \lambda_3 \phi_3^2 \phi_5 + \lambda_4 \phi_4^2 \phi_6 + \lambda_5 (\phi_3^2 + \phi_4^2) + \lambda_6 (\phi_3^3 + \phi_5^2) + \lambda_7 (\phi_4^3 + \phi_6^2) . \tag{3.10}$$

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We did a similar search for this model with symmetry groups $\mathbb{Z}_n$, $2 \leq n \leq 48$. There are some notable differences between this model and the quintic: The superpotential is much more complicated and has much less inner symmetry. Therefore, there are much less solutions of $\mathbb{Z}_n$ charges such that W remains invariant and all other consistency requirements are met. For example, there are no non-trivial solutions for $n = 9, 13, 17, 18, 19, 23, 25, 29, 31, 34, 35, 36, 43, 45$, and the total number of possible solutions in our range is only $5966 + 6208 = 12174$ orbifolds, where the 6208 models all have $\mathbb{Z}_{48}$ symmetry. In total we only get 207 different spectra, which in the plot are not differentiated according to their gauge group.

The plot (Figure 3) shows much less mirror symmetry than the plot for the $(0, 2)$ quintic, but still a certain amount of it. Keeping in mind that solutions are harder to find for this example, and that the naive overall symmetry of W is $\mathbb{Z}_{24}$, we might expect a complete set of solutions only within a huge range $\mathbb{Z}_n$, $n \leq 24^k$, with an unknown power $k \geq 2$. In the case of the quintic our computation abilities were good enough to get all orbifolds which could also have been obtained by modding out twice with basic $\mathbb{Z}_5$ symmetries. In this example, however, we are far from such a degree of completeness. Hence, we might take the appearance of a glance of symmetry in the plot as a hint that mirror symmetry might be true for general $(0, 2)$ models. This would mean that mirror symmetry is a structure not just inherited from $(2, 2)$ models, but much deeper and general.
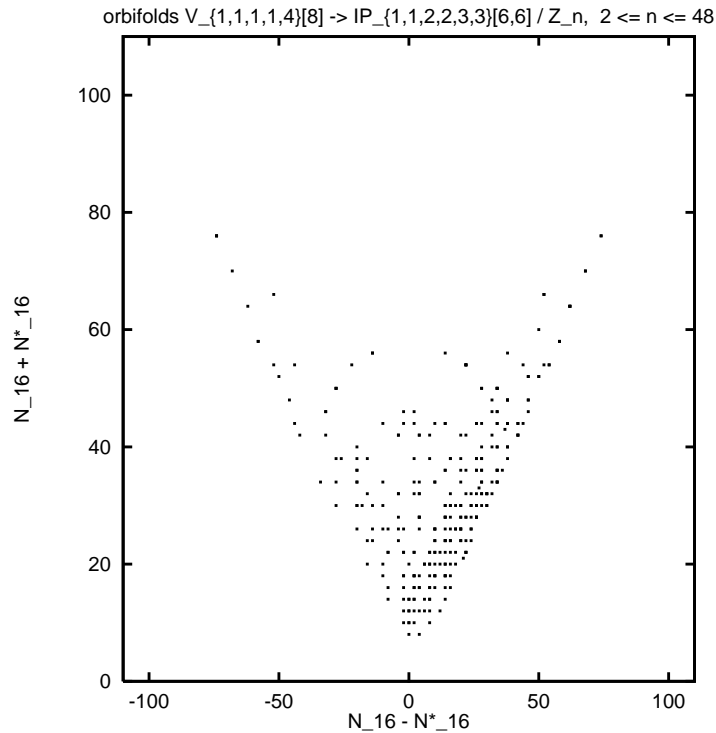
Figure 3: *Orbifolds for an example of a $(0, 2)$ model which is not a $(2, 2)$ descendant.*

4. Yukawa couplings

Using left moving $N = 2$ supersymmetry, it can be shown that the Yukawa coupling $\langle 27, 27, 27 \rangle$ is independent of the Kähler modulus [20][21] and in particular does not receive

world sheet instanton corrections. By choosing a $(2,2)$ model as the internal sector of a type II compactification one recognizes this independence to be equivalent to the fact that for $N=2$ four dimensional theories the hyper and vector multiplets decouple. However, in the general $(0,2)$ case, there is no left moving supersymmetry and one can not in quite generality expect a similar property to hold. Since in the $(2,2)$ case the chiral multiplets in the **27** representation of E_6 are related to the complex moduli by left supersymmetry, the complex moduli space can also be calculated at σ-model tree level. Thus, one gets a complete picture of the complex and Kähler moduli space by looking at the complex moduli space of the original model and its mirror. Thus, $(2,2)$ mirror symmetry is not only an abstract duality but also has far reaching computational consequences.

The question we are facing in this section is, whether similar applications of mirror symmetry hold in the $(0,2)$ case. Due to lack of left supersymmetry the chain of arguments above fails at every single step. So we have to be more modest. We consider models with gauge group $SO(10)$ having a well behaving Landau Ginzburg phase for $r \ll 0$. Then, we know that at the Landau Ginzburg point we can define a chiral ring structure for the Yukawa couplings $\langle 10, 16, 16 \rangle_{ut}$ in the untwisted sector [14]. This means that the chiral multiplets both in the spinor and in the vector representation of $SO(10)$ are given by polynomials in the zero modes ϕ_0^i. Clearly, the coupling depends on the (unknown) normalizations of the vertex operators and the complex and bundle moduli, but nevertheless the chiral ring implies strong selection rules for such couplings to be non-zero. Taking into account that mirror symmetry exchanges generations and anti-generations and that all anti-generations occur only in twisted sectors, one expects that at least some $\langle 10, \overline{16}, \overline{16} \rangle_{tw}$ Yukawas do also have a chiral ring structure. The selection rules then follow from the $\langle 10, 16, 16 \rangle_{ut}$ couplings of the mirror model. A very simple example is the model given in the Calabi-Yau phase by the bundle

$$0 \rightarrow V \rightarrow \bigoplus_{a=1}^{5} \mathcal{O}(1) \rightarrow \mathcal{O}(5) \rightarrow 0 \,, \tag{4.1}$$

over the threefold configuration $\mathbb{P}_{(1,1,1,1,2,2)}[4,4]$. In the Landau-Ginzburg phase, the massless sector contains $N_{16} = 80$ untwisted chiral multiplets which transform in the spinor representation of $SO(10)$. These are given by polynomials in the ϕ_i of degree five modulo the seven constraints of weight four. There are no states transforming in the conjugate spinor representation, and there are $N_{10} = 72_{ut}$ untwisted and $N_{10} = 2_{tw}$ twisted chiral multiplets which transform in the vector representation. The untwisted ones are given by polynomials of degree ten modulo the constraints. The mirror of this model can be written as the Landau-Ginzburg phase of

$$V(51, 64, 60, 80, 65; 360) \longrightarrow \mathbb{P}_{51,60,80,65,128,128}[256, 256] \tag{4.2}$$

with $N_{\overline{16}} = 0$ and $N_{10} = 2_{ut}$ and $N_{10} = 72_{tw}$ from the untwisted and twisted sector, respectively. Mirror symmetry then implies that the Yukawas $\langle 10, \overline{16}, \overline{16} \rangle_{tw}$ of the mirror model (4.2) satisfy selection rules given by the polynomial ring of the original model

$$\mathcal{R} = \left. \frac{\mathcal{C}(\phi_i)}{W_j = F_a = 0} \right. . \tag{4.3}$$

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Since this example is not of a fairly general type and involves plenty of massless fields we will consider a different example in more detail, namely

$$V(1, 1, 3, 5, 5; 15) \longrightarrow \mathbb{P}_{1,1,6,6,5,5}[12, 12] \,. \tag{4.4}$$

The massless spectrum consists of $(N_{16}, N_{\overline{16}}) = (80_{ut}, 8_{tw})$ generations and anti-generations and $N_{10} = 75_{ut} + 7_{tw}$ vectors. We choose the superpotential to be

$$W = \lambda_1 \phi_1^{14} + \lambda_2 \phi_2^{14} + \lambda_3 \phi_3 \phi_4 + \lambda_4 \phi_5^2 + \lambda_6 \phi_6^2 + \lambda_6 \phi_3^2 + \lambda_7 \phi_4^2 \,. \tag{4.5}$$

The mirror is given by taking the $\mathbb{Z}_{15}$ quotient acting as

$$(\vec{q}_i; \vec{q}_a) = \left(\frac{1}{15}, -\frac{1}{15}, 0, 0, 0, 0; -\frac{1}{15}, \frac{1}{15}, 0, 0, 0, 0, 0 \right) \,. \tag{4.6}$$

with spectrum $(N_{16}, N_{\overline{16}}) = (6_{ut} + 2_{tw}, 80_{tw})$ and $N_{10} = 5_{ut} + 77_{tw}$. The untwisted sector of the mirror contains information about the $\langle 10, \overline{16}, \overline{16} \rangle_{tw}$ couplings of the original model. To be more precise, the $N_{16} = 6_{ut}$ untwisted states are represented by polynomials $A = \phi_1^5 \phi_2^5 \phi_{5,6}$ and $B = \phi_1^2 \phi_2^2 \phi_{3,4} \phi_{5,6}$ of degree fifteen and the $N_{10} = 5_{ut}$ untwisted vectors are represented by polynomials $S = \phi_1^{12} \phi_2^{12} \phi_{3,4}$, $T = \phi_1^{10} \phi_2^{10} \phi_5 \phi_6$ and $U = \phi_1^7 \phi_2^7 \phi_{3,4} \phi_5 \phi_6$ of degree thirty. The polynomial ring then tells us that couplings do only have a chance to be non-zero if they are of type TAA or UAB with the indices chosen appropriately. In order to check this, one would have to calculate the $\langle 10, \overline{16}, \overline{16} \rangle_{tw}$ couplings in the original model exactly. Fortunately, for this model a conformal field theory description is known: One starts with the $(1, 1, 3, 13, 13)$ Gepner model and introduces the simple current

$$J = (0 \ 0 \ 0)^2 (0 \ 5 \ 1)(0 \ 0 \ 0)^2 (1)(0) \tag{4.7}$$

into the partition function. Following the discussion in [14] determining the massless states and calculating the Yukawa couplings really shows that there are six anti-generations and five twisted vectors obeying exactly the selection rules above. The exact calculation yields for the couplings

$$TAA = \frac{\Gamma\left(\frac{1}{15}\right) \Gamma^2\left(\frac{3}{5}\right) \Gamma\left(\frac{11}{15}\right)}{\Gamma\left(\frac{4}{15}\right) \Gamma^2\left(\frac{2}{5}\right) \Gamma\left(\frac{14}{15}\right)}, \quad UAB = \frac{\Gamma\left(\frac{1}{15}\right) \Gamma\left(\frac{8}{15}\right) \Gamma\left(\frac{3}{5}\right) \Gamma\left(\frac{4}{5}\right)}{\Gamma\left(\frac{1}{5}\right) \Gamma\left(\frac{2}{5}\right) \Gamma\left(\frac{7}{15}\right) \Gamma\left(\frac{14}{15}\right)} \,. \tag{4.8}$$

This easy example shows that one can indeed learn one bit of information from $(0, 2)$ mirror symmetry. It remains to be seen whether for perhaps a subclass like all linear σ-models stronger statements can be made. As was nicely shown in [11] there are unexpected cancellations in the space-time superpotential so that at least all parameters in a linear σ-model are indeed good moduli. Similar mechanisms are perhaps at work to cancel various corrections for the Yukawa couplings.

Summarizing, we have seen that $(0,2)$ orbifolding is a powerful method to get new $(0,2)$ vacua of the heterotic string. Furthermore, we found strong indications that descendant $(0,2)$ models of $(2,2)$ models feature $(0,2)$ mirror symmetry. Finally, we argued that $(0,2)$ mirror symmetry can be used to extract information about couplings of type $\langle 10, \overline{16}, \overline{16} \rangle_{tw}$ without knowing the exact conformal field theory.

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